Algebra

Worksheets

Verein zur Förderung der Mathematik

Contents

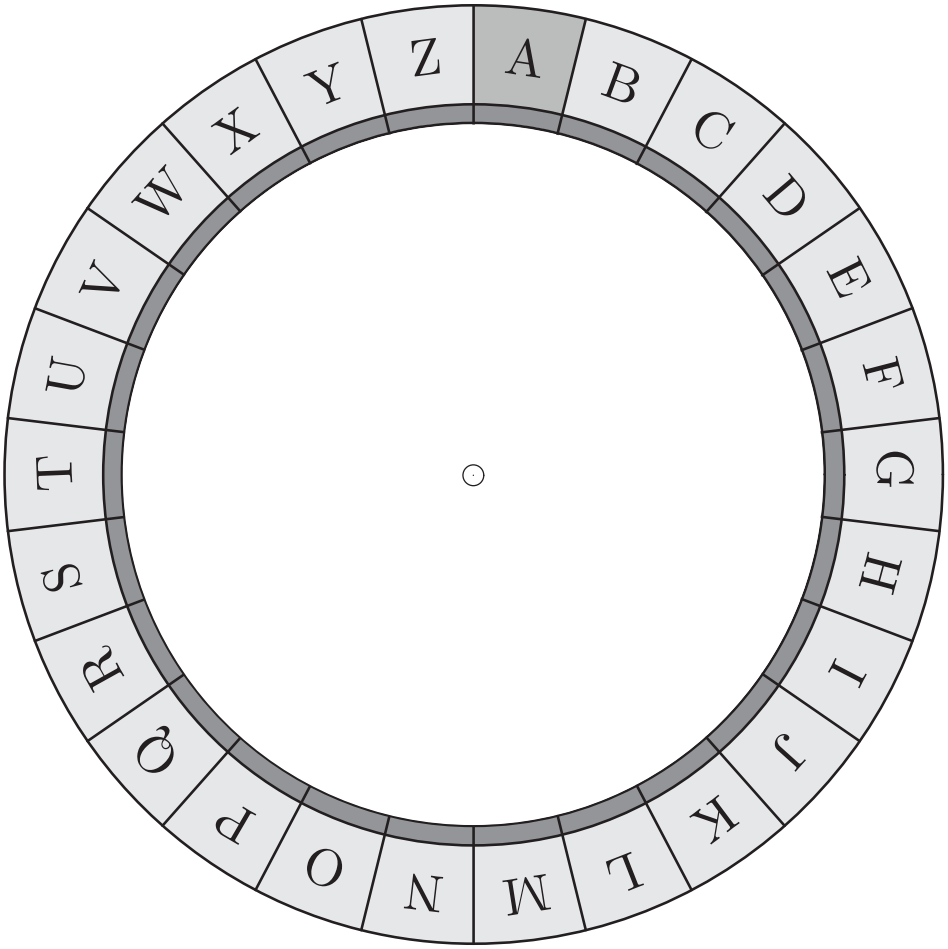
These worksheets are a supplement to the book «Algebra» of the Verein zur Förderung der Mathematik.

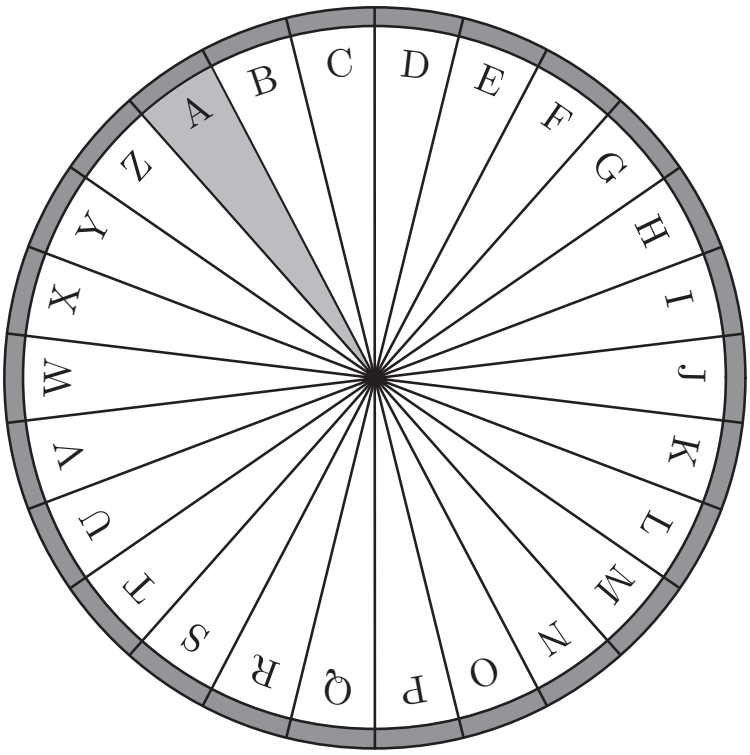
www.mathematikverein.ch

Version of July 2026

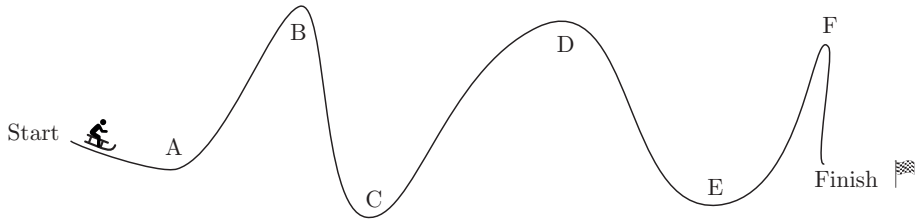
3 Functions

41. - 48. Craft template for a Caesar disc

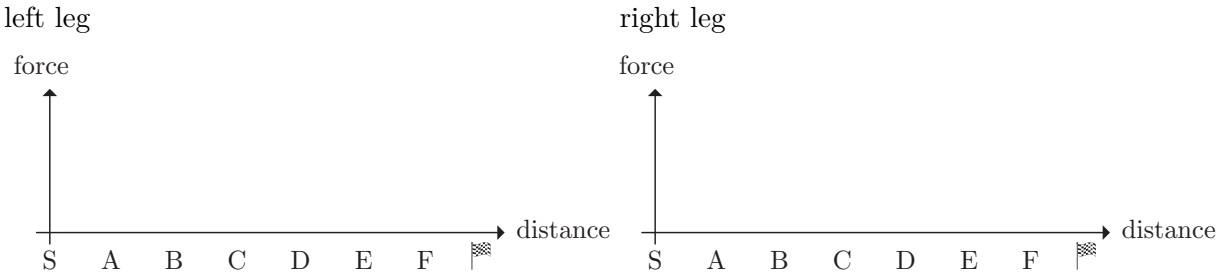




Matthis is riding down a toboggan run on his wooden sledge. The illustration shows the run from a bird's eye view. This type of sledge is steered using only your feet. The harder you press one leg into the snow, the more the sledge turns in that direction.

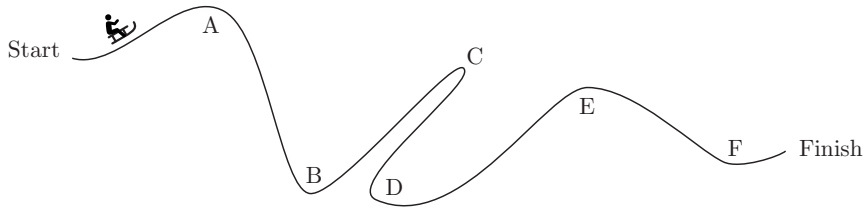


49. a) In the force-distance diagrams draw the approximate force exerted for the ...

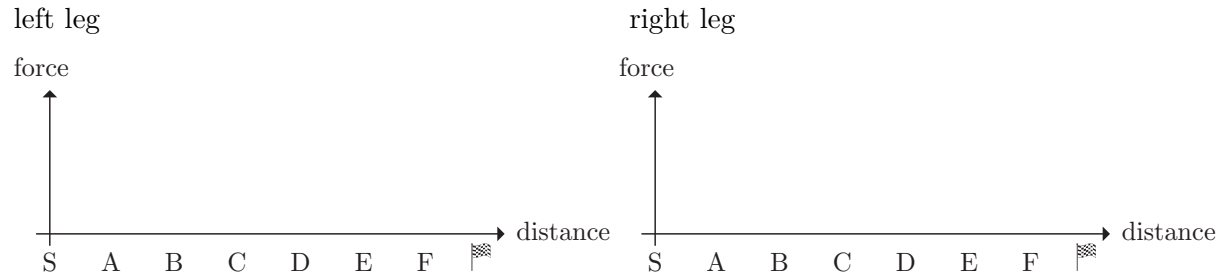


b) Draw a possible speed-distance diagram for the toboggan run above.

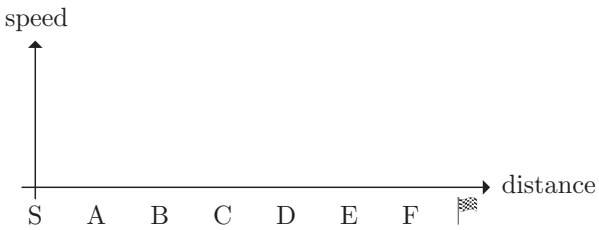
Solve the previous task with the following toboggan run:



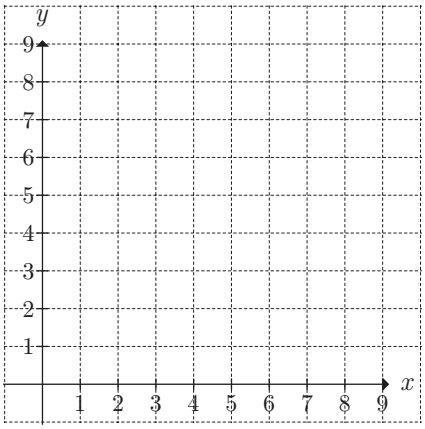
50. a) In the force-distance diagrams draw the approximate force exerted for the ...



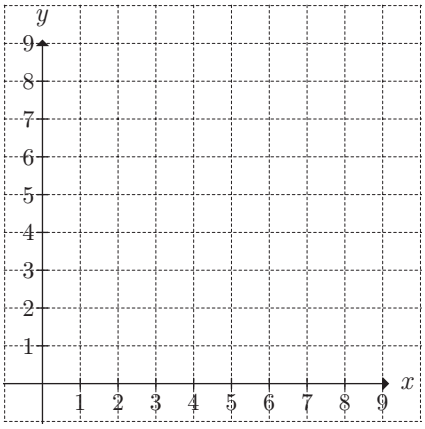
b) Draw a possible speed-distance diagram for the toboggan run above.



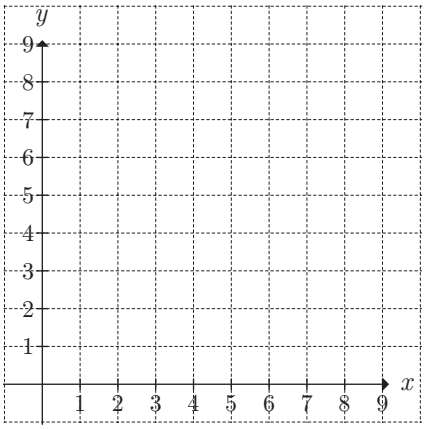
55. a) $y = 2x$



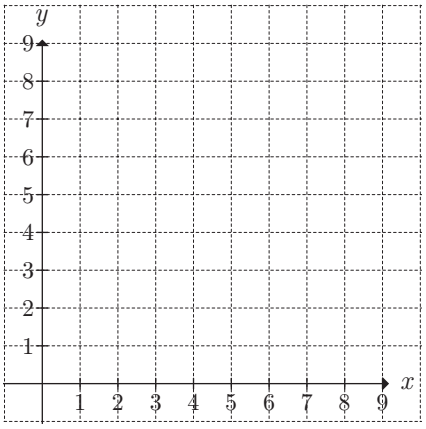
b) $y = 1.5x$



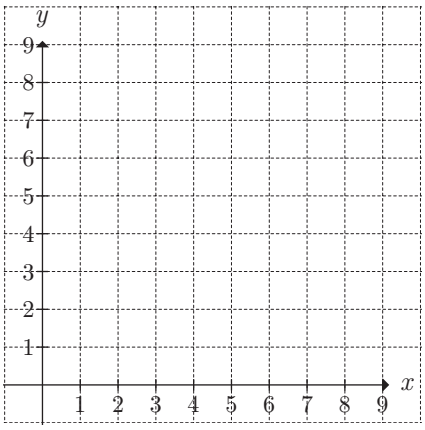
c) $y = 0.5x + 2$



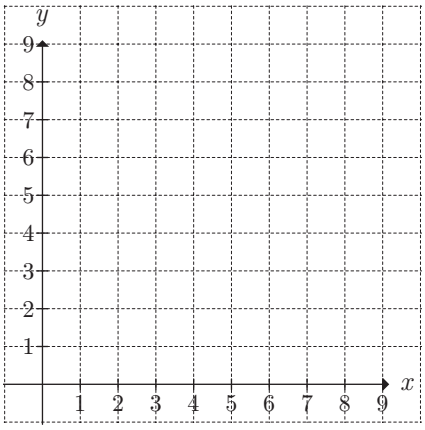
d) $y = 3x + 1$



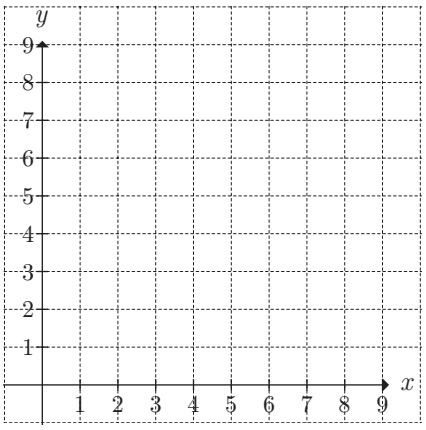
56. a) $y = 3x$



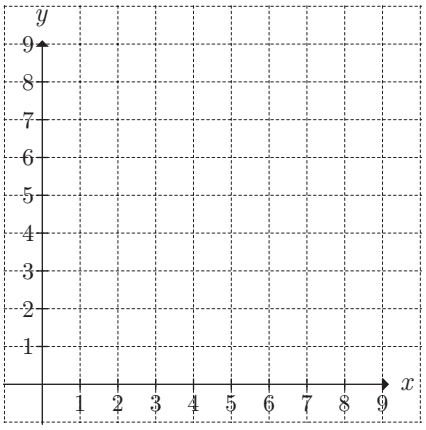
b) $y = 0.5x$



c) $y = 1.5x - 1$



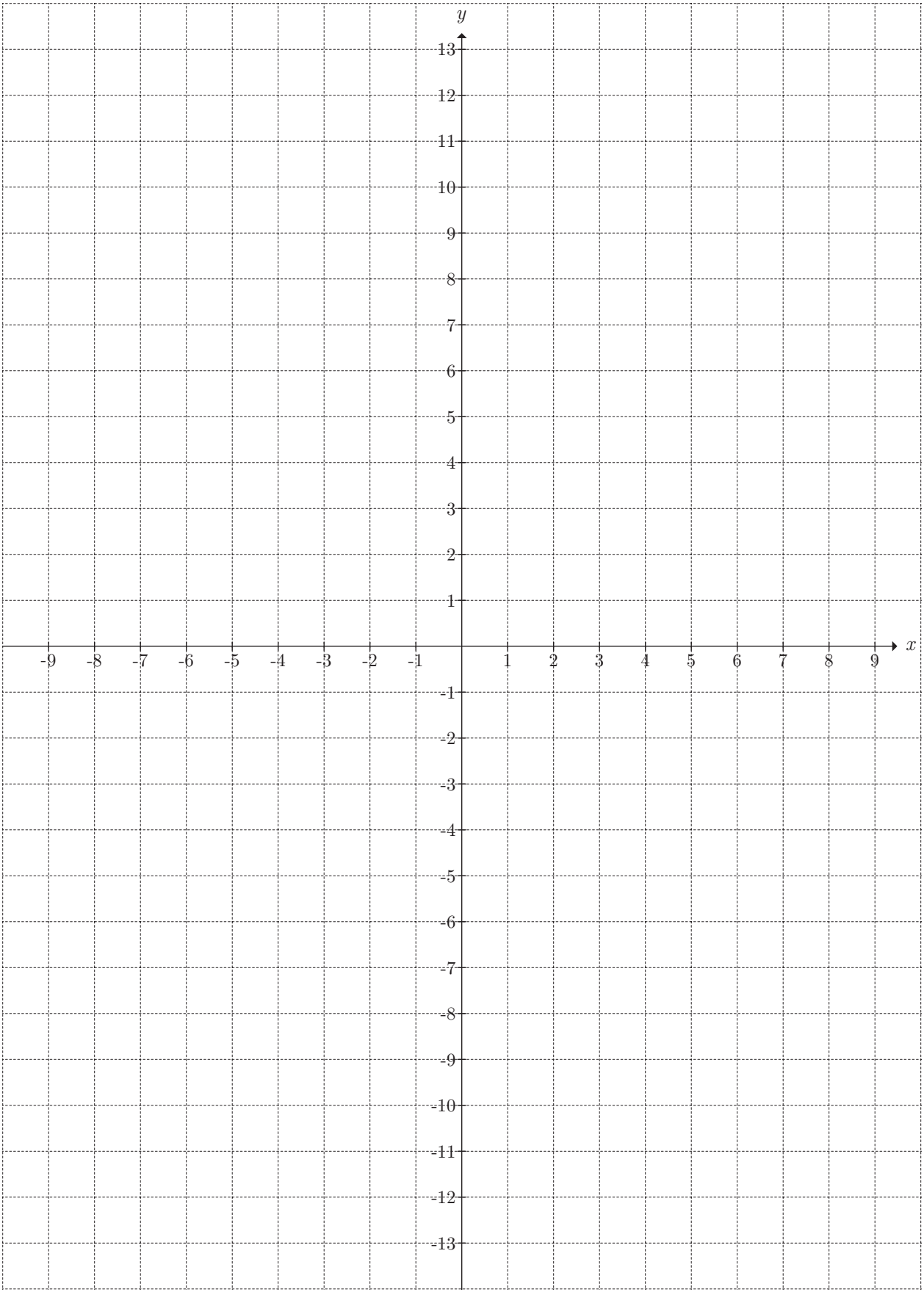
d) $y = 4x + 2$



59. Create a suitable table of values and draw the corresponding graph.

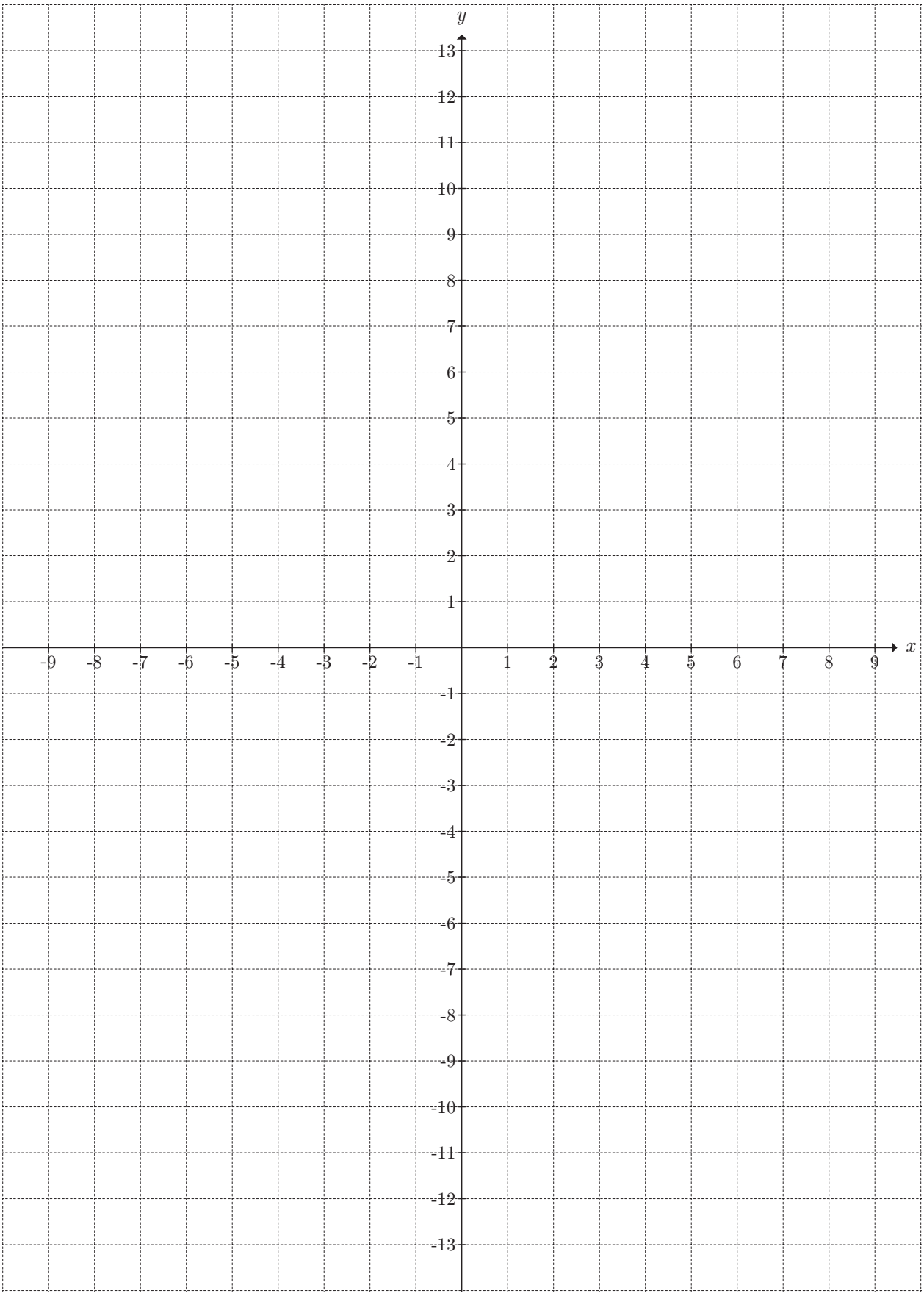
a) $y = 2x + 1$

x															
y															



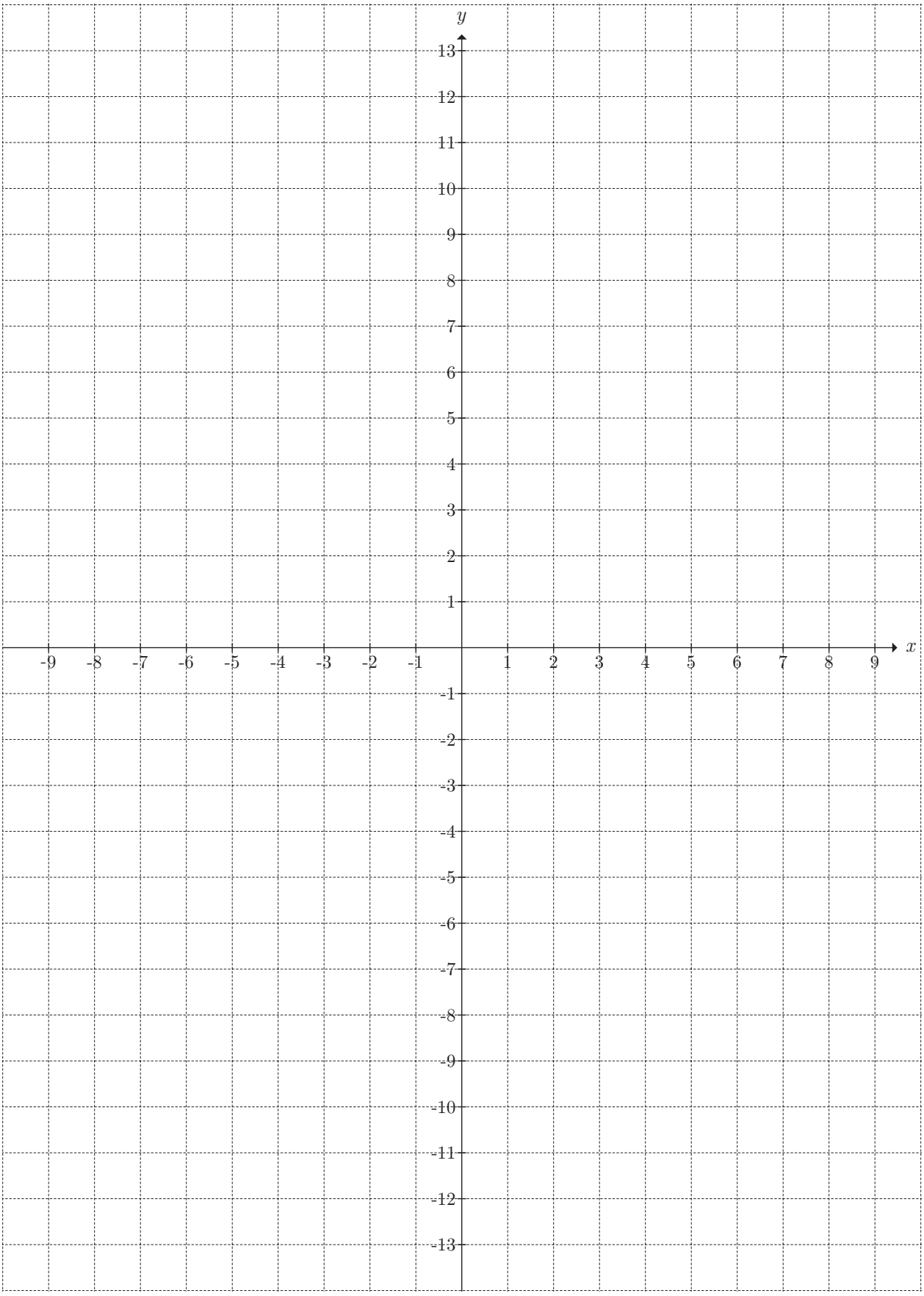
b) $y = x^2 - 4$

x															
y															



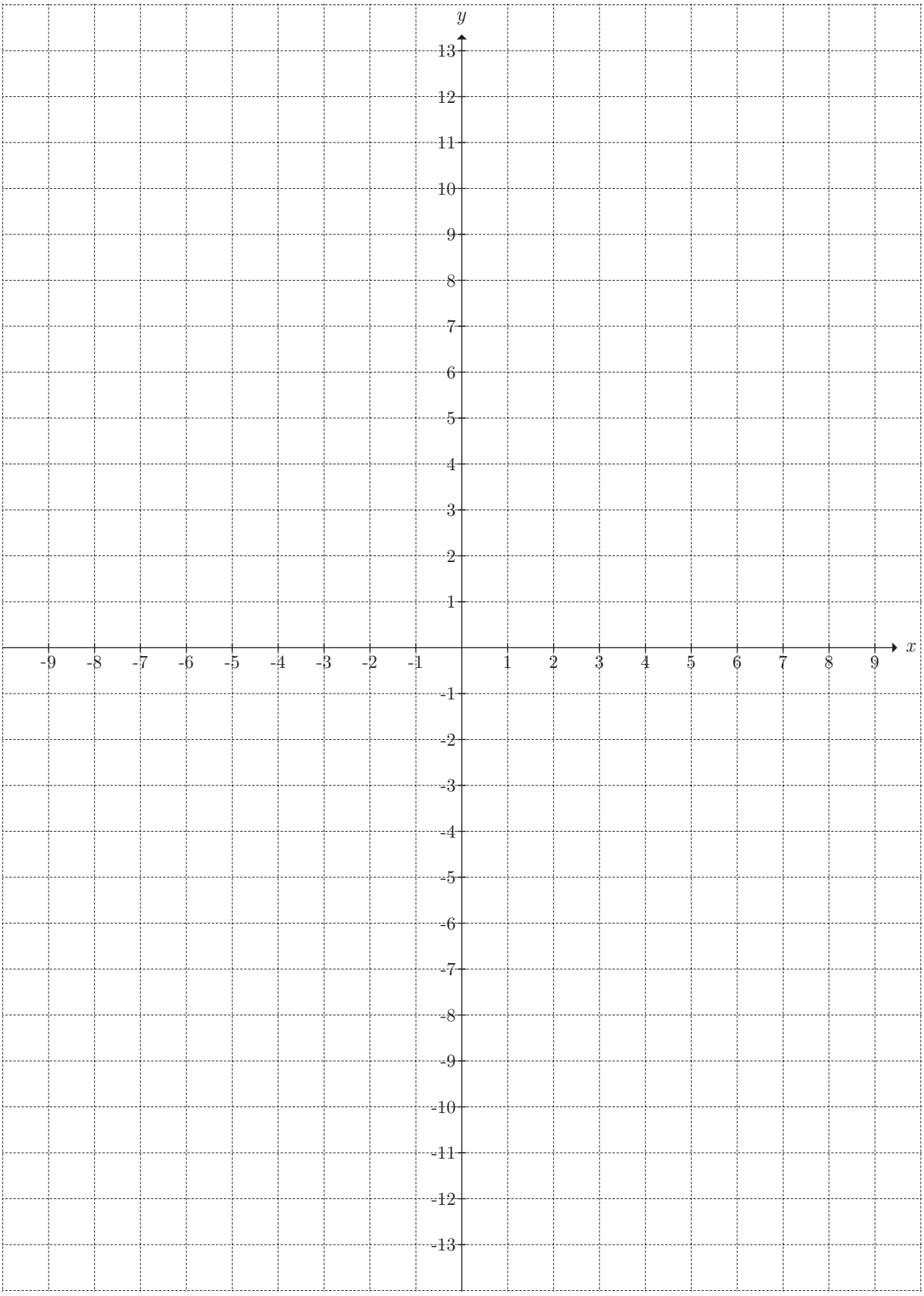
c) $y = -x^2 + 3x$

x															
y															



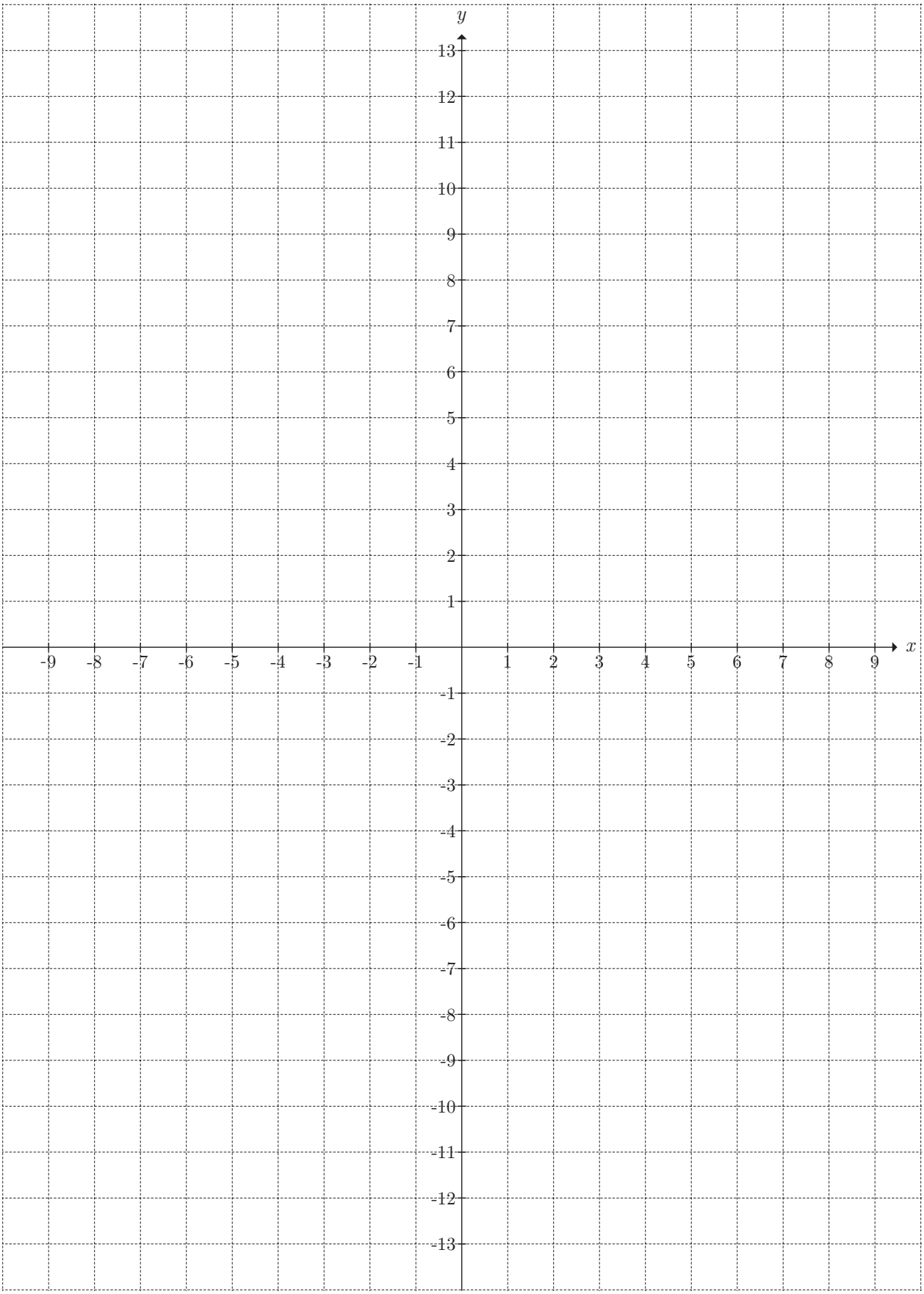
d) $y = x^3 - 3x^2$

x															
y															



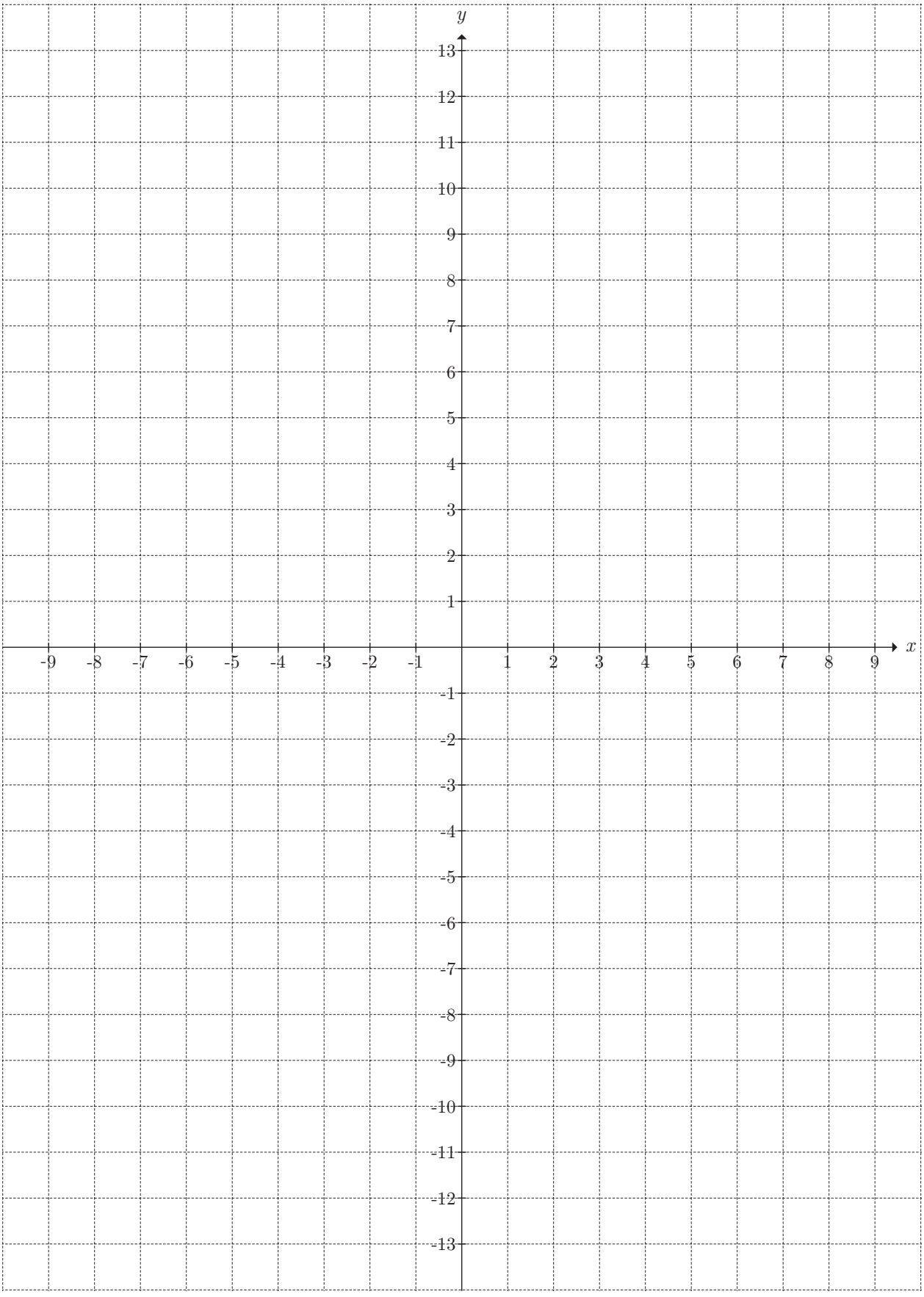
e) $y = 1 - \frac{2}{3}x$

x															
y															



f) $y = 2x^2 - \frac{1}{2}x^3$

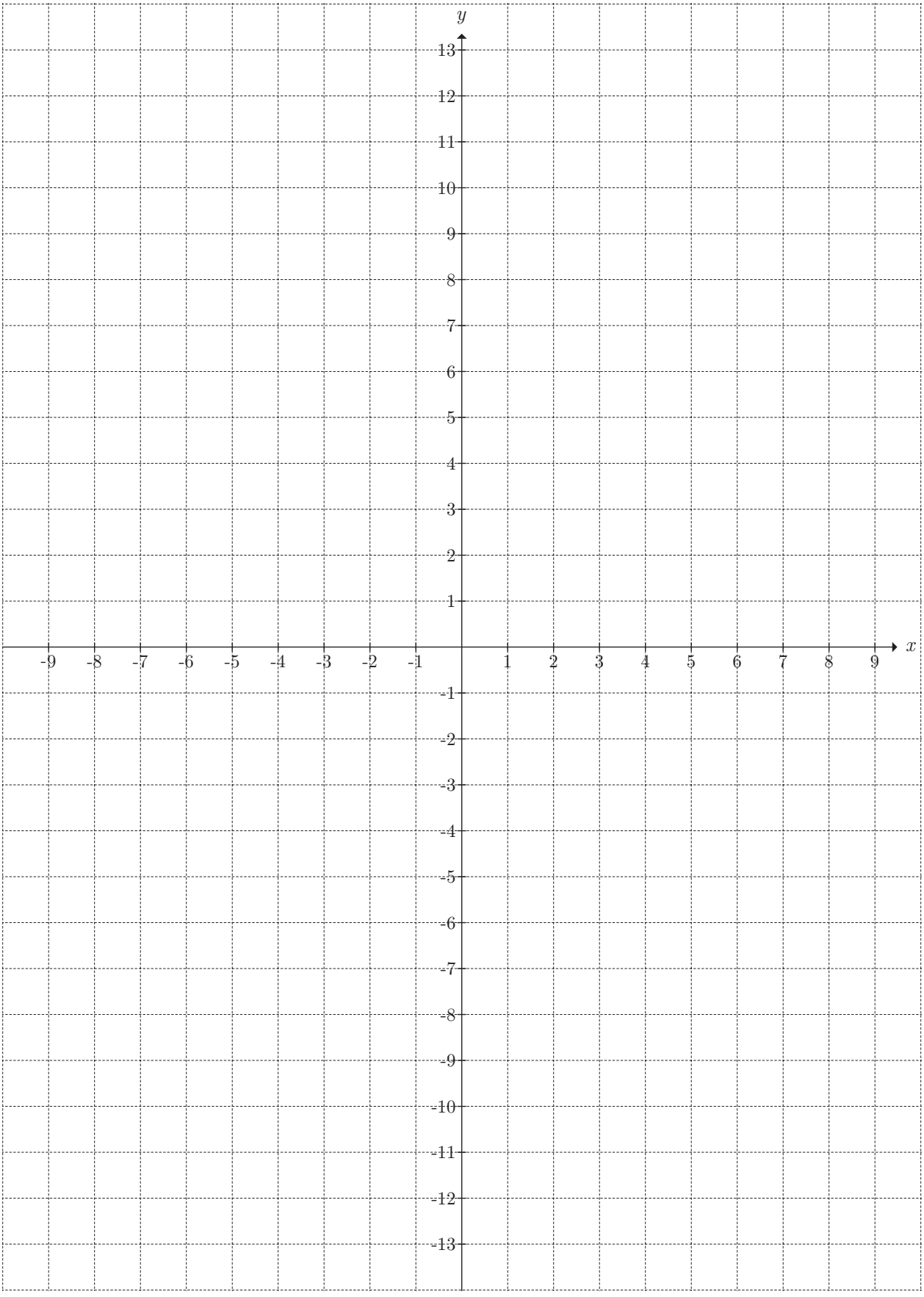
x															
y															



60. Create a suitable table of values and draw the corresponding graph.

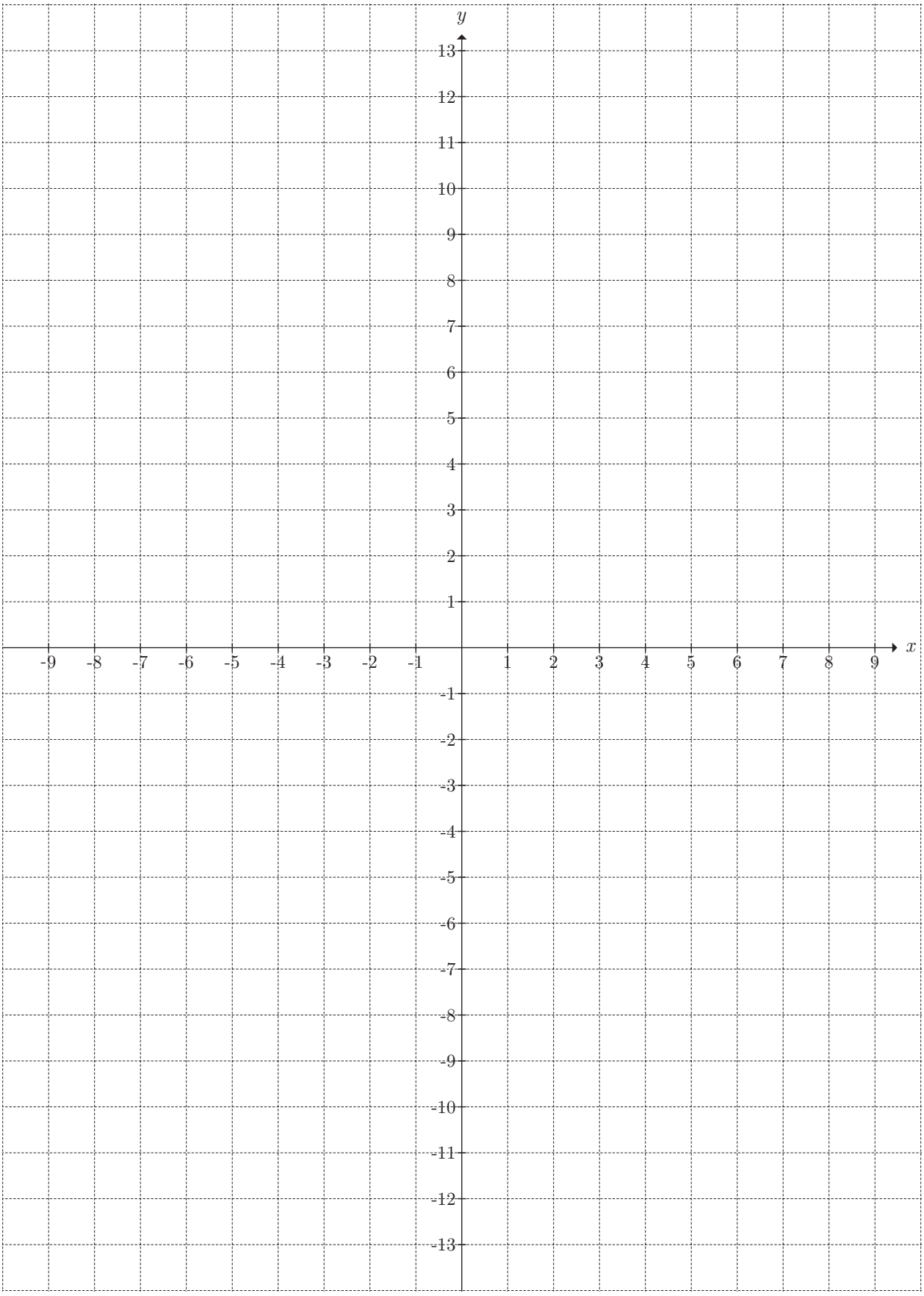
a) $y = -\frac{1}{2}x - 3$

x															
y															



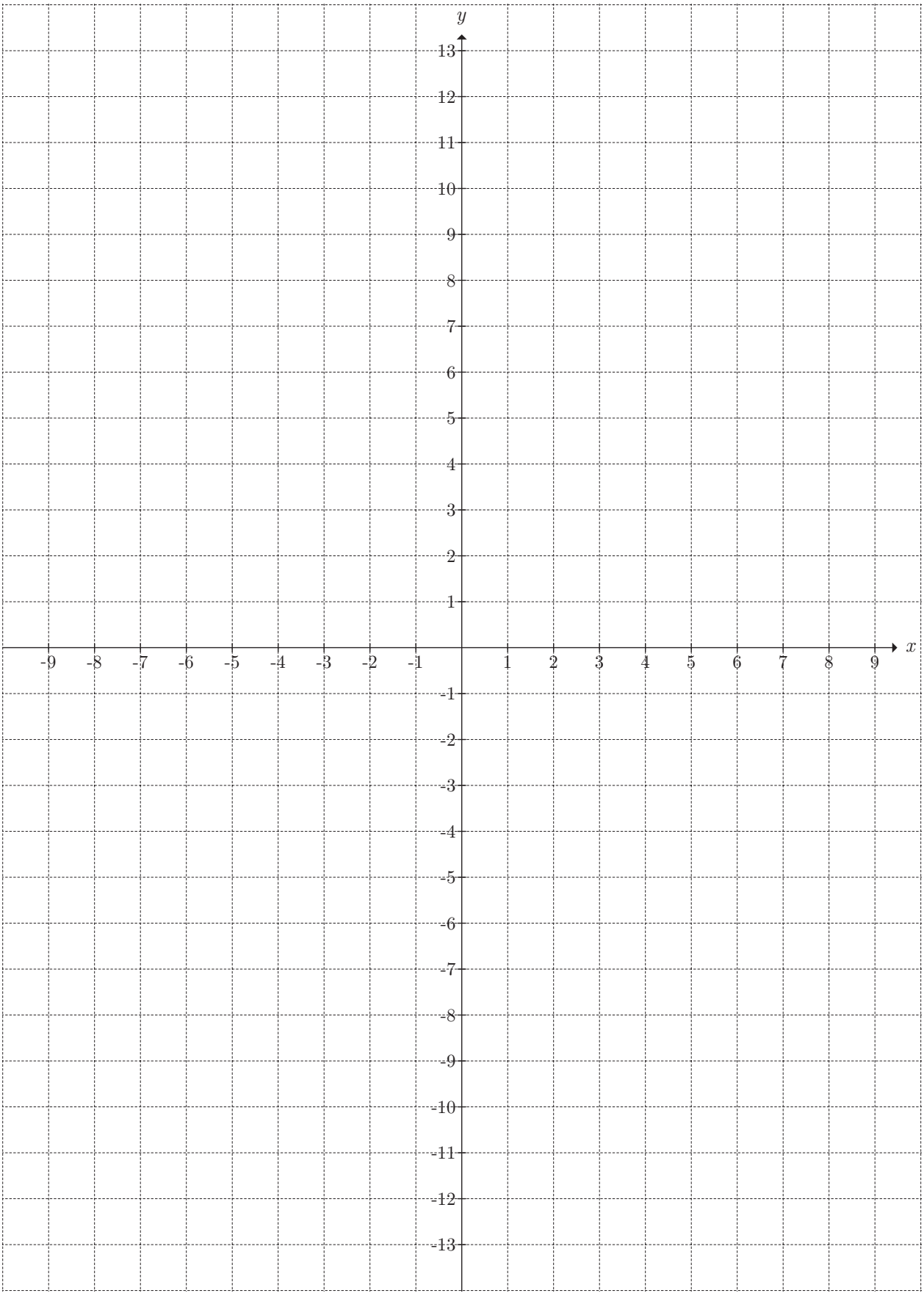
b) $y = -x^2 + 5$

x															
y															



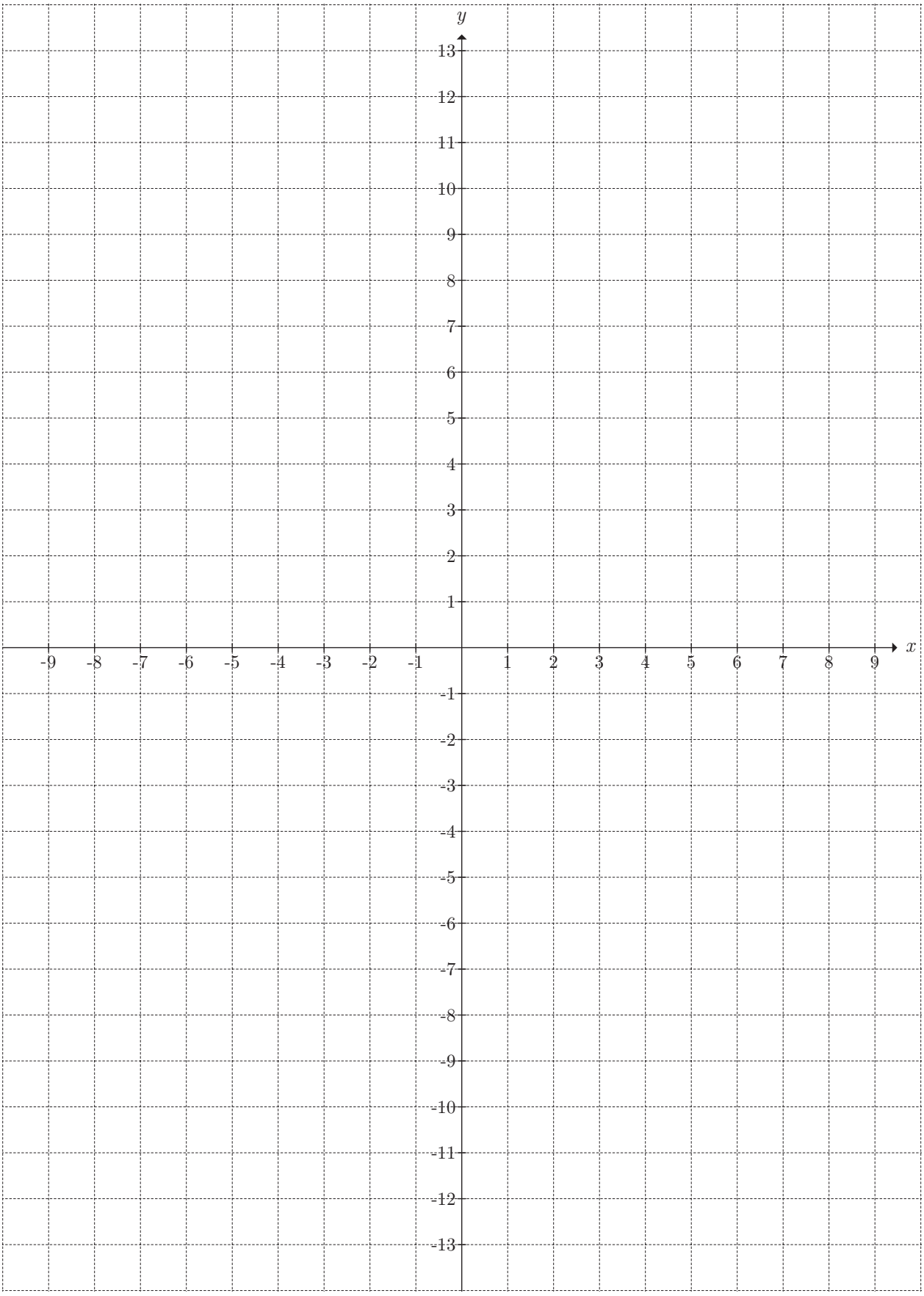
c) $y = 4x^2 - x^3 - 4$

x															
y															



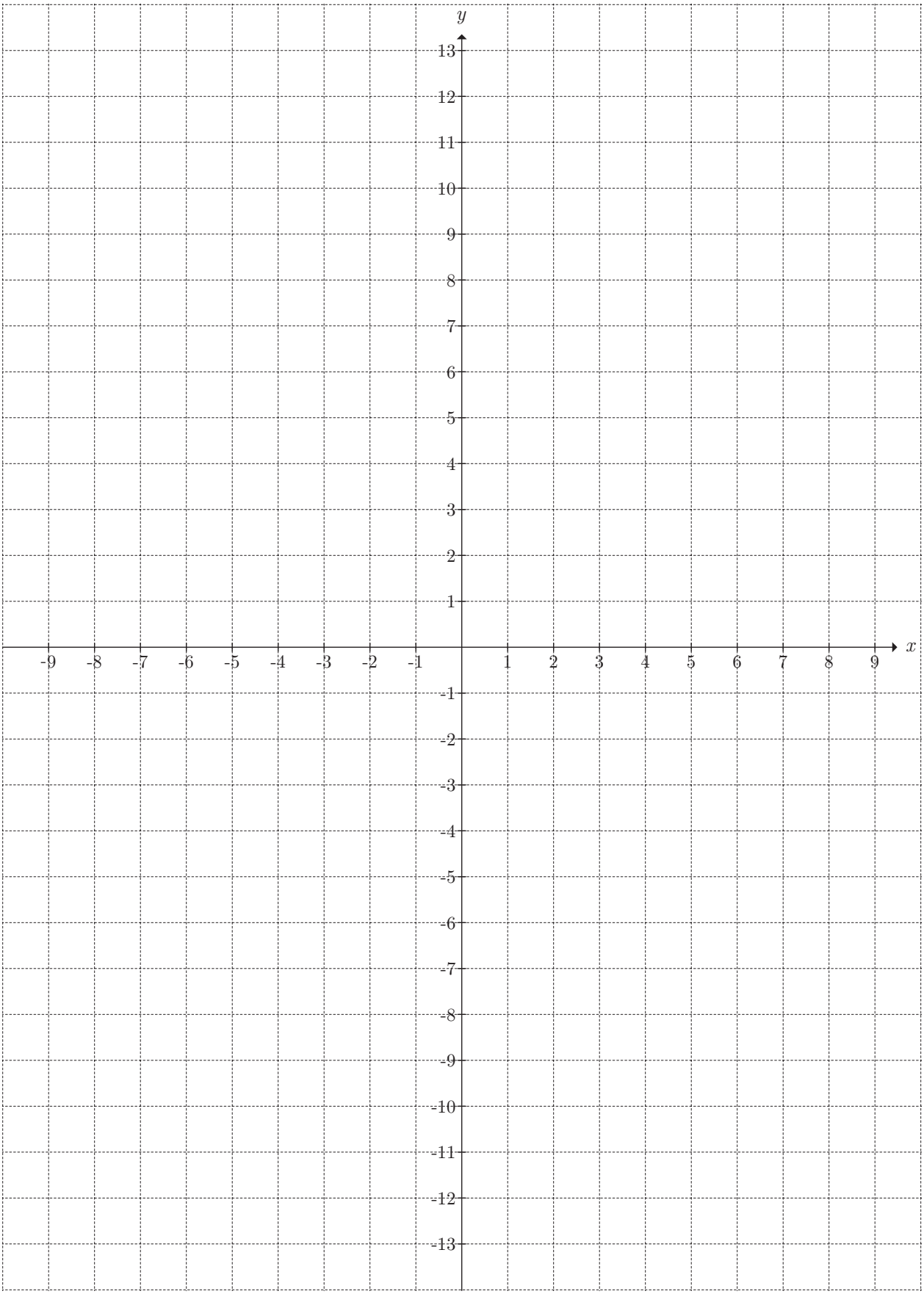
d) $y = -4 + 2x$

x															
y															



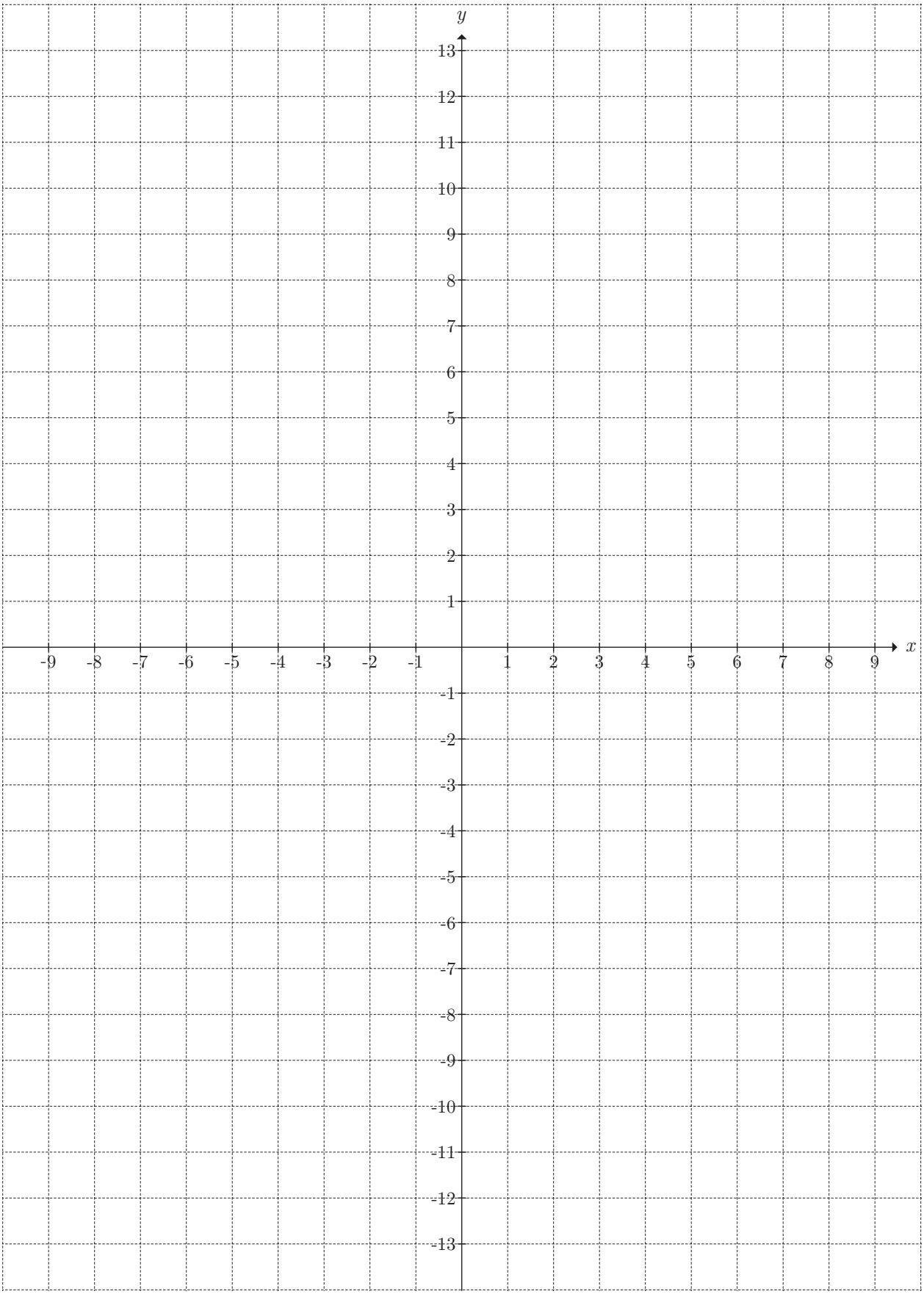
e) $y = \frac{1}{2}x^2 + x - 5$

x															
y															



f) $y = \frac{1}{2}x^3 - \frac{3}{2}x^2$

x															
y															

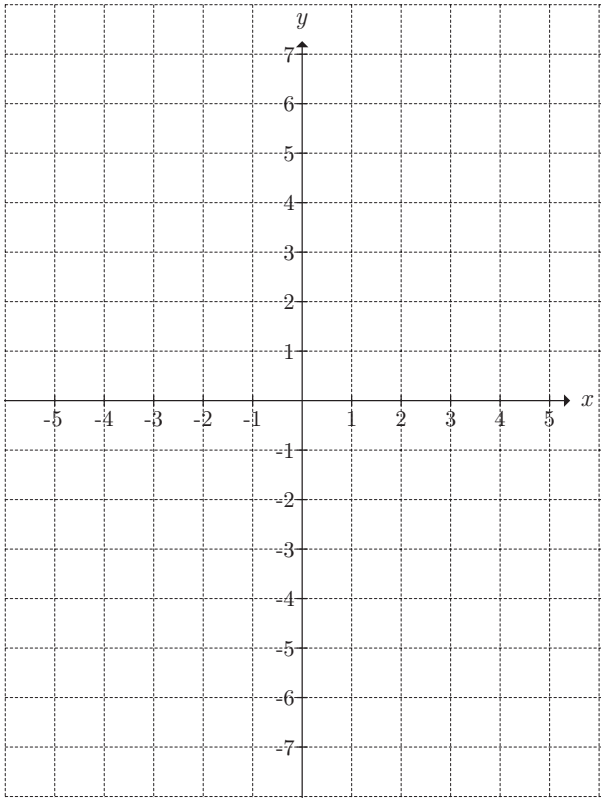


4 Linear Functions

9. Draw the graphs of the proportionalities. First, create a suitable table of values.

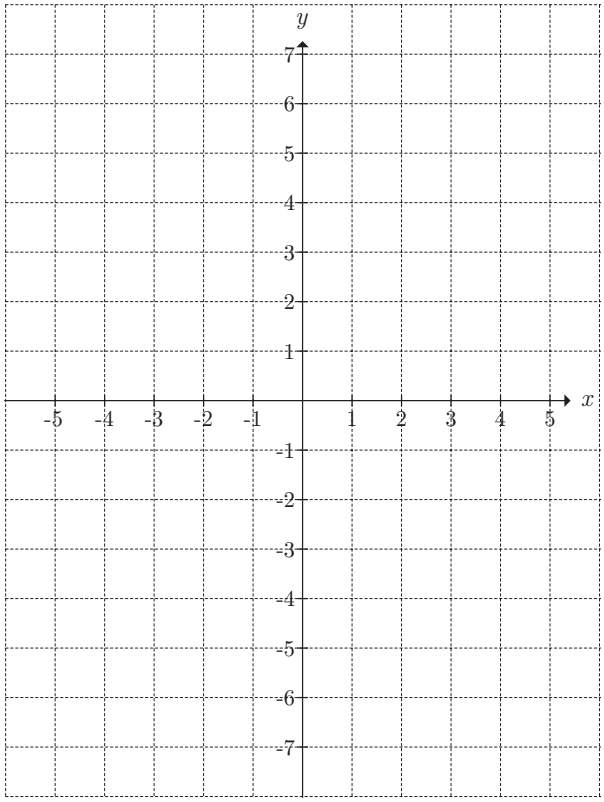
a) $y = 2x$

x							
y							



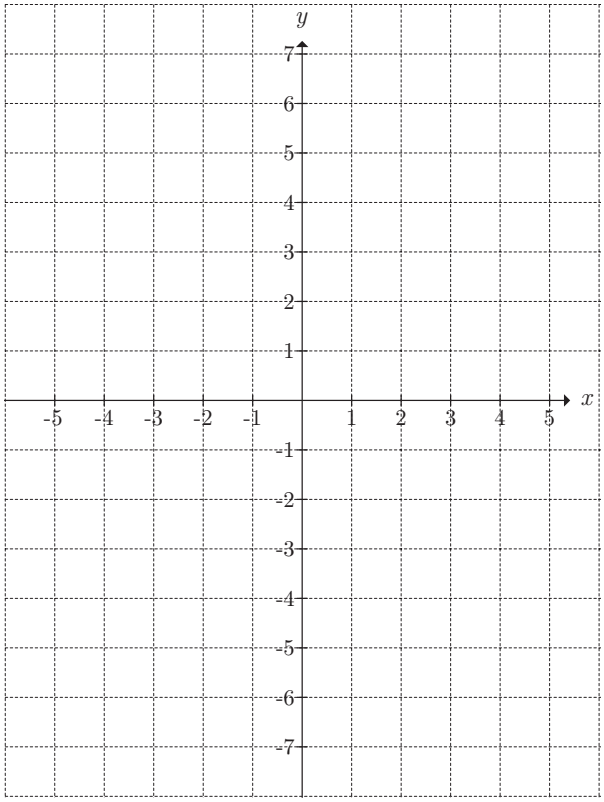
c) $y = \frac{1}{2}x$

x							
y							



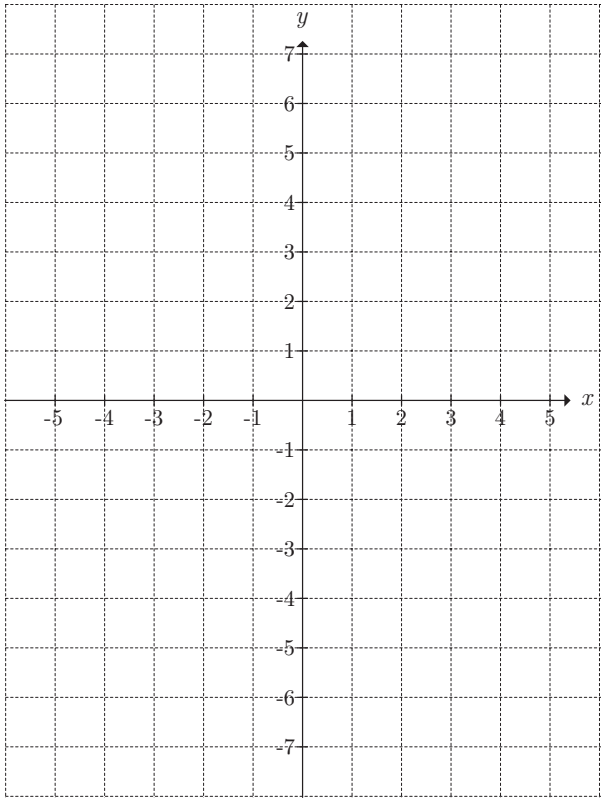
b) $y = -4x$

x							
y							



d) $y = -\frac{2}{3}x$

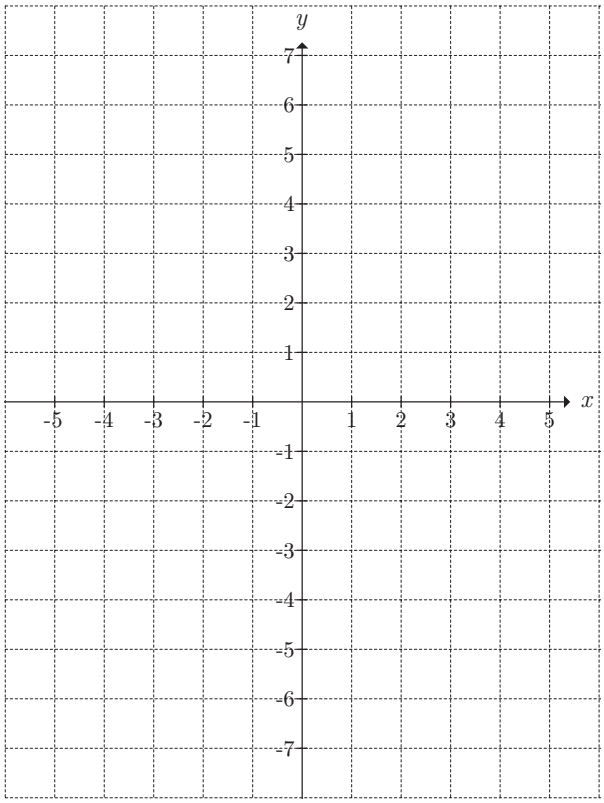
x							
y							



10. Draw the graphs of the proportionalities. First, create a suitable table of values.

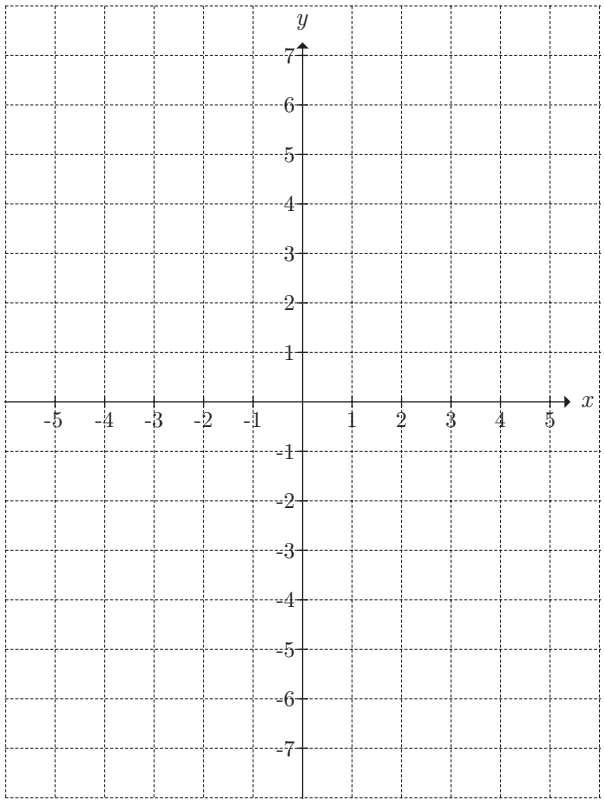
a) $y = -3x$

x							
y							



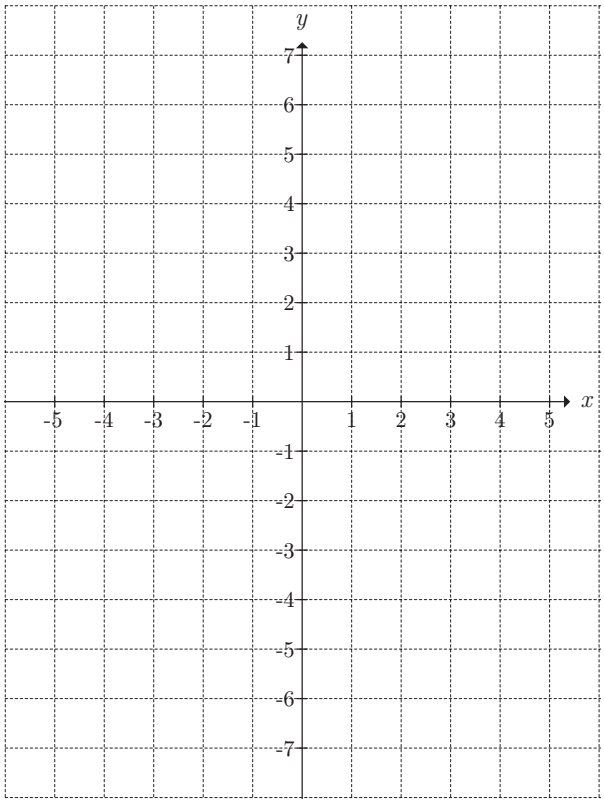
c) $y = 5x$

x							
y							



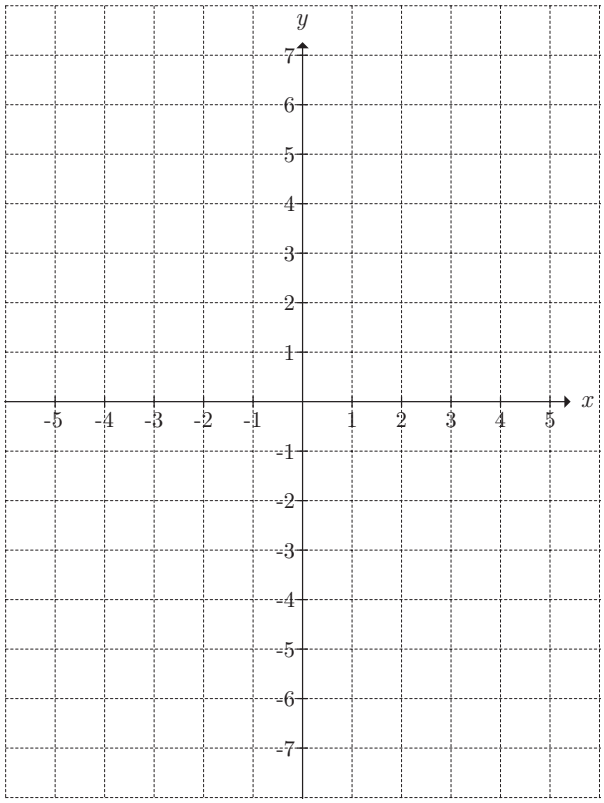
b) $y = \frac{1}{3}x$

x							
y							



d) $y = -\frac{3}{2}x$

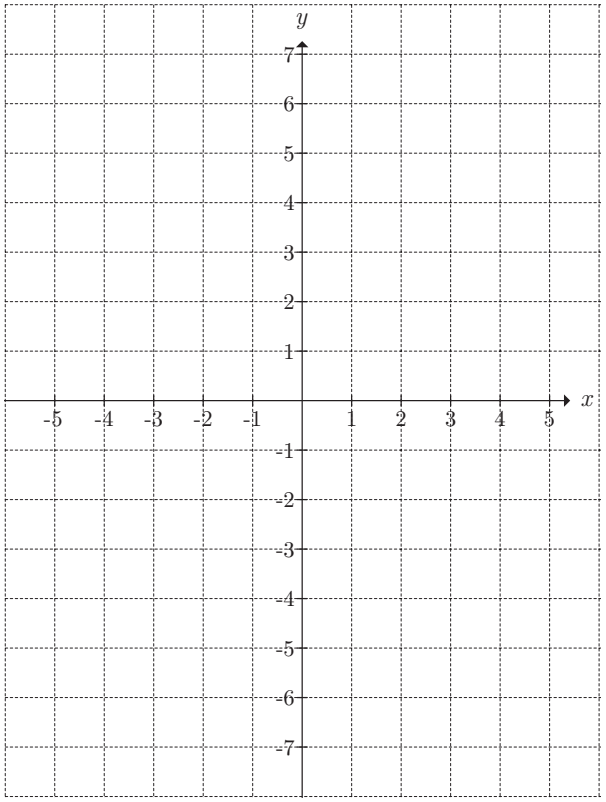
x							
y							



11. Draw the graphs of the inverse proportionalities. First, create a suitable table of values.

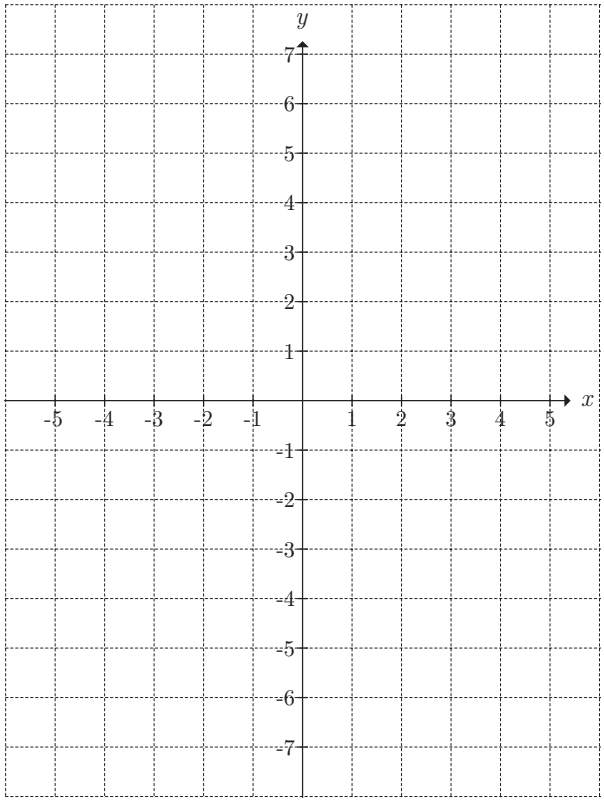
a) $y = \frac{1}{x}$

x							
y							



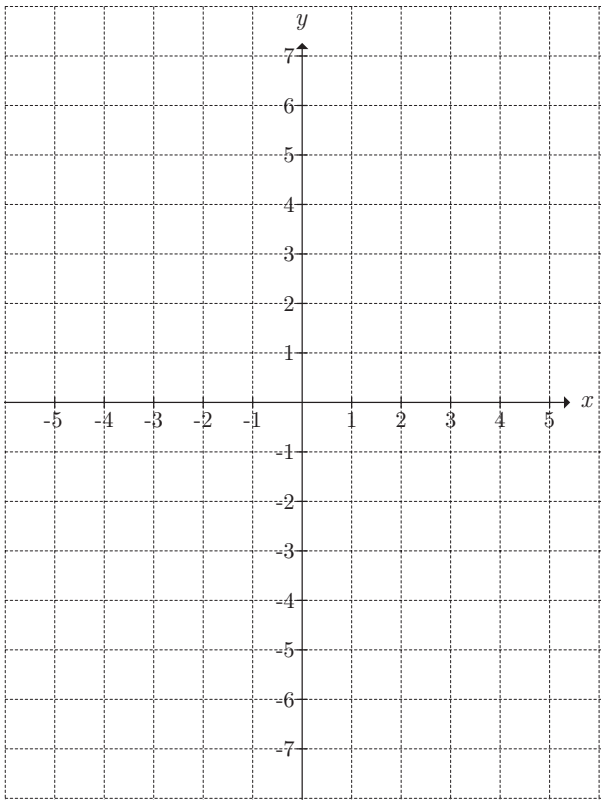
c) $y = -\frac{2}{3x}$

x							
y							



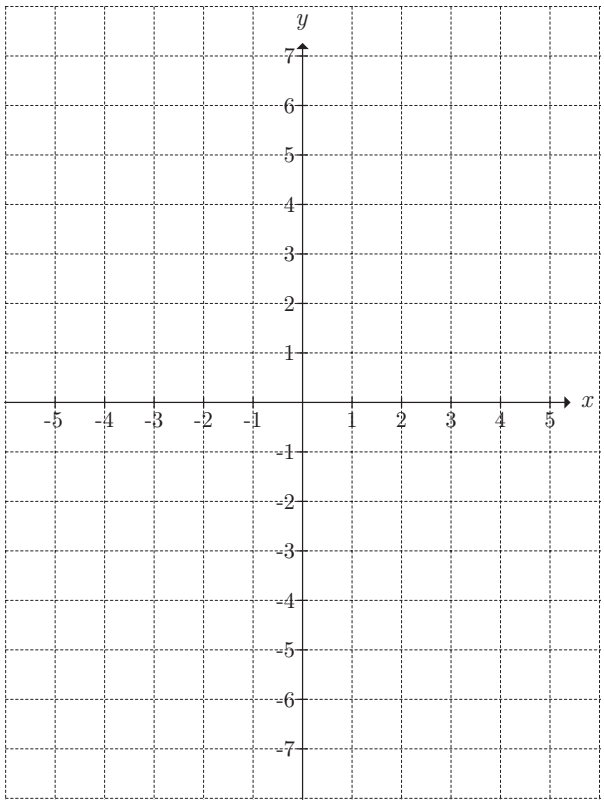
b) $y = -\frac{3}{x}$

x							
y							



d) $y = \frac{4}{5x}$

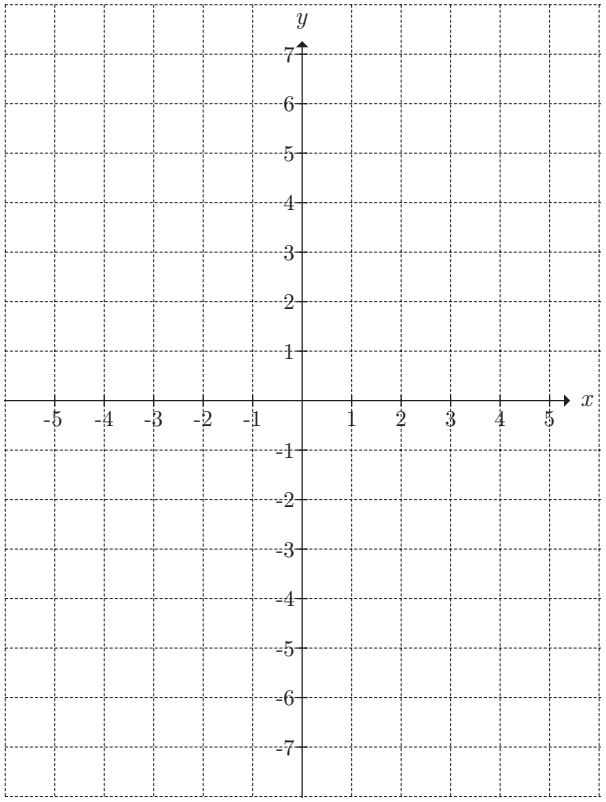
x							
y							



12. Draw the graphs of the inverse proportionalities. First, create a suitable table of values.

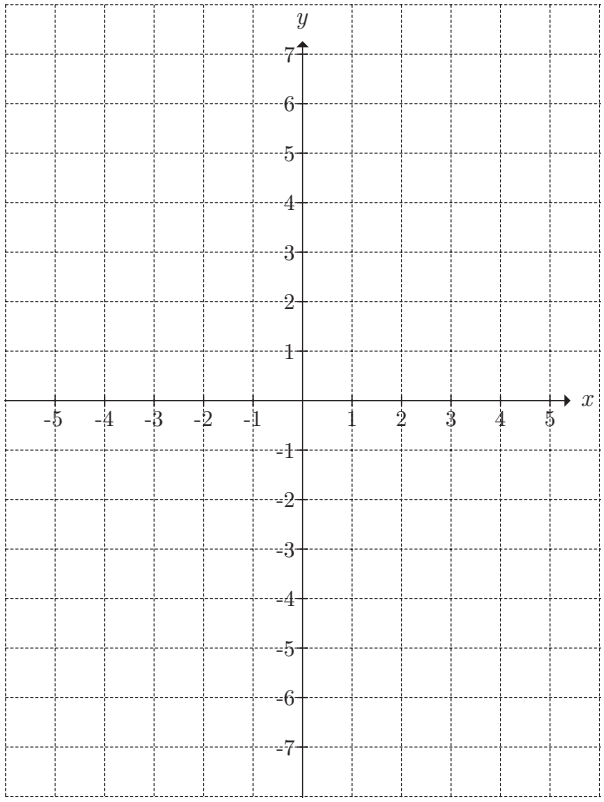
a) $y = \frac{2}{x}$

x							
y							



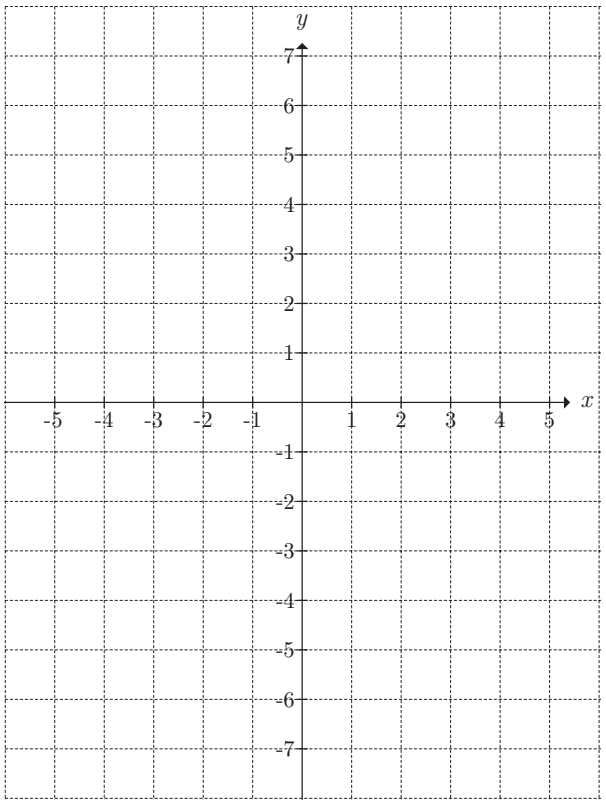
c) $y = -\frac{4}{x}$

x							
y							



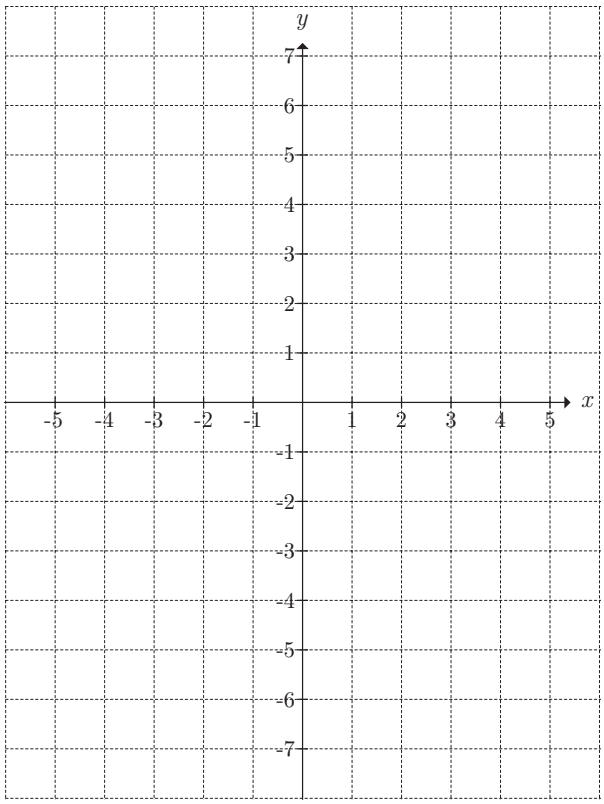
b) $y = -\frac{3}{2x}$

x							
y							



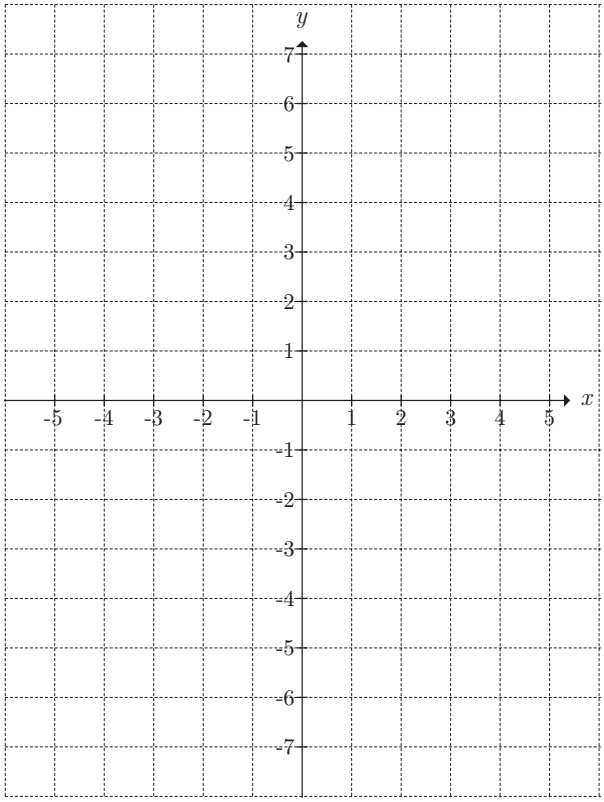
d) $y = \frac{3}{5x}$

x							
y							

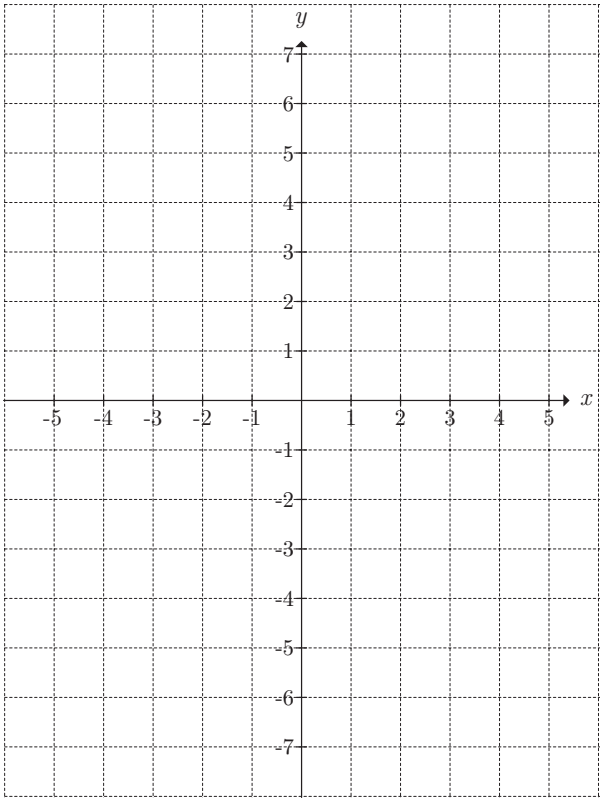


47. Draw the graphs of the linear functions.

a) $f: y = 2x + 3$, $g: y = -x + 1$,
$h: y = \frac{1}{3}x + 4$, $i: y = -\frac{2}{3}x - 3$

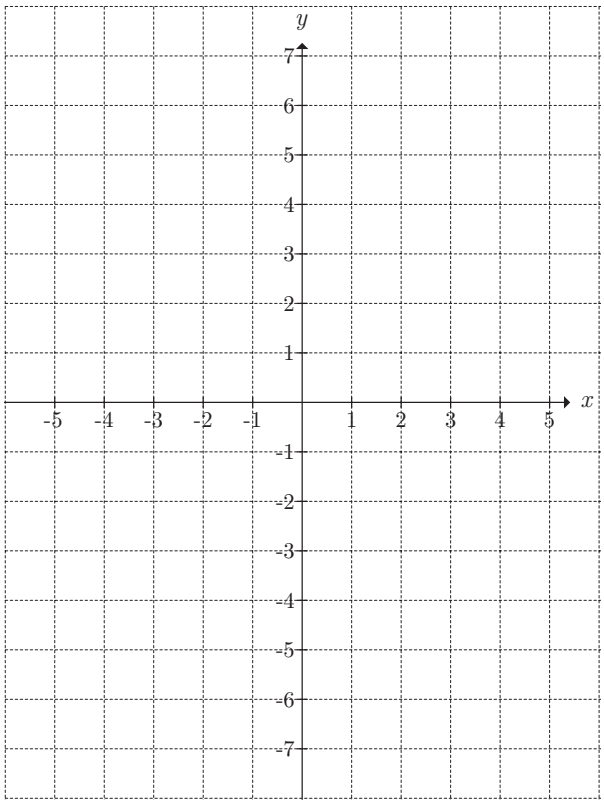


b) $f: y = 3x - 5$, $g: y = -\frac{1}{3}x + 4$,
$h: y = 2$, $i: y = -\frac{3}{4}x - 2$

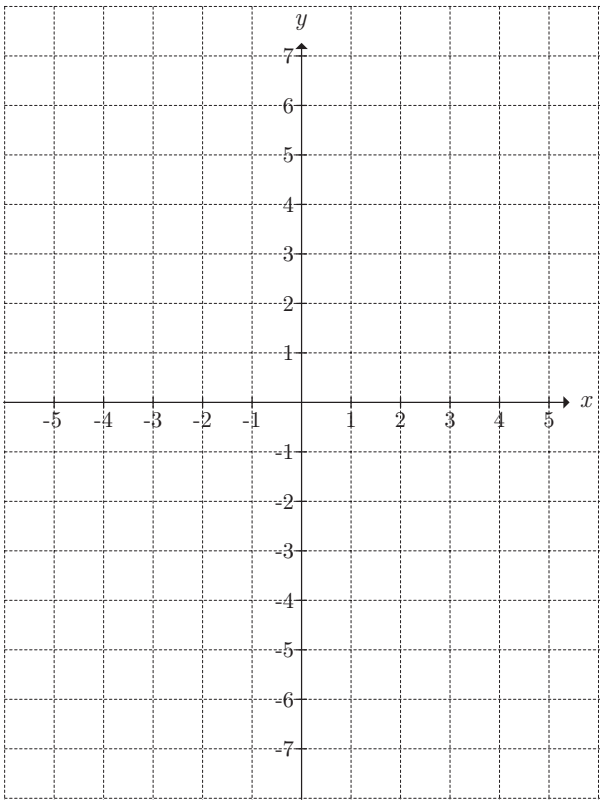


48. Draw the graphs of the linear functions.

a) $f: y = -x + 5$, $g: y = -5x - 4$,
$h: y = \frac{1}{4}x + 2$, $i: y = -5$

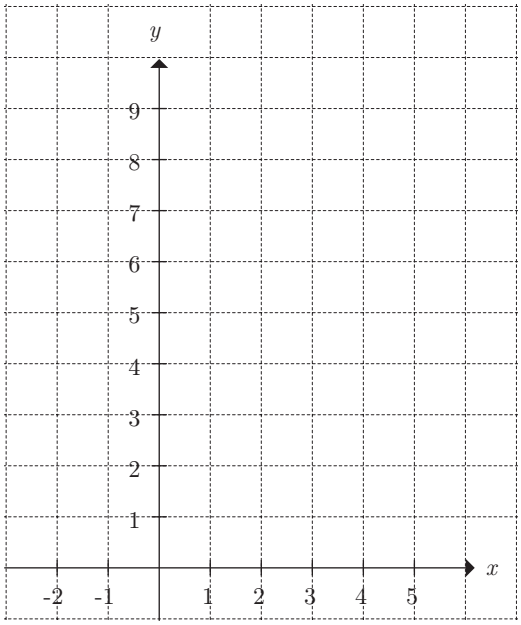


b) $f: y = x + 3$, $g: y = -\frac{1}{2}x + 2$,
$h: y = -\frac{2}{3}x - 4$, $i: y = \frac{7}{4}x - 5$

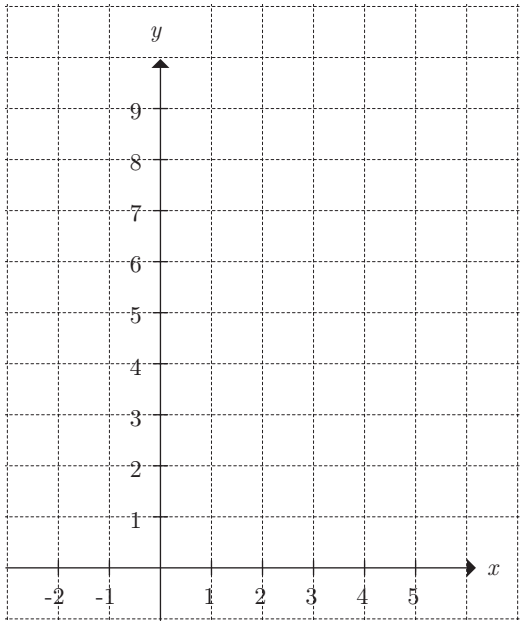


87. Sketch the graph of the piecewise defined function.

a) $y = \begin{cases} x+3 & \text{if} \quad x < 3 \\ 2x & \text{if} \quad x \geq 3 \end{cases}$

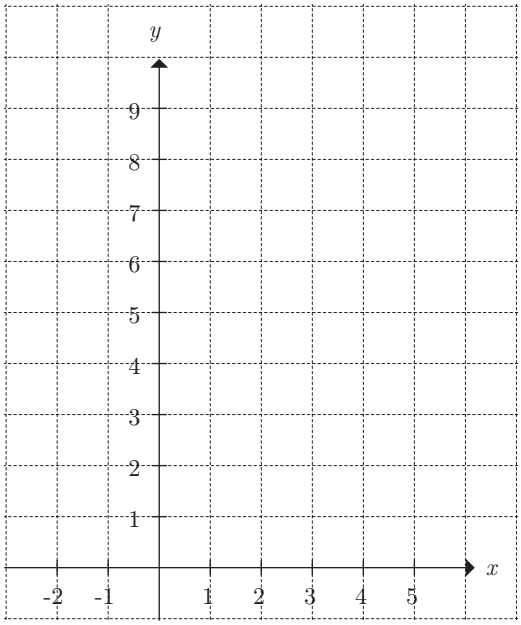


b) $y = \begin{cases} -4x+9 & \text{if} \quad x \leq 2 \\ 4x-7 & \text{if} \quad x > 2 \end{cases}$

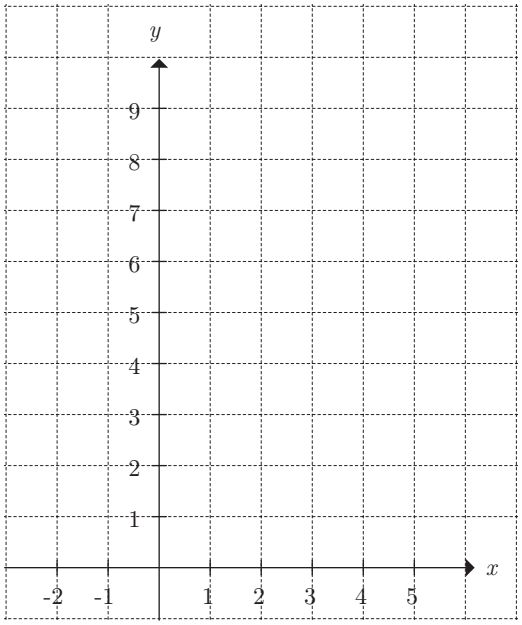


88. Sketch the graph of the piecewise defined function.

a) $y = \begin{cases} 2x-1 & \text{if} \quad x < 2 \\ -\frac{1}{2}x+4 & \text{if} \quad x \geq 2 \end{cases}$

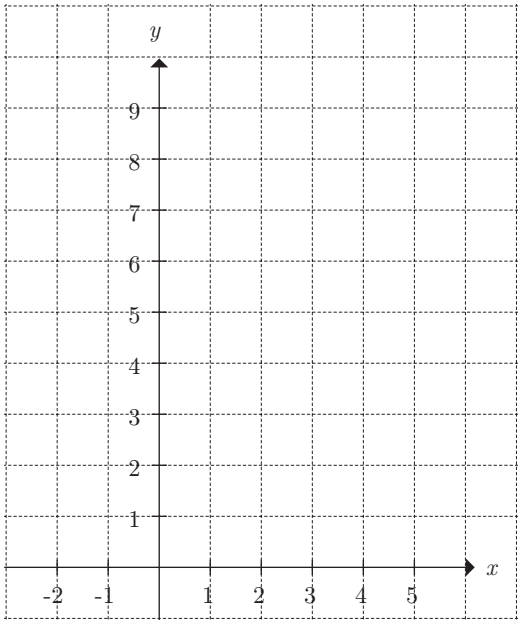


b) $y = \begin{cases} -x+1 & \text{if} \quad x \leq 0 \\ \frac{2}{3}x+1 & \text{if} \quad x > 0 \end{cases}$

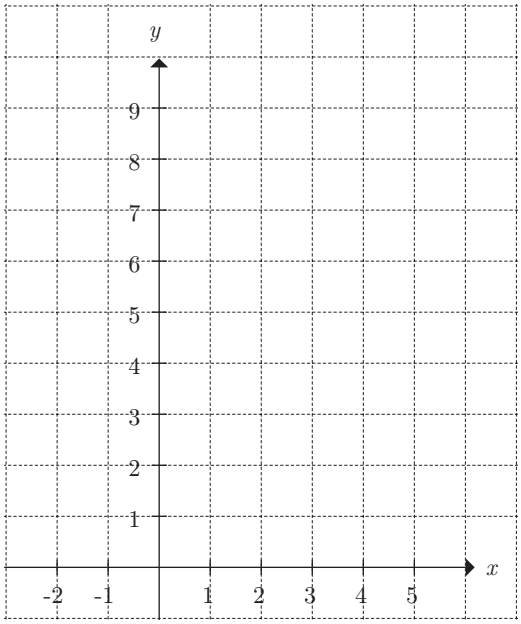


89. Sketch the graph of the piecewise defined function. Use a filled dot if the endpoint belongs to the function and an empty dot if the endpoint does not belong to the function.

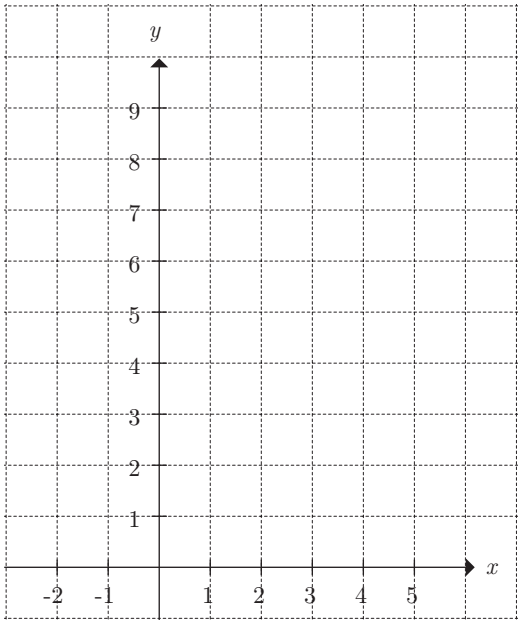
a) $y = \begin{cases} 2 & \text{if} \quad x < 3 \\ 5 & \text{if} \quad x \geq 3 \end{cases}$



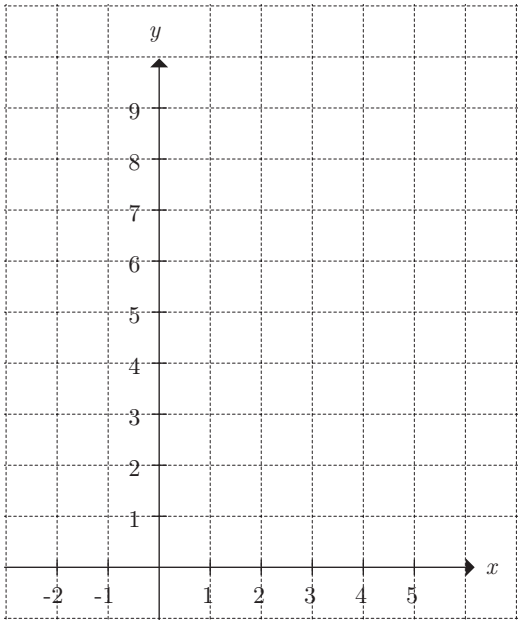
b) $y = \begin{cases} 2 & \text{if} \quad x \leq 3 \\ 5 & \text{if} \quad x > 3 \end{cases}$



c) $y = \begin{cases} 2x + 2 & \text{if} \quad x < 1 \\ 4 & \text{if} \quad 1 \leq x < 4 \\ 2x - 4 & \text{if} \quad x \geq 4 \end{cases}$

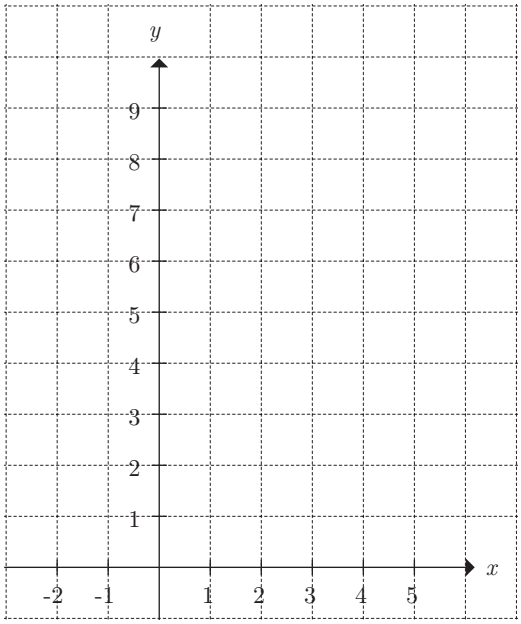


d) $y = \begin{cases} -x + 4 & \text{if} \quad x \leq 1 \\ -x + 7 & \text{if} \quad 1 < x < 4 \\ -x + 10 & \text{if} \quad x \geq 4 \end{cases}$

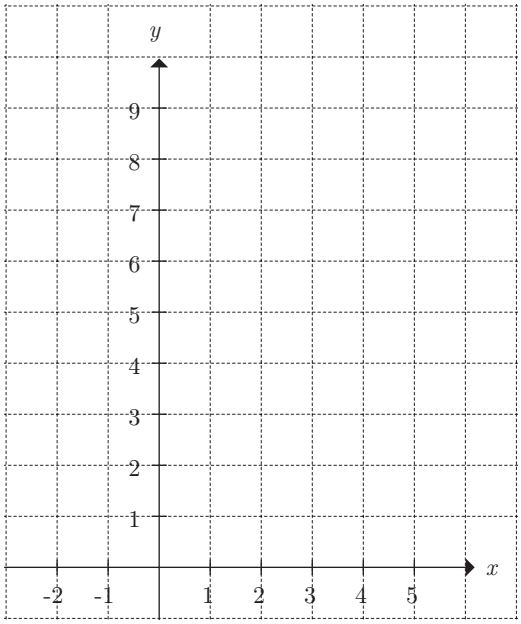


90. Sketch the graph of the piecewise defined function. Use a filled dot if the endpoint belongs to the function and an empty dot if the endpoint does not belong to the function.

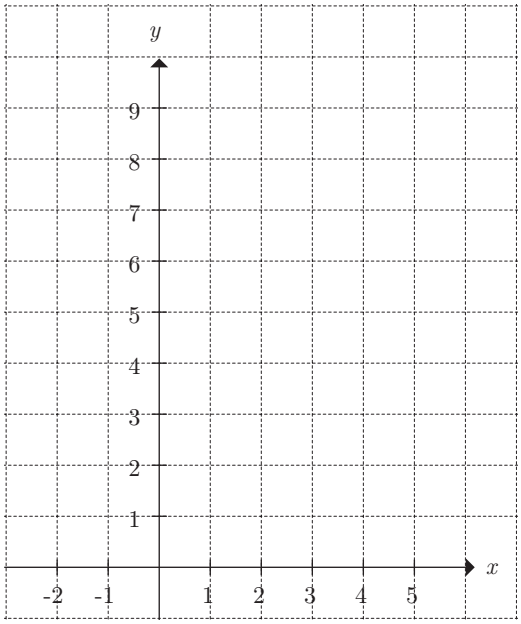
a) $y = \begin{cases} 8 & \text{if} \quad x \leq 2 \\ 3 & \text{if} \quad x > 2 \end{cases}$



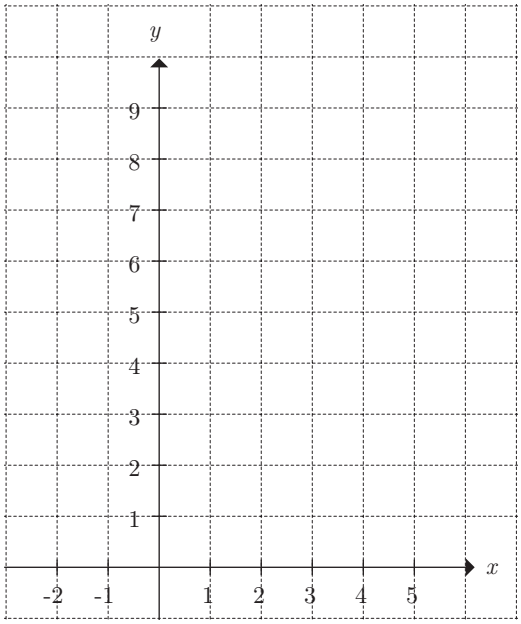
b) $y = \begin{cases} 8 & \text{if} \quad x < 2 \\ 3 & \text{if} \quad x \geq 2 \end{cases}$



c) $y = \begin{cases} 3x + 9 & \text{if} \quad x < 0 \\ 9 & \text{if} \quad 0 \leq x < 4 \\ -3x + 21 & \text{if} \quad x \geq 4 \end{cases}$

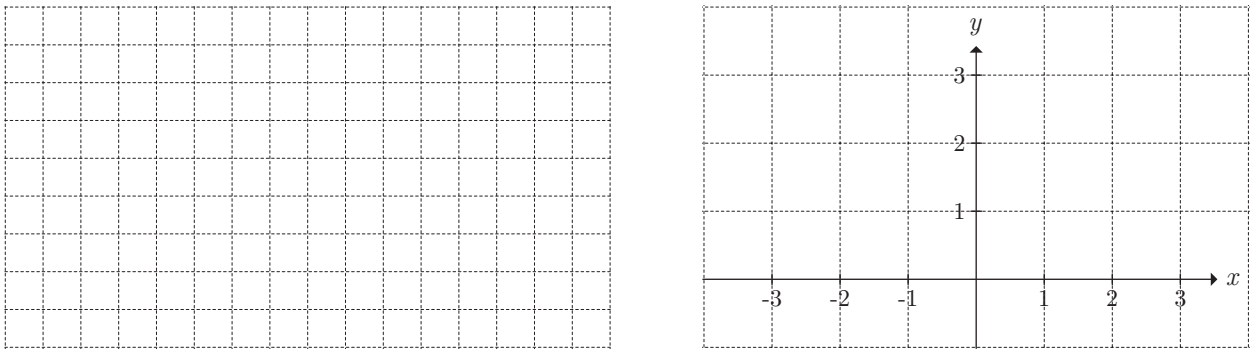


d) $y = \begin{cases} x + 5 & \text{if} \quad x < 0 \\ x + 1 & \text{if} \quad 0 \leq x \leq 5 \\ x - 2 & \text{if} \quad x > 5 \end{cases}$

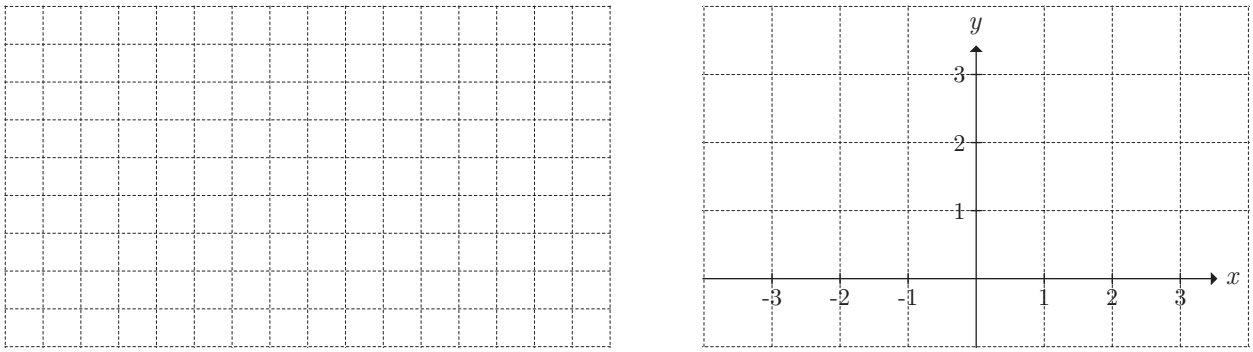


93. Draw the graph and write it as a piecewise defined function.

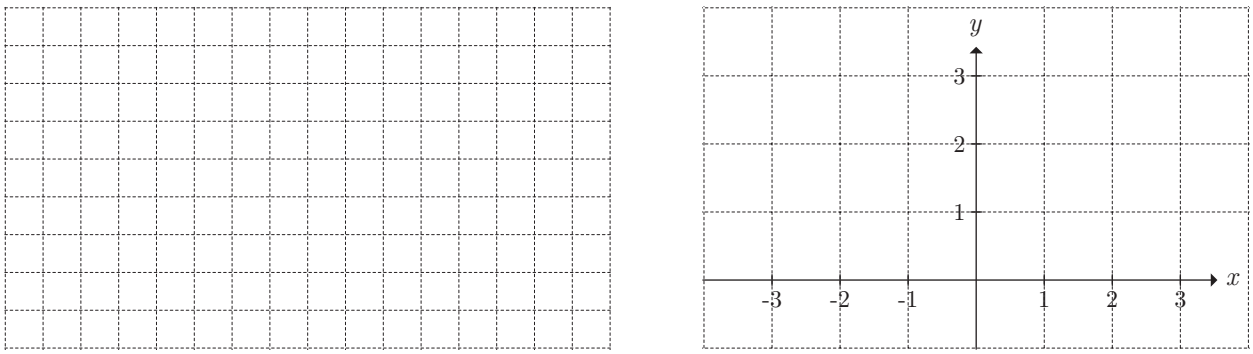
a) $y = |x|$



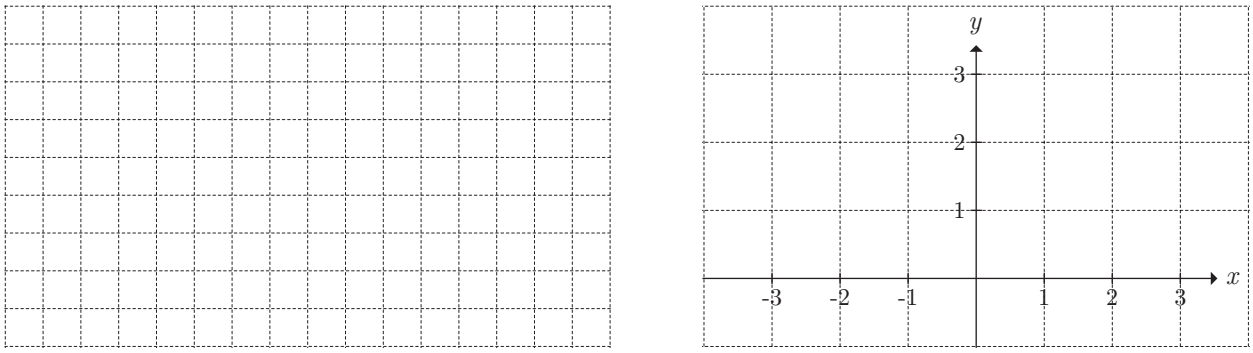
b) $y = |x| - 2$



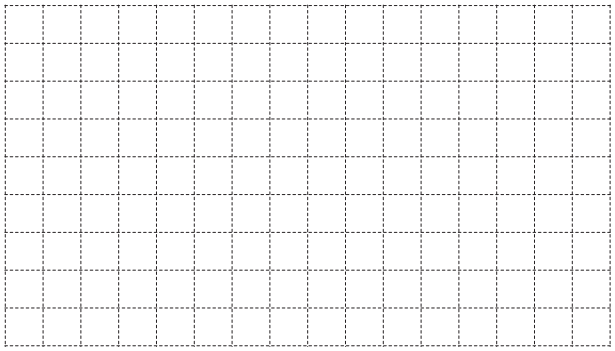
c) $y = |x - 1|$

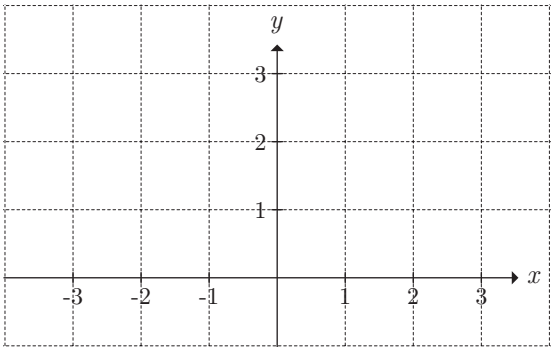


d) $y = x + |x|$

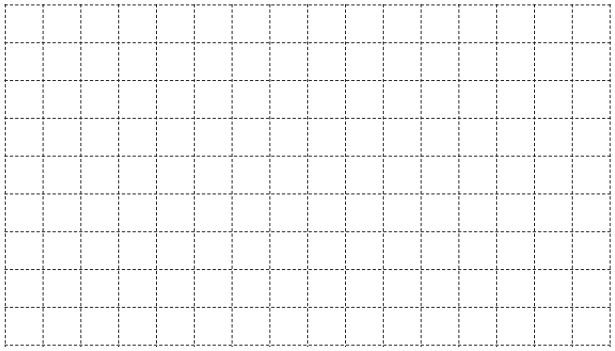


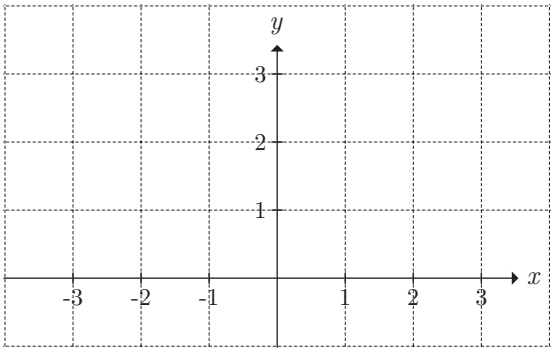
e) $y = 2x - |x|$





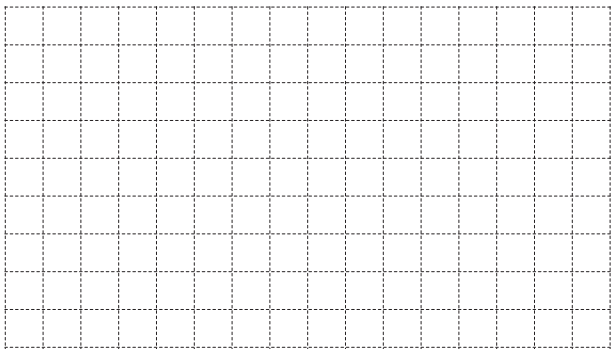
f) $y = |x + 1| + x$

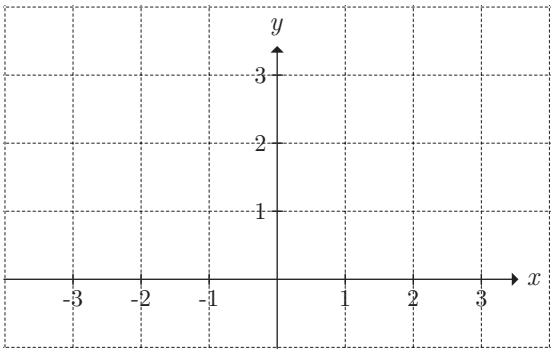




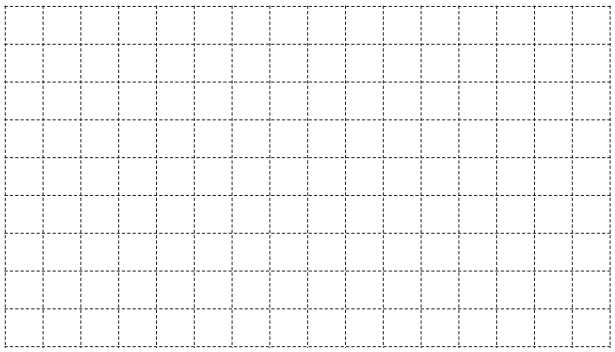
94. Draw the graph and write it as a piecewise defined function.

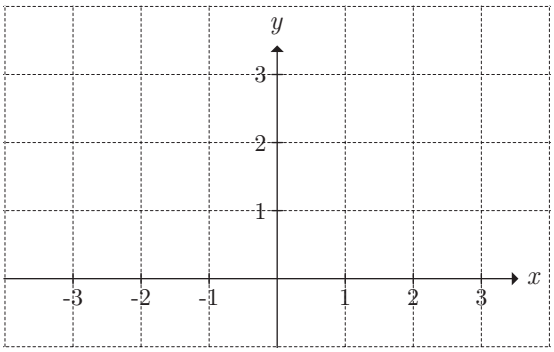
a) $y = |2x|$



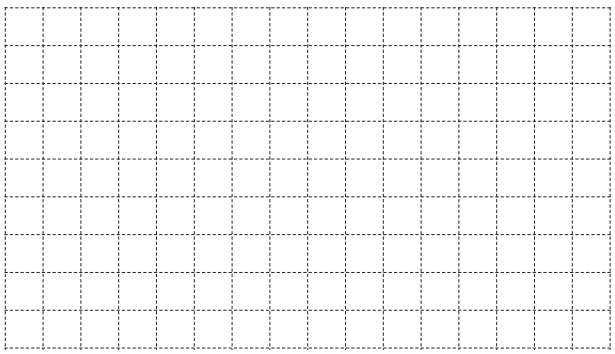


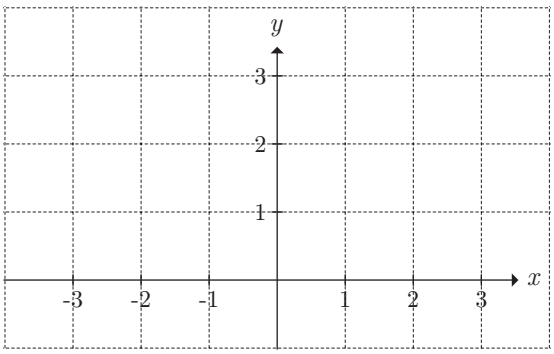
b) $y = |x + 2|$



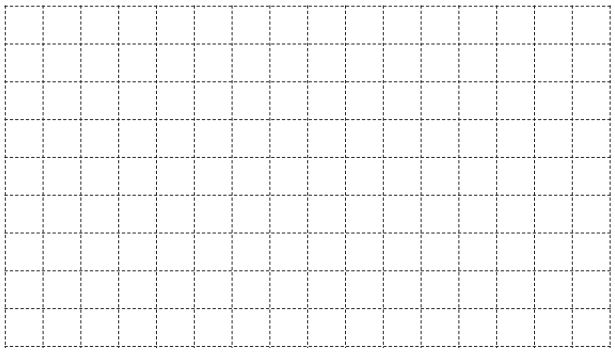


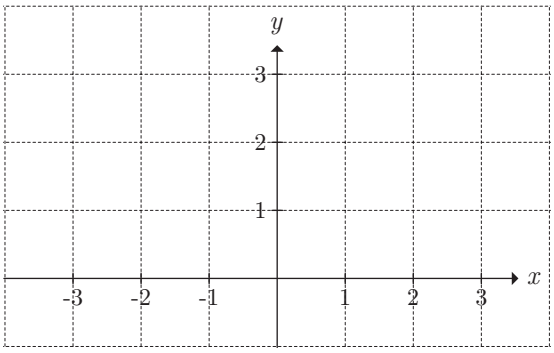
c) $y = -|x| + 1$



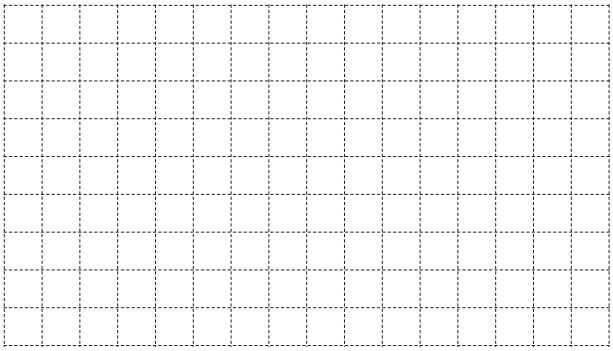


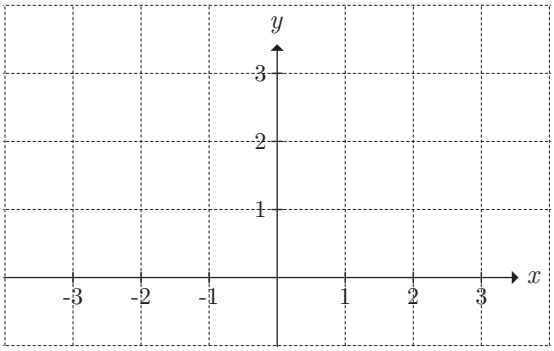
d) $y = |x| - x$



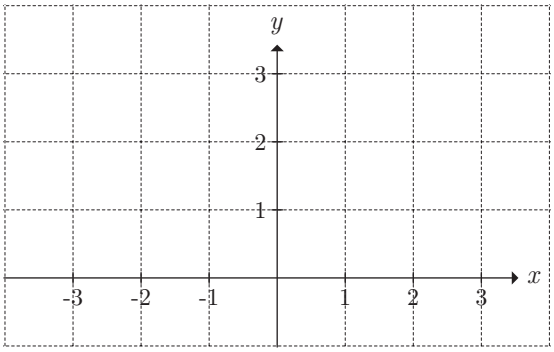


e) $y = \frac{1}{2}|x - 2| - 1$



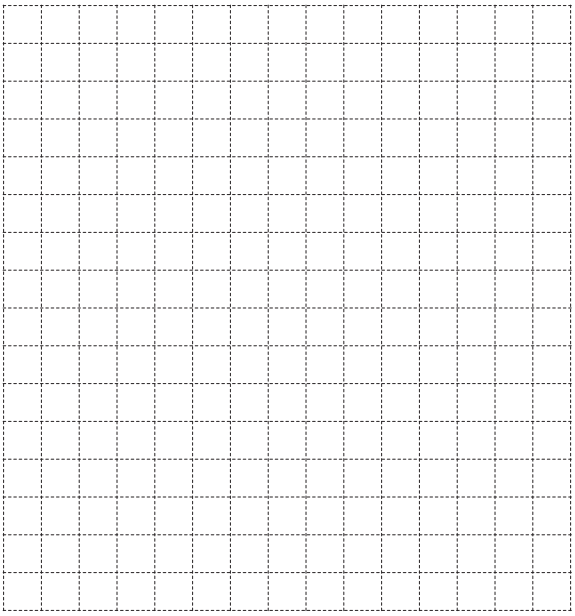


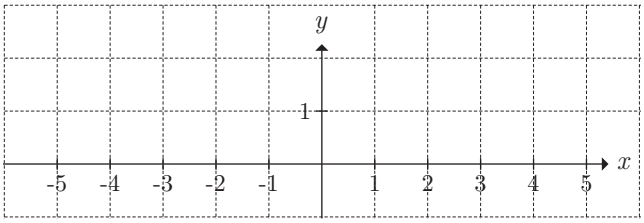
f) $y = |x - 1| + 2x$



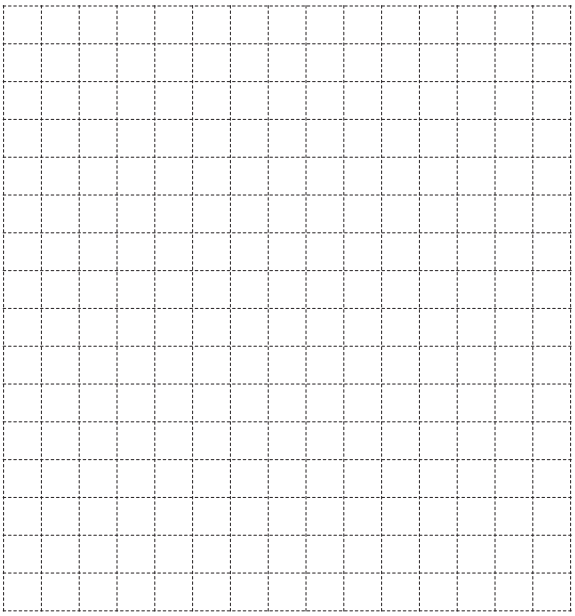
95. Draw the graph and write it as a piecewise defined function.

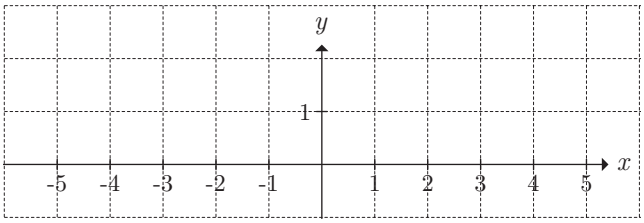
a) $y = \big|\, 2 - |x| \big|$





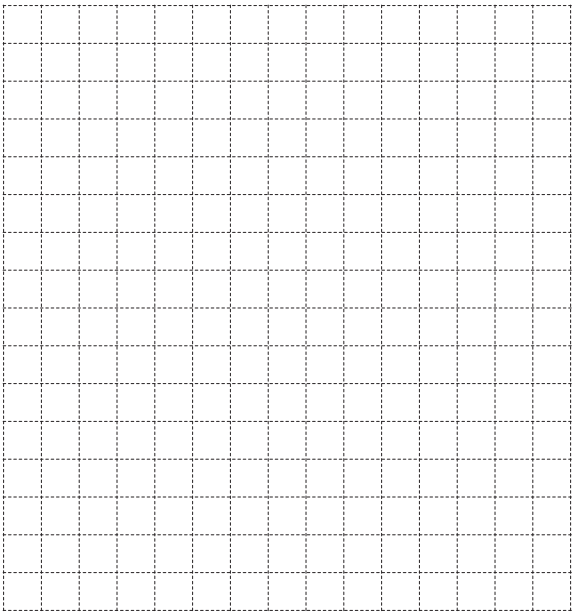
b) $y = \Big|\, |\,|x| - 1\big| - 2\Big|$

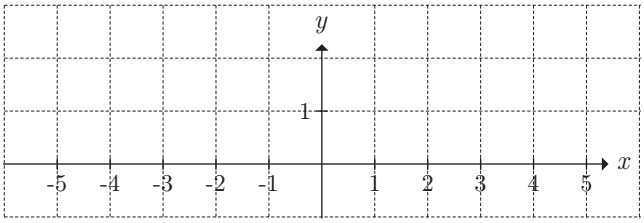




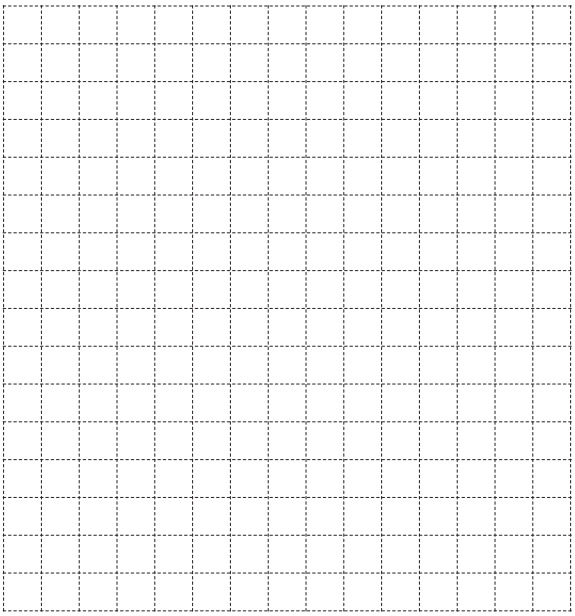
96. Draw the graph and write it as a piecewise defined function.

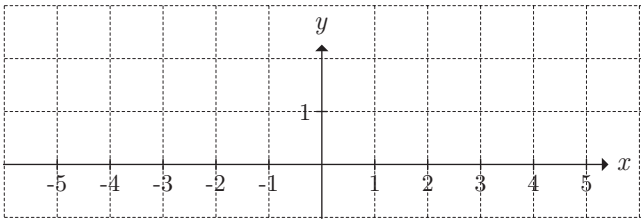
a) $y = \big|\,|x-4|-2\big|$





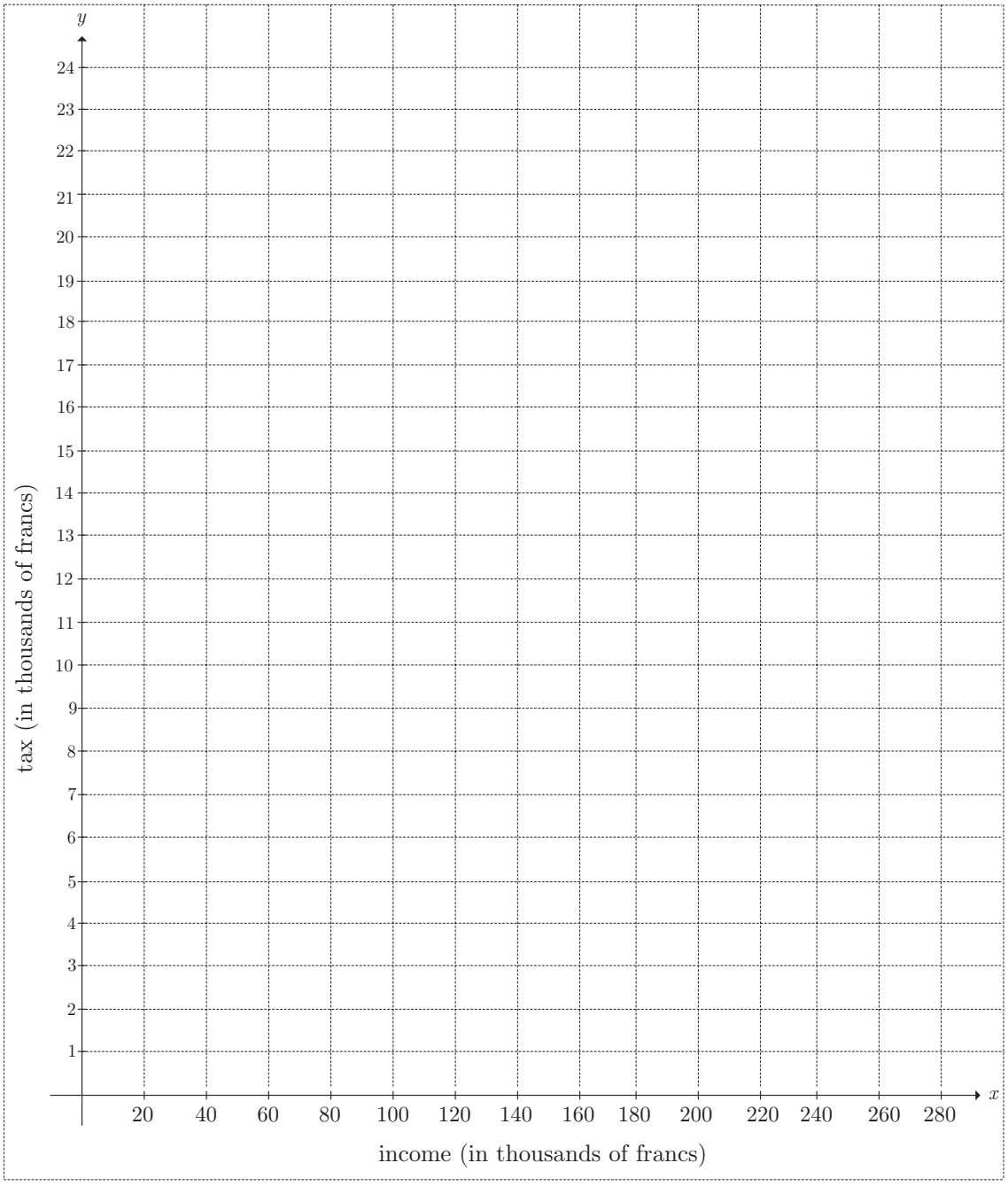
b) $y = \Big|\,|1-|x||-1\Big|$





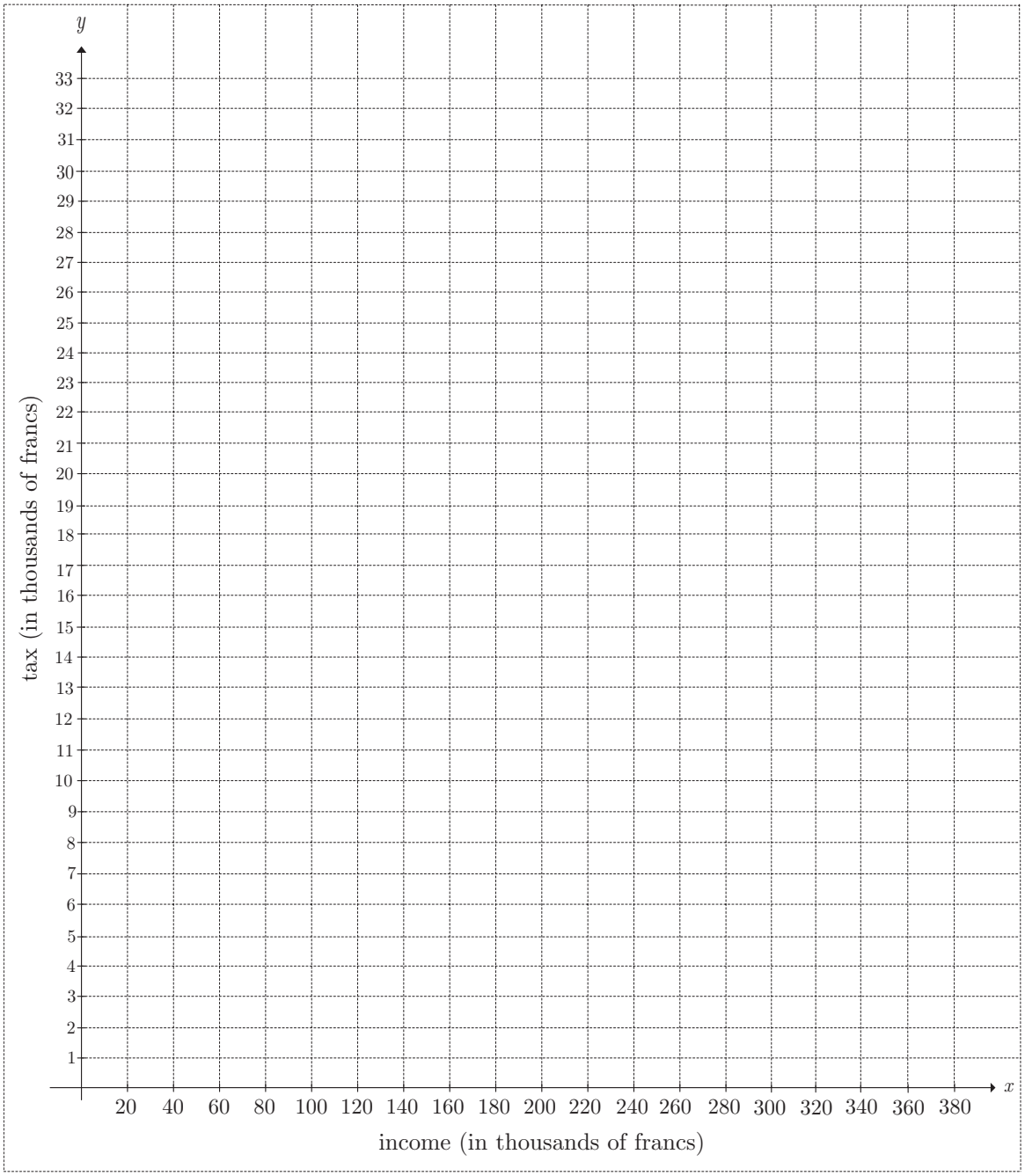
109. Draw the graph of the piecewise linear tax calculation function for unmarried persons:

	Income	tax			
up to	6900.–	0.–	+	0.–	
more than	6900.–	0.–	+	2 %	from additional income
more than	11 800.–	98.–	+	3 %	from additional income
more than	16 600.–	242.–	+	4 %	from additional income
more than	24 500.–	558.–	+	5 %	from additional income
more than	34 100.–	1038.–	+	6 %	from additional income
more than	45 100.–	1698.–	+	7 %	from additional income
more than	58 000.–	2601.–	+	8 %	from additional income
more than	75 400.–	3993.–	+	9 %	from additional income
more than	109 000.–	7017.–	+	10 %	from additional income
more than	142 200.–	10 337.–	+	11 %	from additional income
more than	194 900.–	16 134.–	+	12 %	from additional income
more than	263 300.–	24 342.–	+	13 %	from additional income



110. Draw the graph of the piecewise linear tax calculation function for married persons:

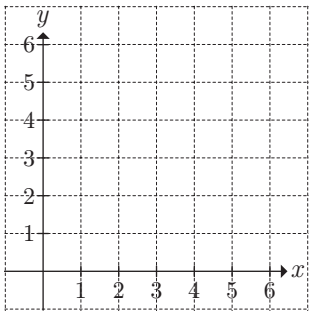
	income	tax			
up to	13 900.–	0.–	+	0.–	
more than	13 900.–	0.–	+	2 %	from additional income
more than	20 200.–	126.–	+	3 %	from additional income
more than	28 200.–	366.–	+	4 %	from additional income
more than	37 900.–	754.–	+	5 %	from additional income
more than	49 000.–	1309.–	+	6 %	from additional income
more than	63 300.–	2167.–	+	7 %	from additional income
more than	95 100.–	4393.–	+	8 %	from additional income
more than	127 000.–	6945.–	+	9 %	from additional income
more than	174 900.–	11 256.–	+	10 %	from additional income
more than	232 100.–	16 976.–	+	11 %	from additional income
more than	294 200.–	23 807.–	+	12 %	from additional income
more than	365 800.–	32 399.–	+	13 %	from additional income



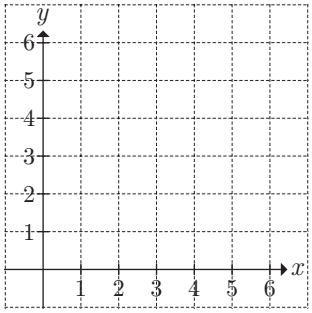
5 Systems of Linear Equations

Solve the system of equations using a graphical representation. The solutions have integer coordinates and are located in the first quadrant.

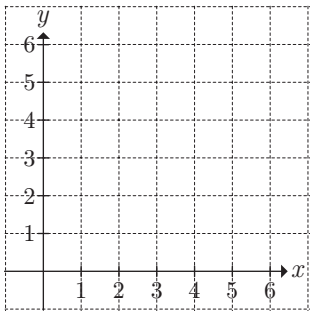
25. a) $\left|\begin{array}{l} 2x+3y=12 \\ 2x-4y=-2 \end{array}\right|$



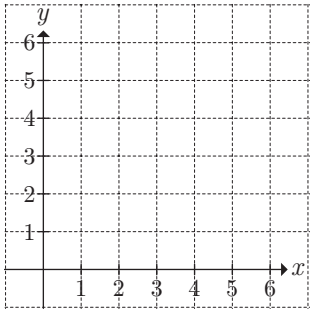
b) $\left|\begin{array}{l} x-2y=2 \\ x+y=5 \end{array}\right|$



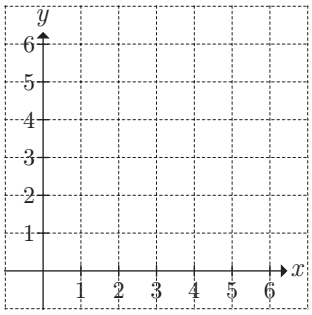
c) $\left|\begin{array}{l} 5y-2x=20 \\ y-x=1 \end{array}\right|$



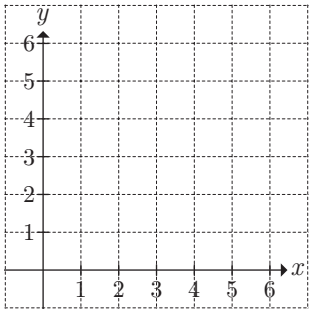
d) $\left|\begin{array}{l} x+2y=10 \\ 4y-x=8 \end{array}\right|$



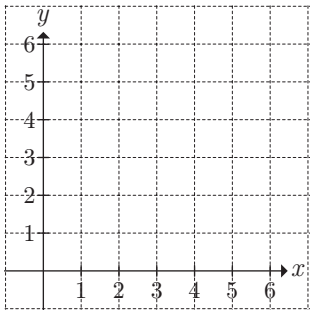
26. a) $\left|\begin{array}{l} x-y=-1 \\ 2x+y=4 \end{array}\right|$



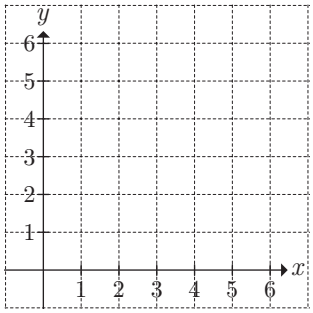
b) $\left|\begin{array}{l} x+3y=12 \\ 3y-x=6 \end{array}\right|$



c) $\left|\begin{array}{l} x-y=2 \\ x+3y=14 \end{array}\right|$

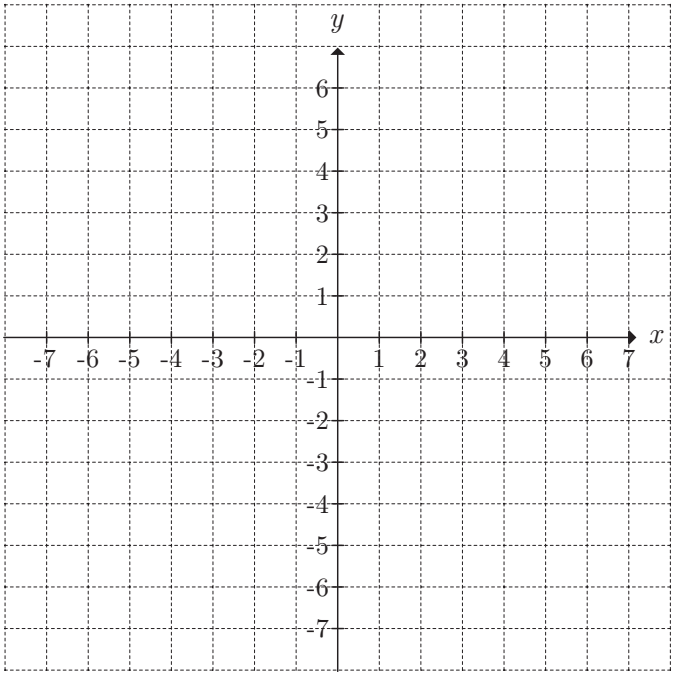


d) $\left|\begin{array}{l} 2y-3x=4 \\ x+2y=12 \end{array}\right|$

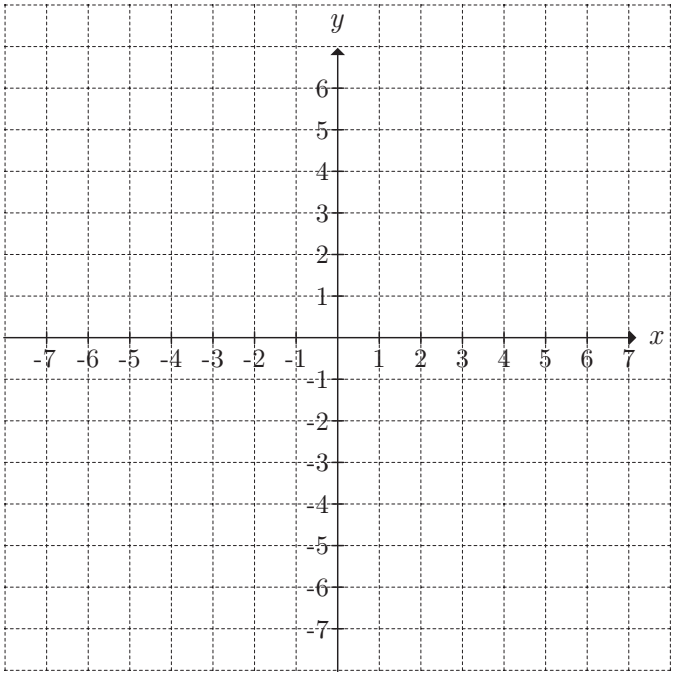


Solve using a graphical representation. The coordinates of the solutions are integers. Draw the straight lines for a) and b) in the same coordinate system.

27. a) $\left|\begin{array}{l} x - 3y = 6 \\ y - 2x = 8 \end{array}\right|$ b) $\left|\begin{array}{l} x + 4y = 13 \\ 5y - x = 32 \end{array}\right|$

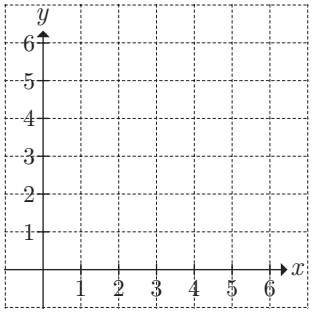


28. a) $\left|\begin{array}{l} 2x + y = 4 \\ x - 5y = 13 \end{array}\right|$ b) $\left|\begin{array}{l} 5y - x = 4 \\ x + 2y = -4 \end{array}\right|$

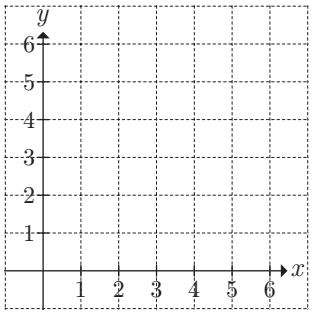


Solve the system of equations using a graphical representation. Estimate the non-integer coordinates of the solution.

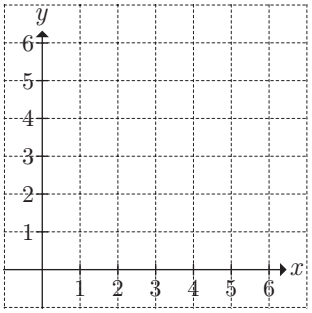
29. a) $\left|\begin{array}{r} 3x+5y=15 \\ 3x-5y=3 \end{array}\right|$



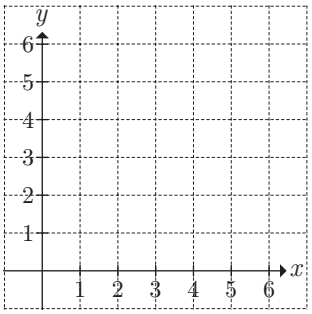
b) $\left|\begin{array}{r} x+y=5 \\ 5y-x=5 \end{array}\right|$



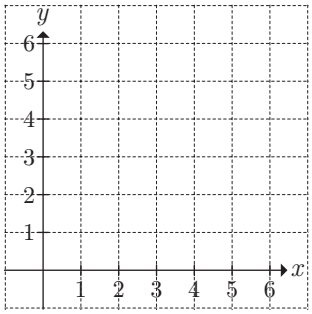
c) $\left|\begin{array}{r} y-x=1 \\ x+2y=6 \end{array}\right|$



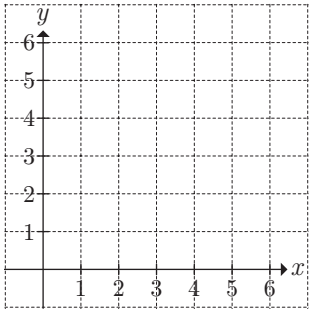
d) $\left|\begin{array}{r} x+2y=5 \\ 3y-2x=6 \end{array}\right|$



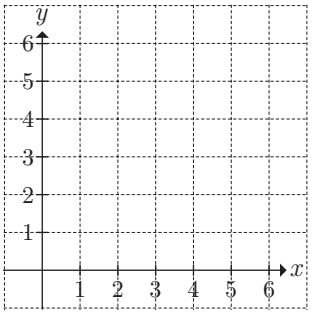
30. a) $\left|\begin{array}{r} 2x-y=6 \\ 2y-x=4 \end{array}\right|$



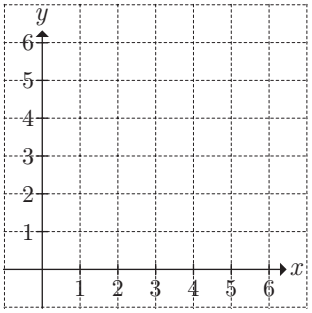
b) $\left|\begin{array}{r} 4x+y=12 \\ 4y-x=8 \end{array}\right|$



c) $\left|\begin{array}{r} 3x+4y=15 \\ x+3y=6 \end{array}\right|$

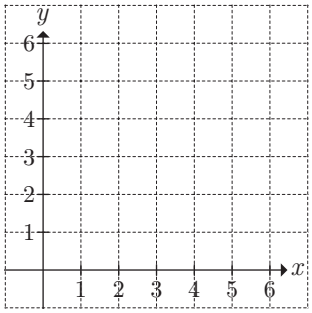


d) $\left|\begin{array}{r} 2x+y=4 \\ 3y-x=3 \end{array}\right|$

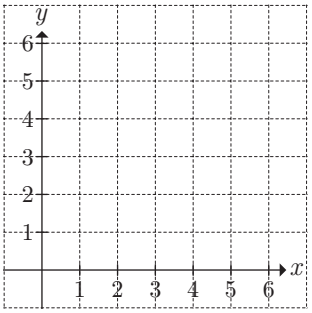


Do the three equations have a common solution? Find out using their graphs.

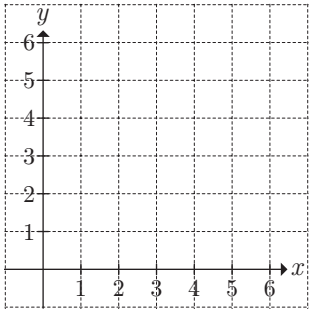
31. a) $\left|\begin{array}{r} x+y=6 \\ 5y-3x=2 \\ x+2y=9 \end{array}\right|$



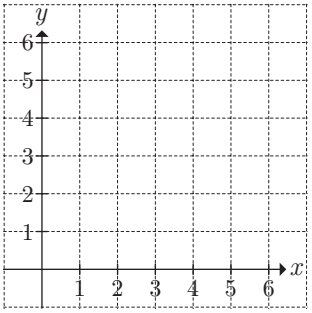
b) $\left|\begin{array}{r} x+y=5 \\ x-y=2 \\ 7y+x=14 \end{array}\right|$



32. a) $\left|\begin{array}{r} 3x+5y=23 \\ x-y=3 \\ 5y-x=4 \end{array}\right|$

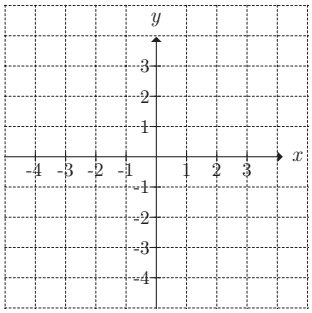


b) $\left|\begin{array}{r} 3x+5y=18 \\ y+4x=12 \\ 3x-2y=1 \end{array}\right|$

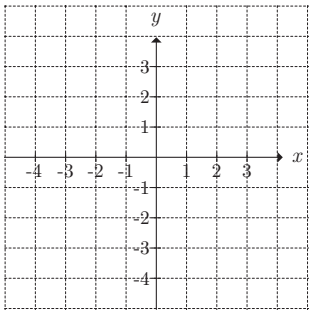


Shade the set of points described by the system of inequalities.

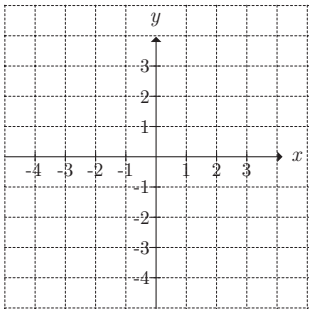
37. a) $\left|\begin{array}{l} y < x + 2 \\ y > -x - 1 \\ x < 3 \end{array}\right|$



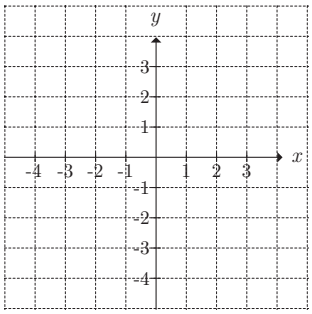
b) $\left|\begin{array}{r} x + y < 3 \\ -2x + y < 1 \\ y + 2 > 0 \end{array}\right|$



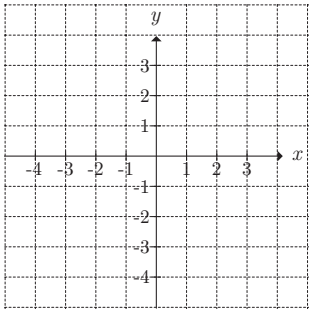
c) $\left|\begin{array}{r} x + 2y < 6 \\ 2x - 3y < 3 \\ y < 4 \end{array}\right|$



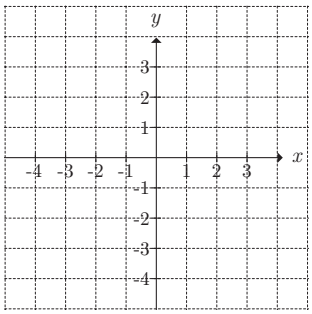
d) $\left|\begin{array}{l} 3y - 2x < 3 \\ x + 2y > -2 \\ x > 1 \end{array}\right|$



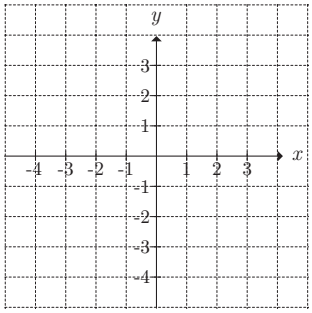
38. a) $\left|\begin{array}{l} y > x - 2 \\ y > -x - 3 \\ y < 1 \end{array}\right|$



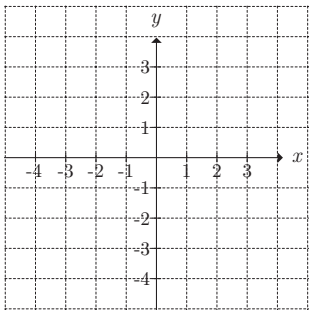
b) $\left|\begin{array}{r} x - 2y < 4 \\ 2x + 3y < 9 \\ x > -2 \end{array}\right|$



c) $\left|\begin{array}{l} 5x - y < 4 \\ x - 4y < 8 \\ y < 3 \end{array}\right|$

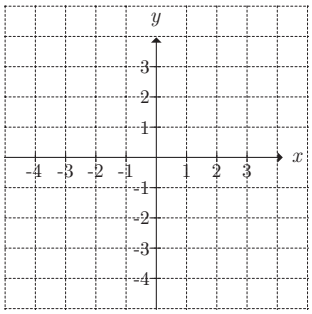


d) $\left|\begin{array}{l} x - 3y < 12 \\ x + y < 2 \\ y - 3x < 3 \end{array}\right|$

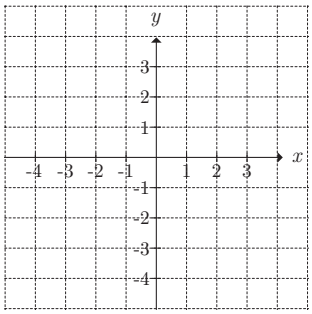


Shade the set of points described by the system of inequalities.

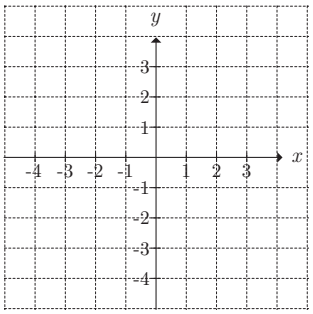
39. a)
$$\left|\begin{array}{r} y-x<2 \\ y<3 \\ y>-2 \\ x<4 \end{array}\right|$$



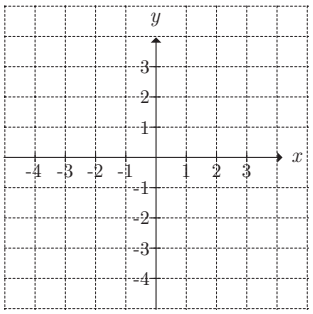
b)
$$\left|\begin{array}{r} 4y-x<12 \\ 2x-y<4 \\ 5x+2y>-2 \\ y>-1 \end{array}\right|$$



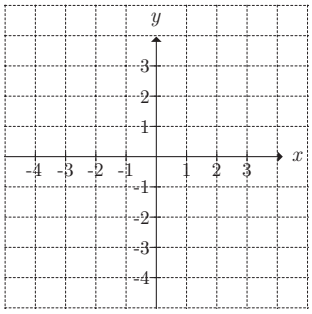
c)
$$\left|\begin{array}{r} -2<y<2 \\ -x<4 \\ 0<y-x<4 \\ 2y+x<3 \end{array}\right|$$



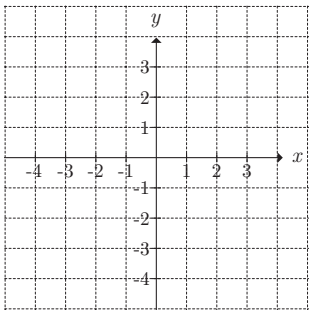
d)
$$\left|\begin{array}{r} 0<y-x<5 \\ x+3>0 \\ x+3y>0 \\ y<3 \end{array}\right|$$



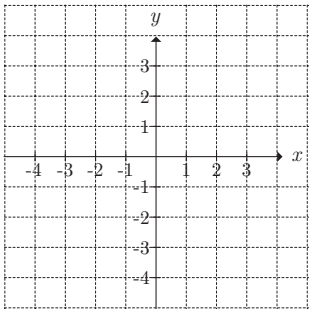
40. a)
$$\left|\begin{array}{r} x+y<1 \\ x-y<5 \\ y>-4 \\ x>-3 \end{array}\right|$$



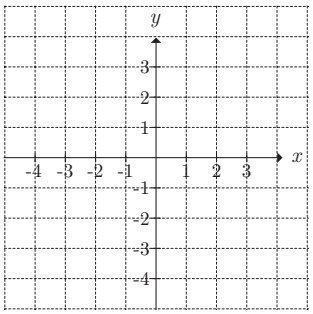
b)
$$\left|\begin{array}{r} x-3y<12 \\ x+y<1 \\ 3y-2x<6 \\ x>-3 \end{array}\right|$$



c)
$$\left|\begin{array}{r} -3<x<4 \\ -3<y<2 \\ y-x<3 \\ x-3y<9 \end{array}\right|$$



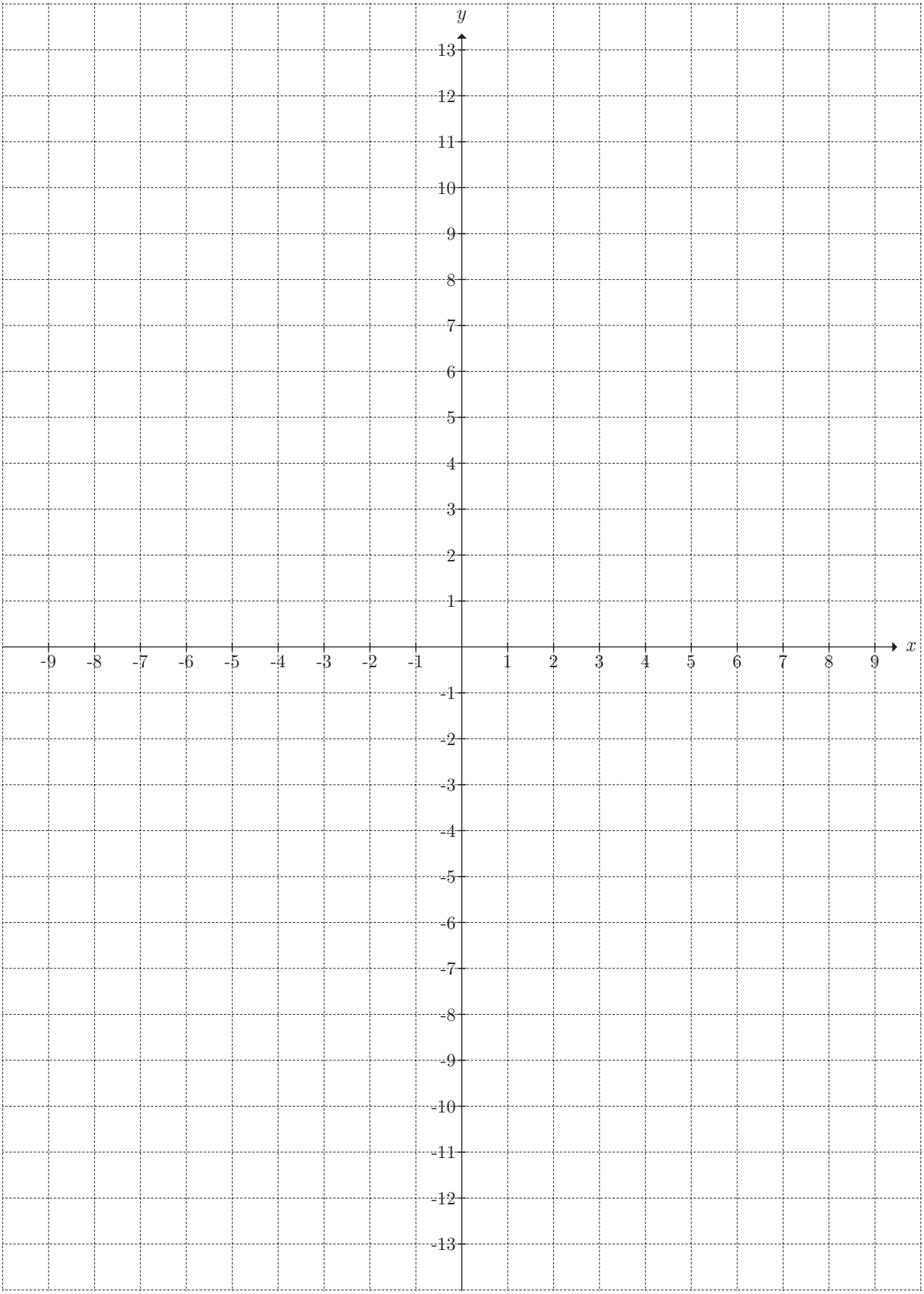
d)
$$\left|\begin{array}{r} |x|<3 \\ x-2y<9 \\ -2x-y<5 \\ y<2 \end{array}\right|$$



7 Quadratic Functions

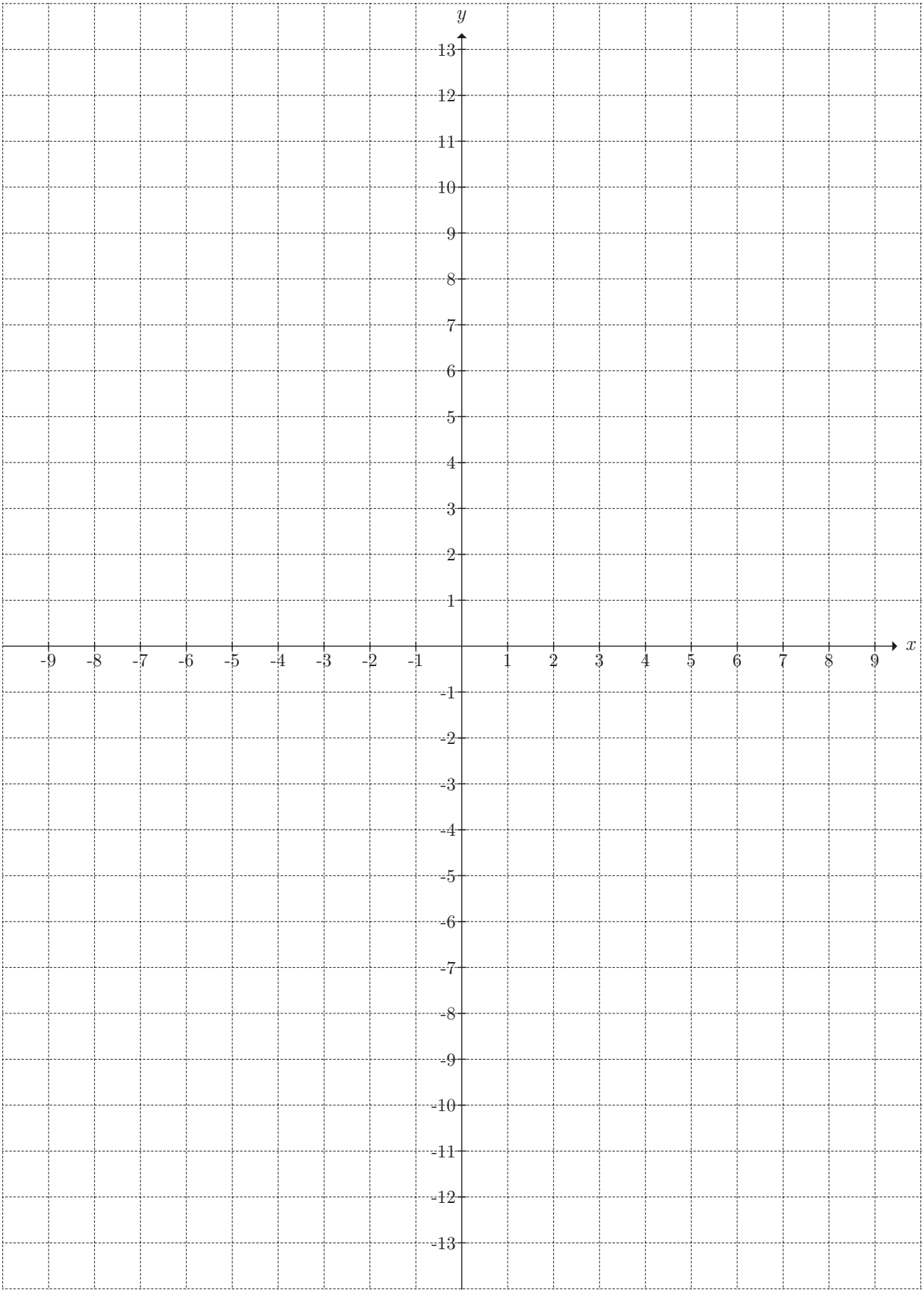
3. Sketch the graphs of the quadratic functions.

$f_1(x) = 2x^2$, $f_2(x) = x^2 - 1$, $f_3(x) = -\frac{1}{2}x^2 + 2$, $f_4(x) = \frac{1}{3}x^2 - 4$



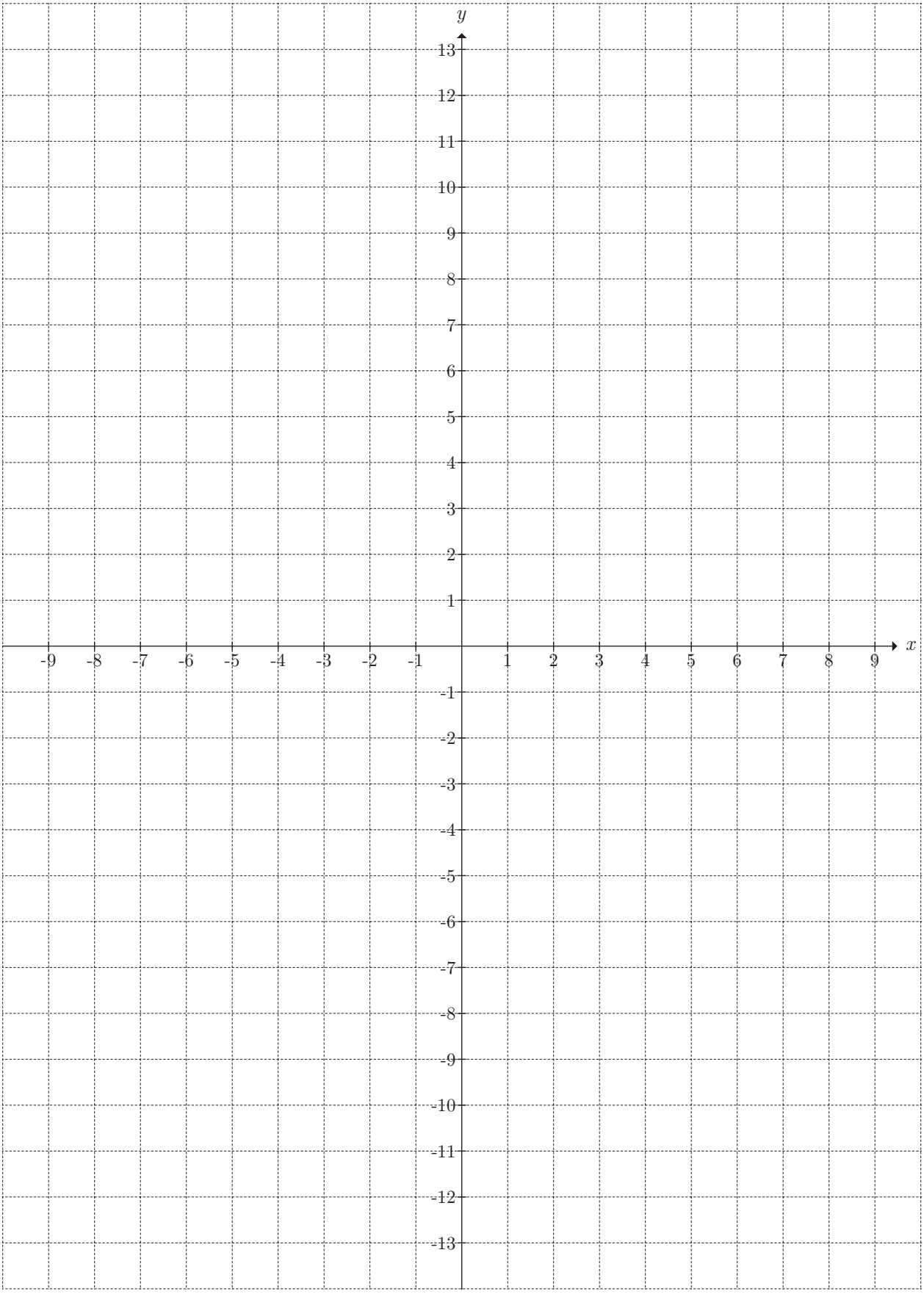
4. Sketch the graphs of the quadratic functions.

$f_1(x) = \frac{1}{2}x^2$, $f_2(x) = -x^2 + 3$, $f_3(x) = \frac{1}{2}x^2 + 1$, $f_4(x) = 3x^2 - 6$



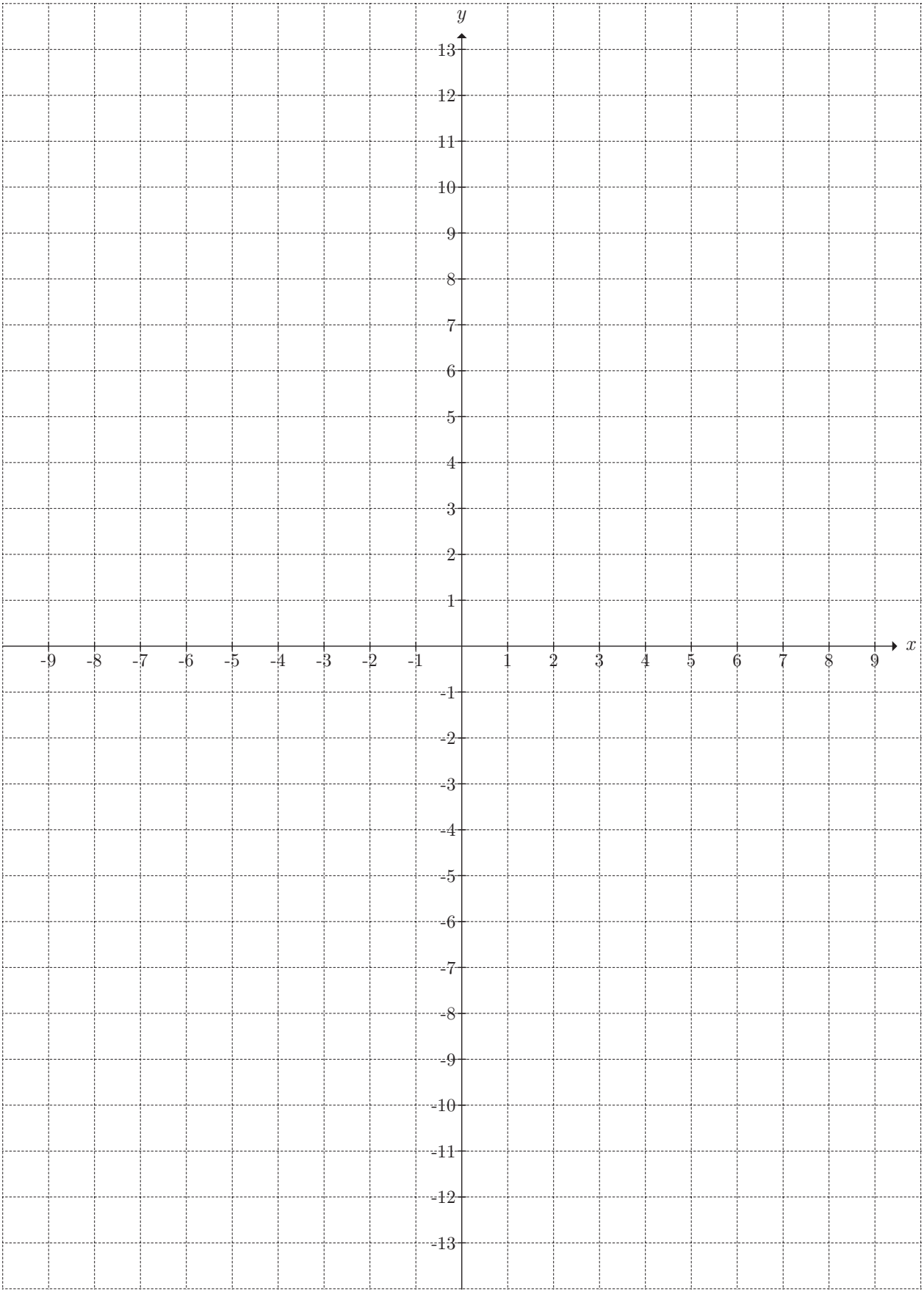
5. Sketch the graphs of the quadratic functions.

$f_1(x) = -(x-3)^2$, $f_2(x) = (x+1)^2 - 2$, $f_3(x) = -\frac{1}{2}(x+2)^2$, $f_4(x) = 3(x-4)^2$



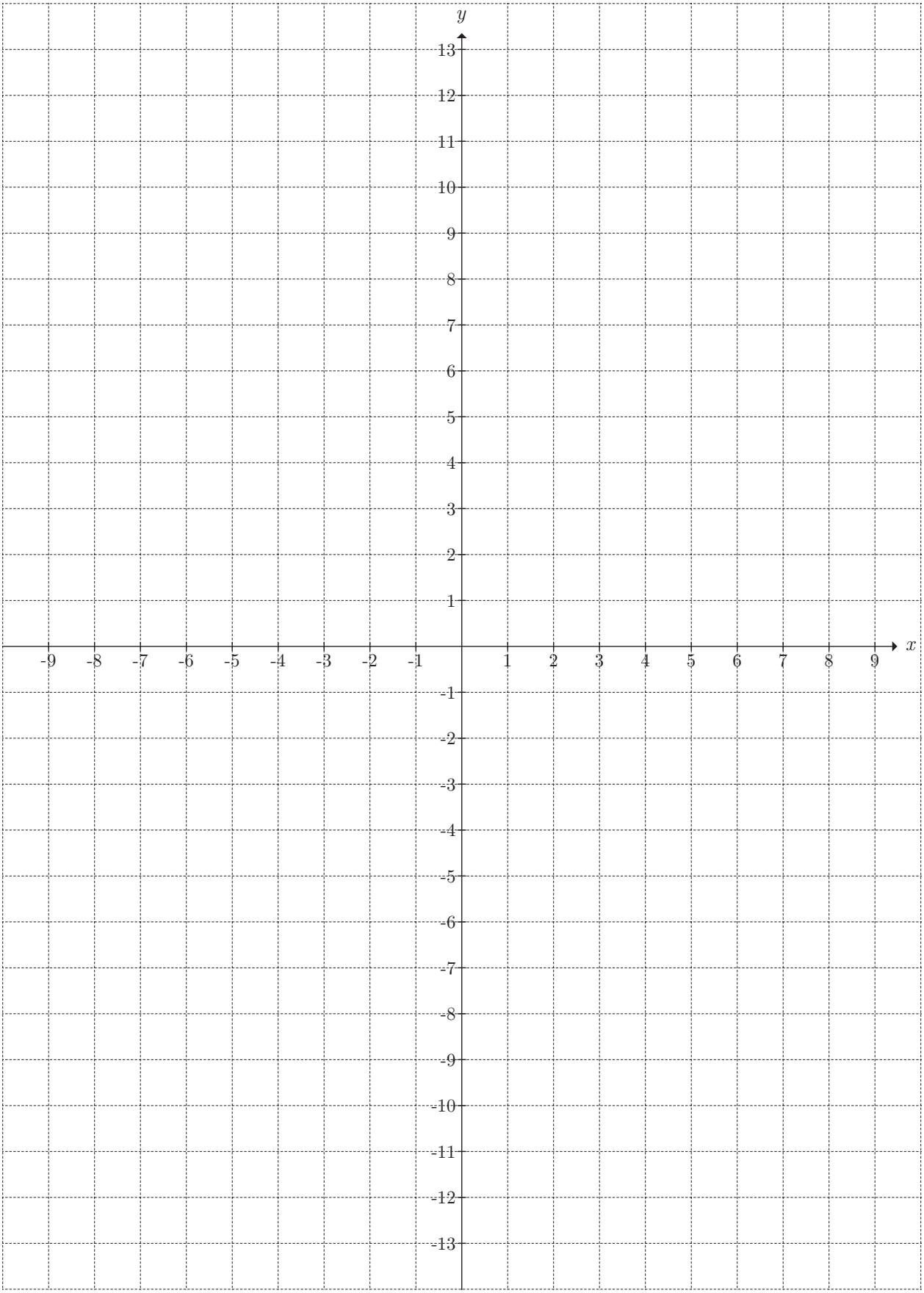
6. Sketch the graphs of the quadratic functions.

$f_1(x) = (x+5)^2$, $f_2(x) = -(x-4)^2$, $f_3(x) = 2(x+3)^2 - 1$, $f_4(x) = -\frac{1}{5}(x+6)^2 - 3$



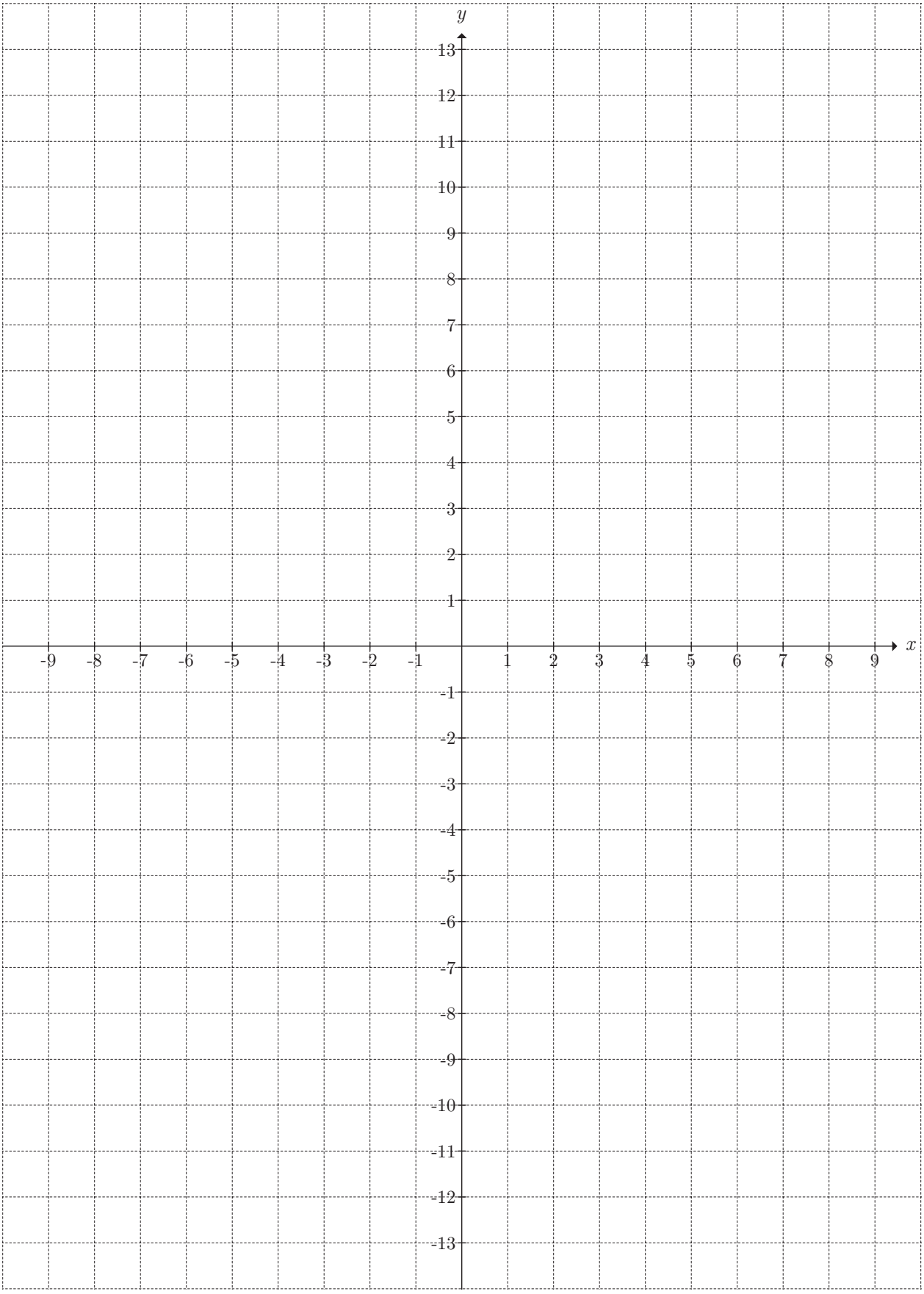
7. Sketch the graphs of the quadratic functions.

$f_1(x) = -(x-4)^2 + 1, \qquad f_2(x) = (x+1)^2 - 3, \qquad f_3(x) = \frac{1}{2}(x+2)^2 - 1, \qquad f_4(x) = -2x^2 + 3$



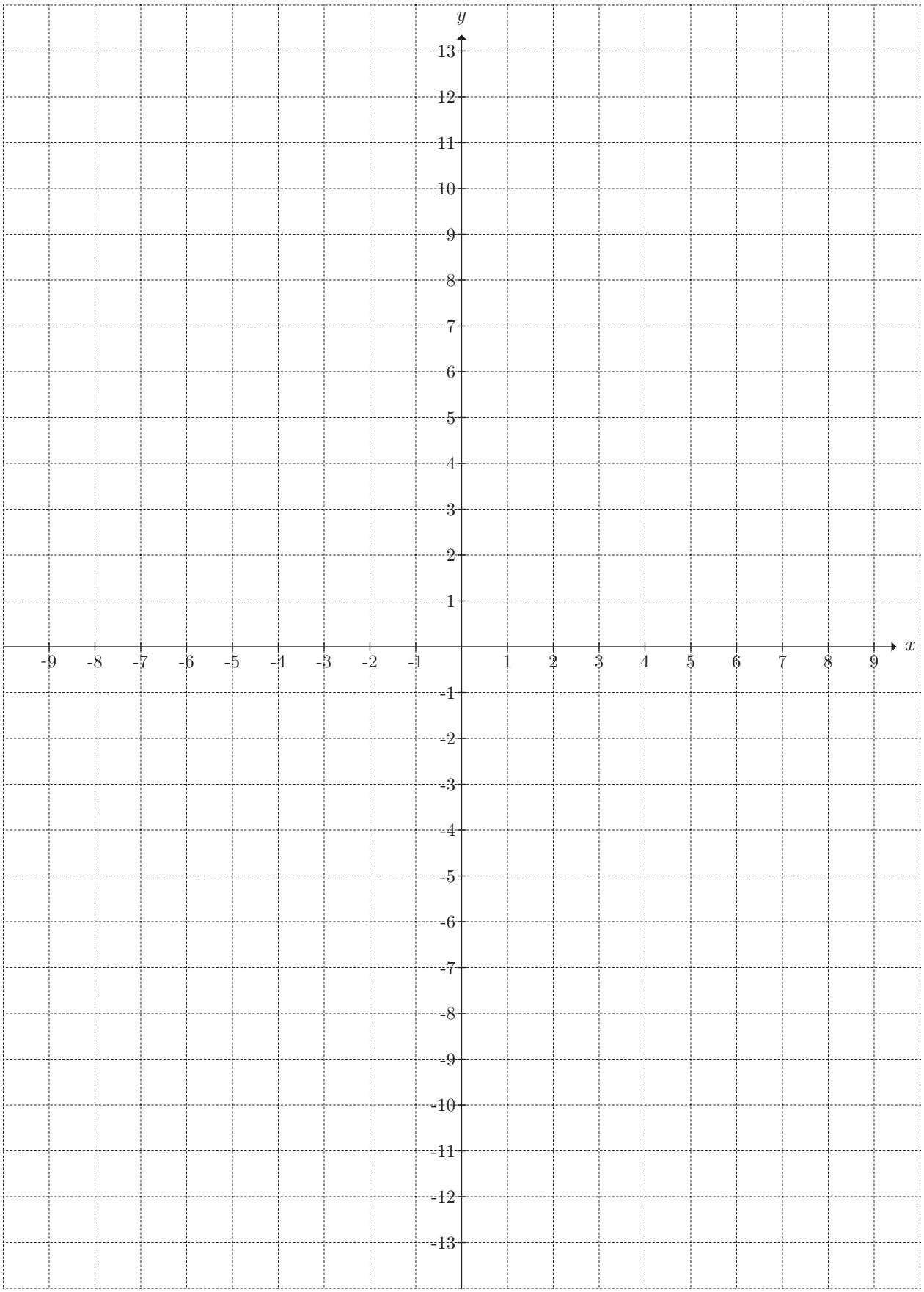
8. Sketch the graphs of the quadratic functions.

$f_1(x) = (x+5)^2 - 2, \quad f_2(x) = -(x+4)^2 + 5, \quad f_3(x) = \frac{1}{5}(x+1)^2 - 4, \quad f_4(x) = 3(x+3)^2 - 6$



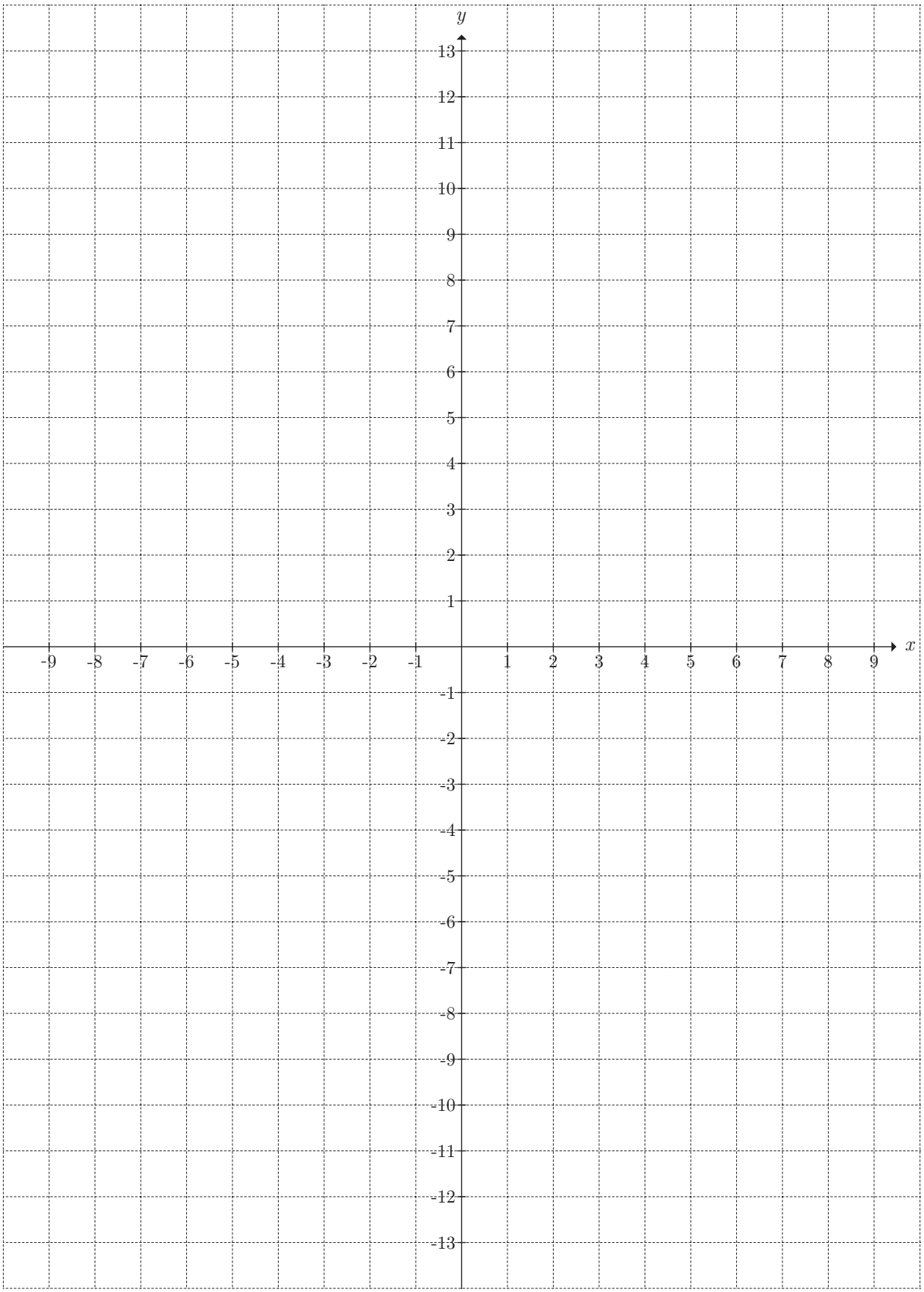
27. Sketch the graph by first converting the equation into vertex form. Also note the coordinates of the vertex.

$f_1(x) = 2x^2 - 8x + 3, \quad f_2(x) = -\frac{3}{2}x^2 - 6x - 2, \quad f_3(x) = -x^2 - 6x - 3, \quad f_4(x) = \frac{1}{2}x^2 - 4x + 9$



28. Sketch the graph by first converting the equation into vertex form. Also note the coordinates of the vertex.

$f_1(x) = -2x^2 - 4x, \quad f_2(x) = \frac{3}{4}x^2 + 6x + 12, \quad f_3(x) = -\frac{1}{2}x^2 + 3x - \frac{1}{2}, \quad f_4(x) = \frac{2}{3}x^2 - 8x + 20$

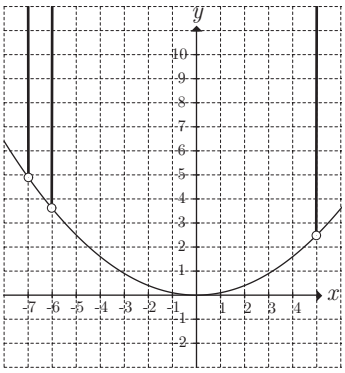


137. Given is the graph of the parabola $y = 0.1x^2$.

a) Calculate the focus F and the directrix ℓ and plot both.

b) Vertical rays are drawn that strike the parabola. They are reflected there. According to the law of reflection, the angle of incidence of the ray relative to the tangent of the parabola at the point of impact is equal to the angle of reflection. Draw the path of all three rays after reflection.
What do you notice?

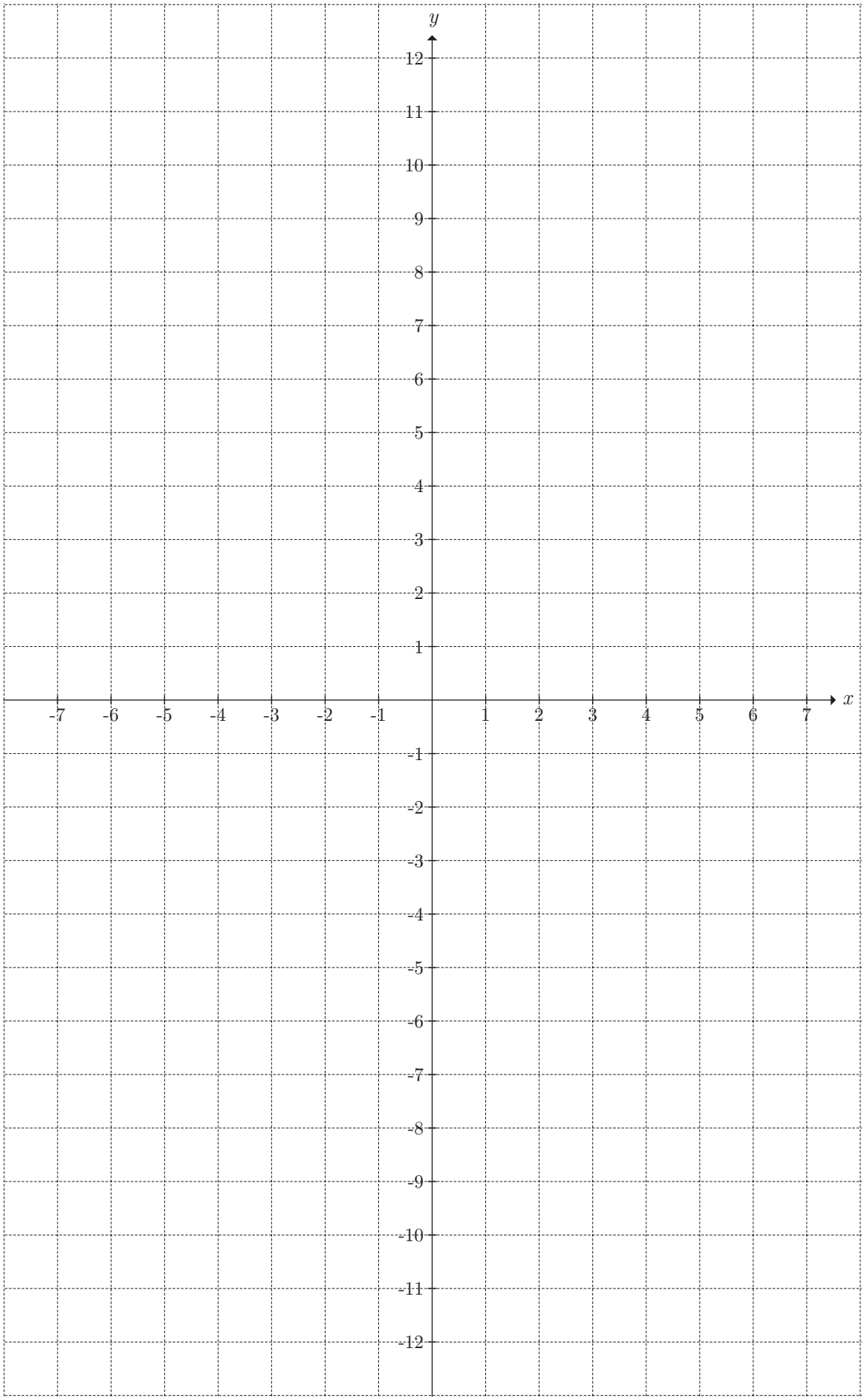
c) Extend the reflected rays so that they hit the parabola again and perform another reflection. How do these doubly reflected rays travel?



9 Power Functions

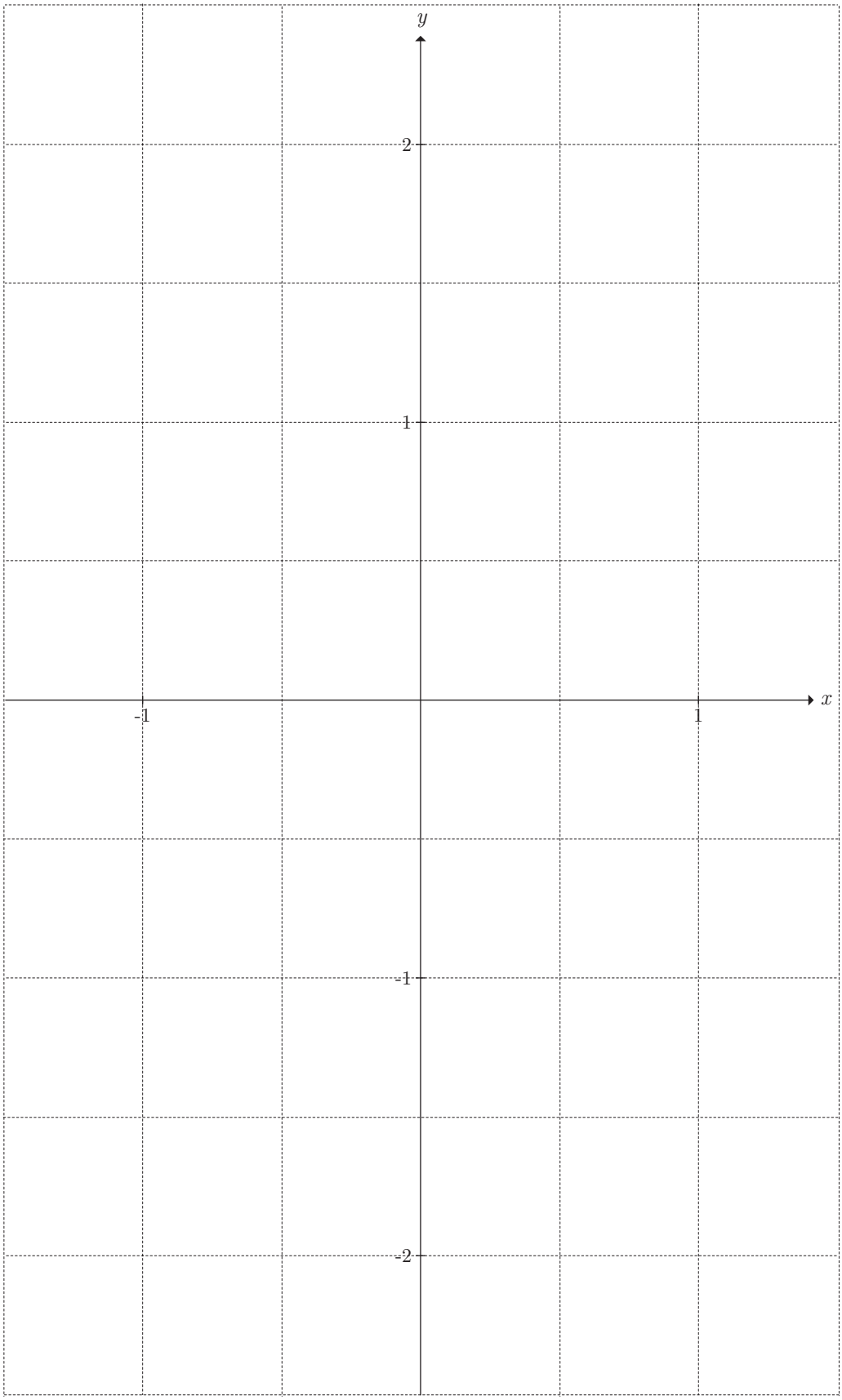
1. Sketch the graphs of the following functions in the same coordinate system.

   $y = x^2$, $y = x^3$, $y = x^4$, $y = x^5$



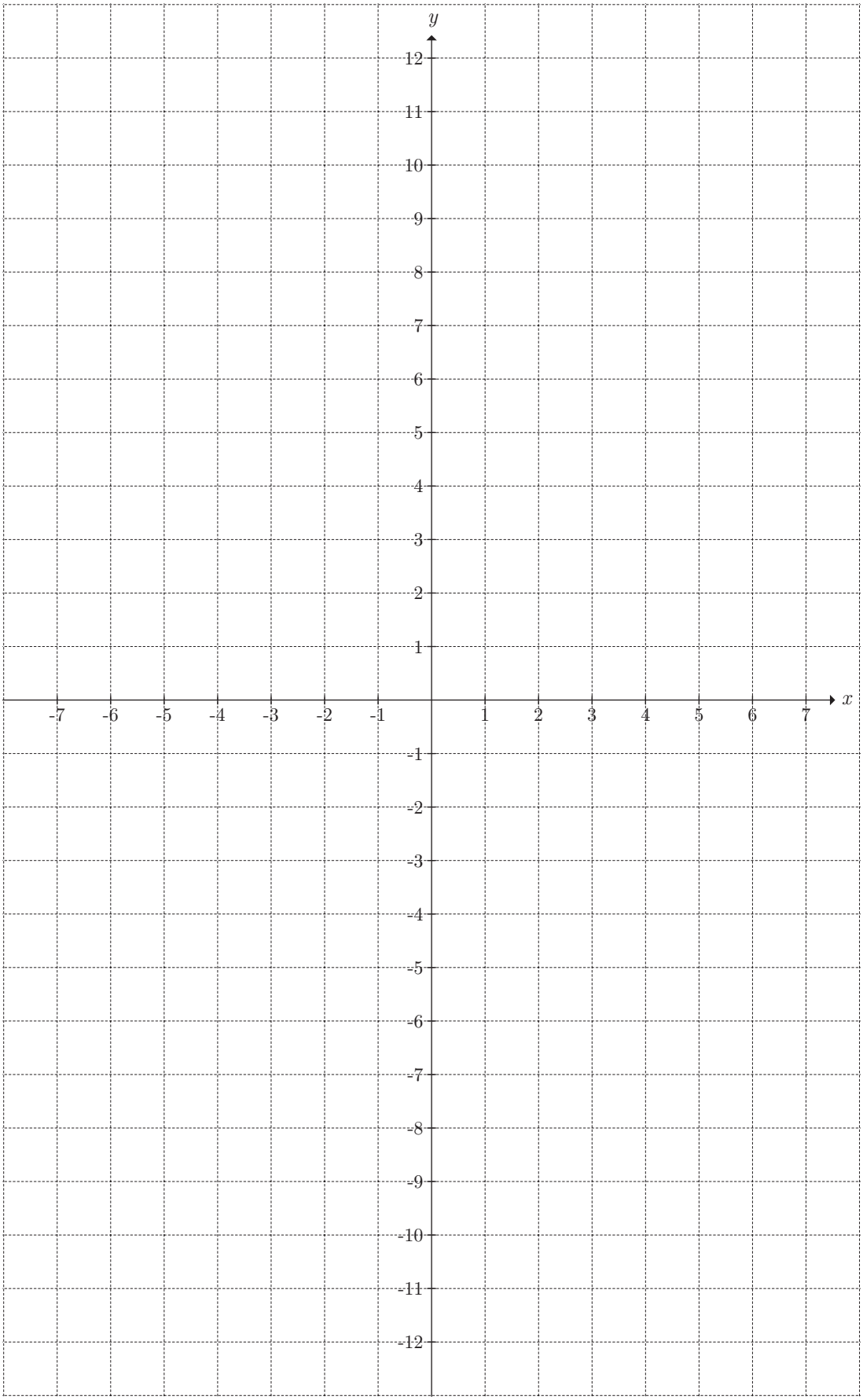
2. Sketch the graphs of the following functions in the same coordinate system.

   $y = x^3$, $y = x^4$, $y = x^5$, $y = x^6$



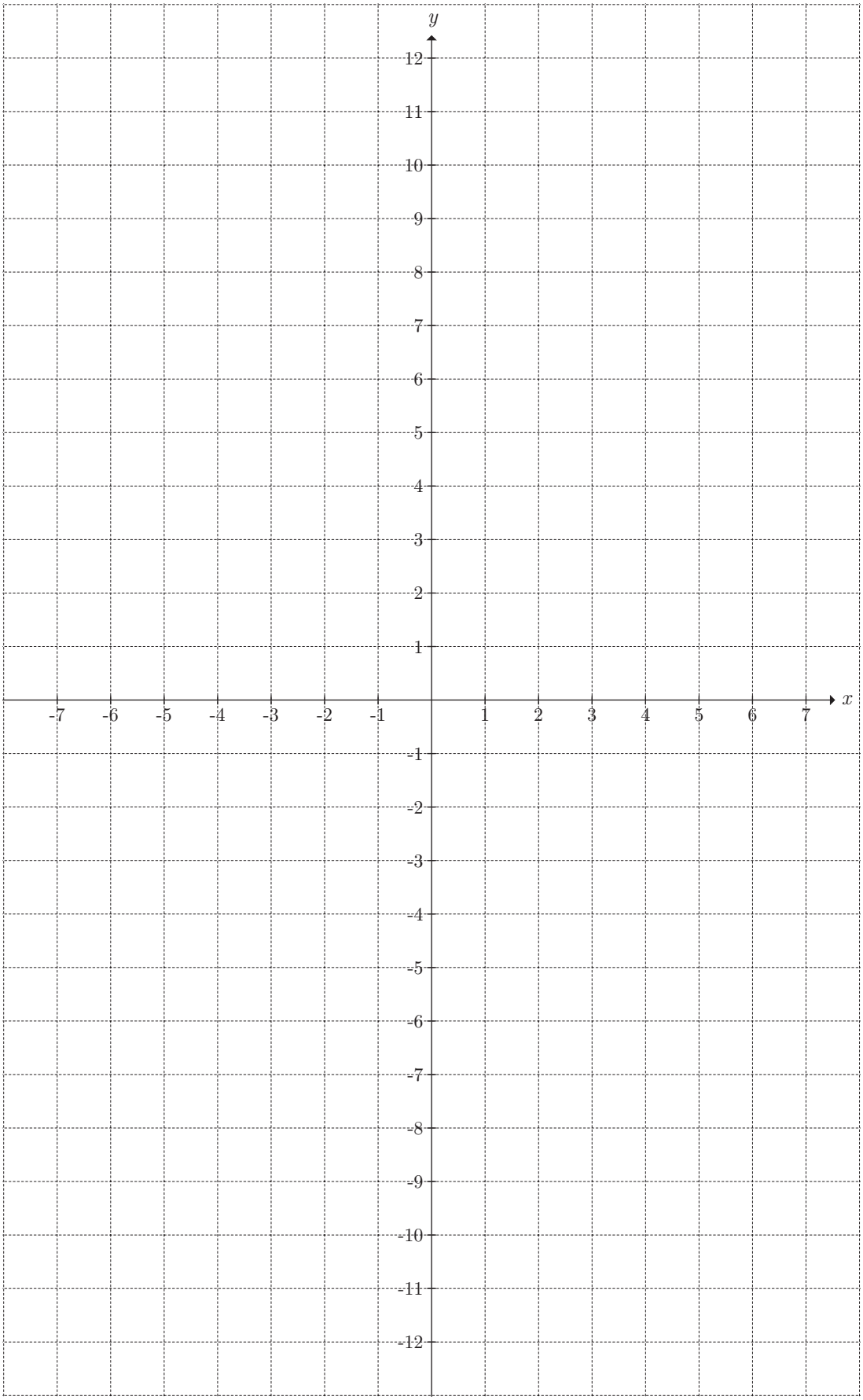
17. Sketch the graphs of the following functions in the same coordinate system.

$y = x^{-1}$, $y = x^{-2}$, $y = x^{-3}$, $y = x^{-4}$



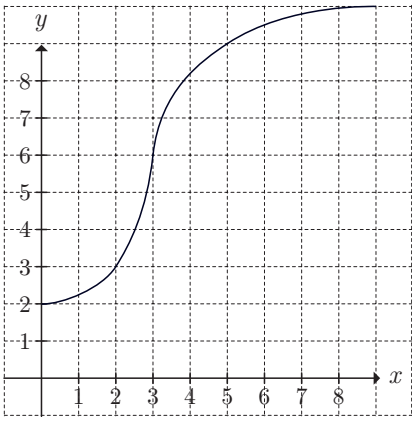
18. Sketch the graphs of the following functions in the same coordinate system.

$y = x^{-1}$, $y = x^{-2}$, $y = x^{-5}$, $y = x^{-6}$

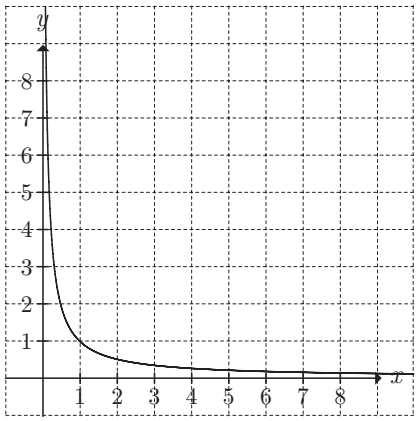


39. Sketch the graph of the inverse function for the given function.

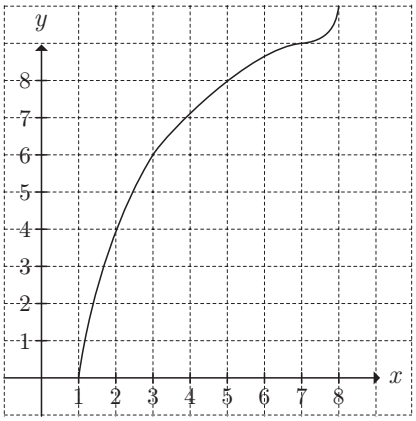
a)



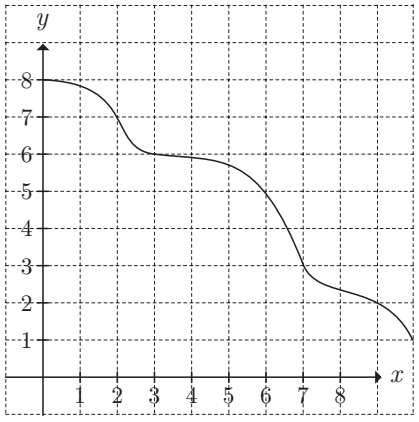
b)



c)

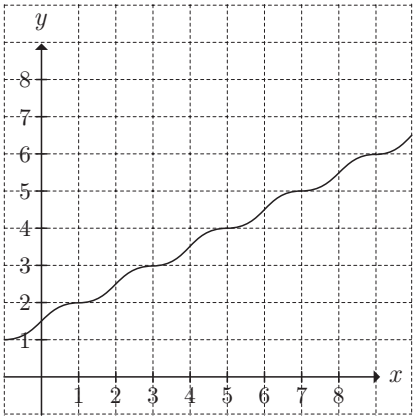


d)

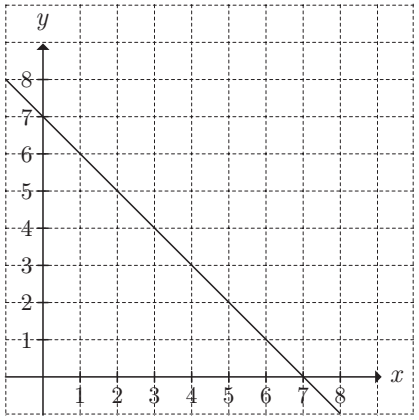


40. Sketch the graph of the inverse function for the given function.

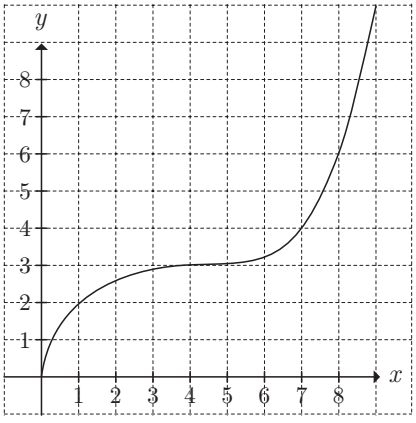
a)



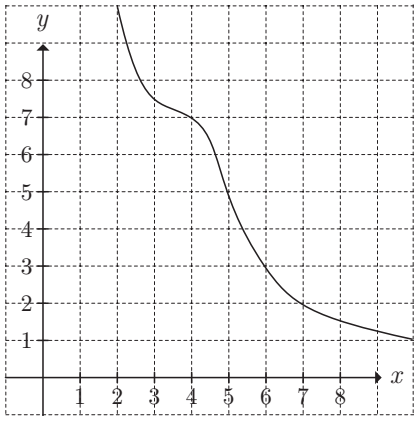
b)



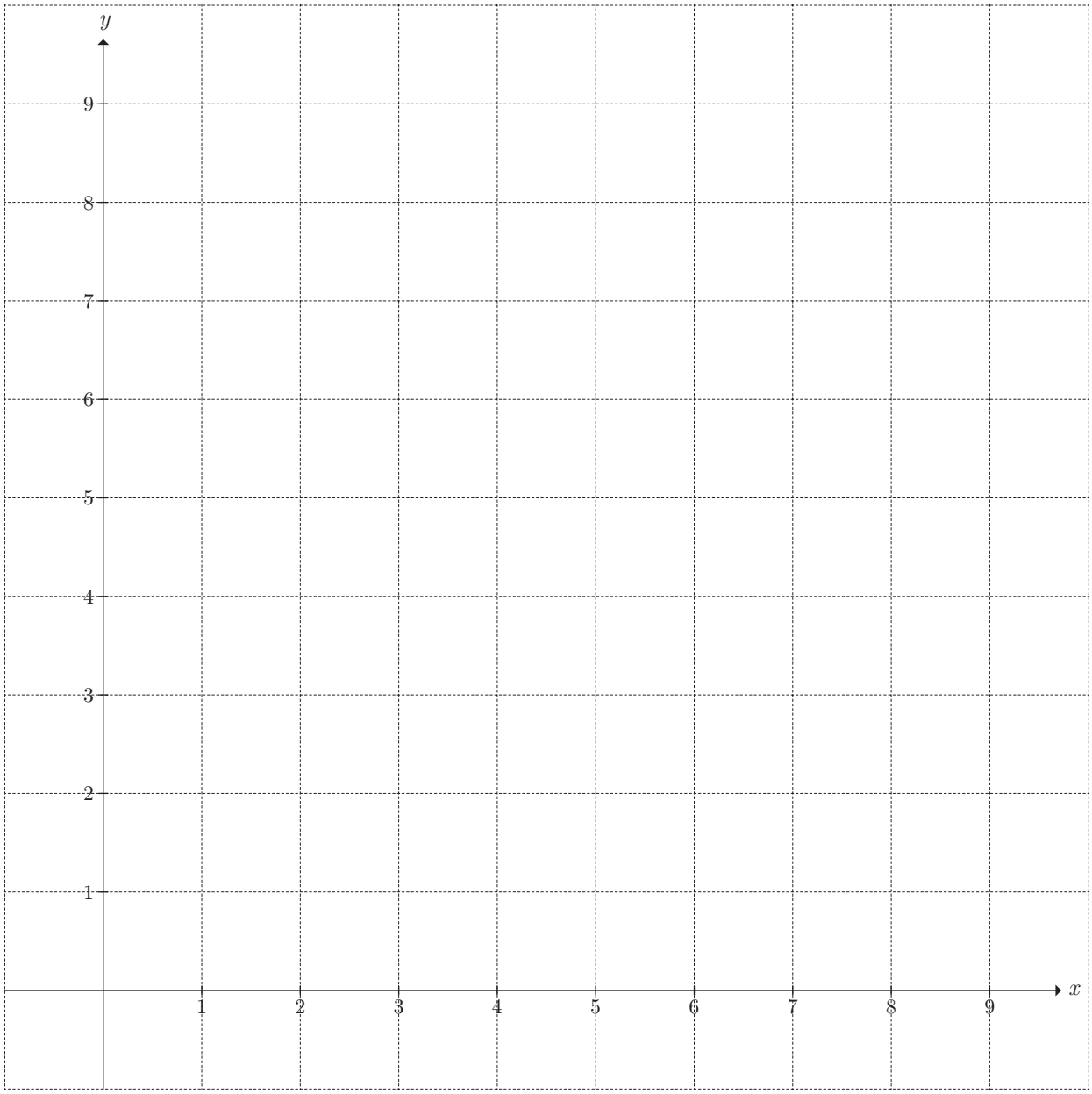
c)



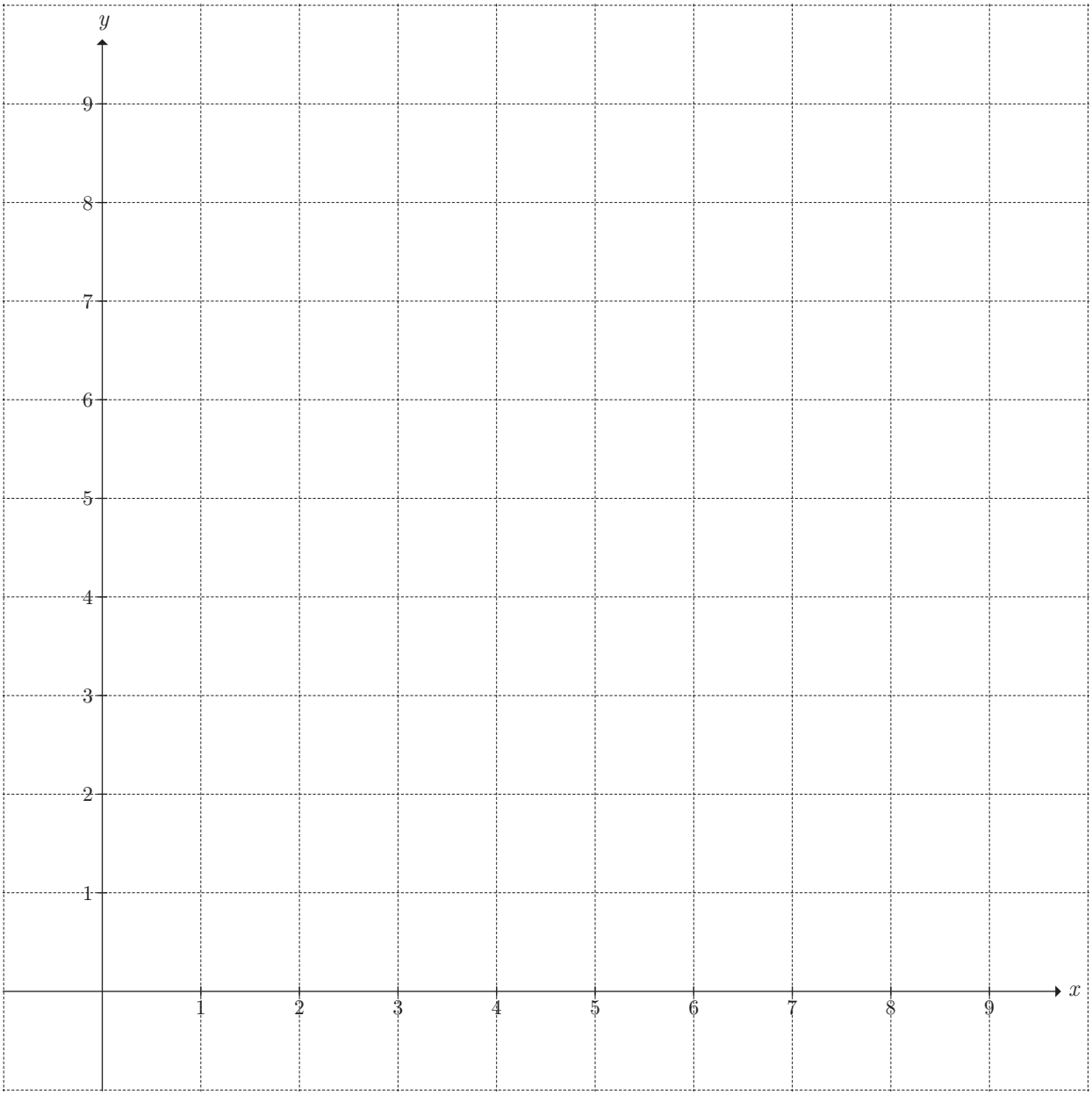
d)



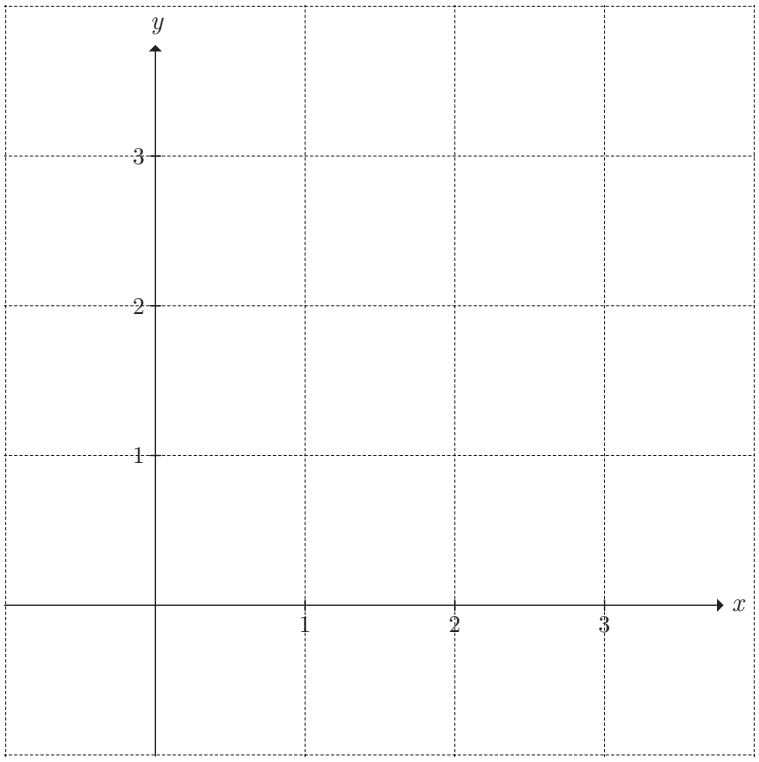
45. In the same coordinate system for $\mathbb{X} = \mathbb{R}_0^+$ draw the graphs of the functions $y = x^2$ and $y = \sqrt{x}$



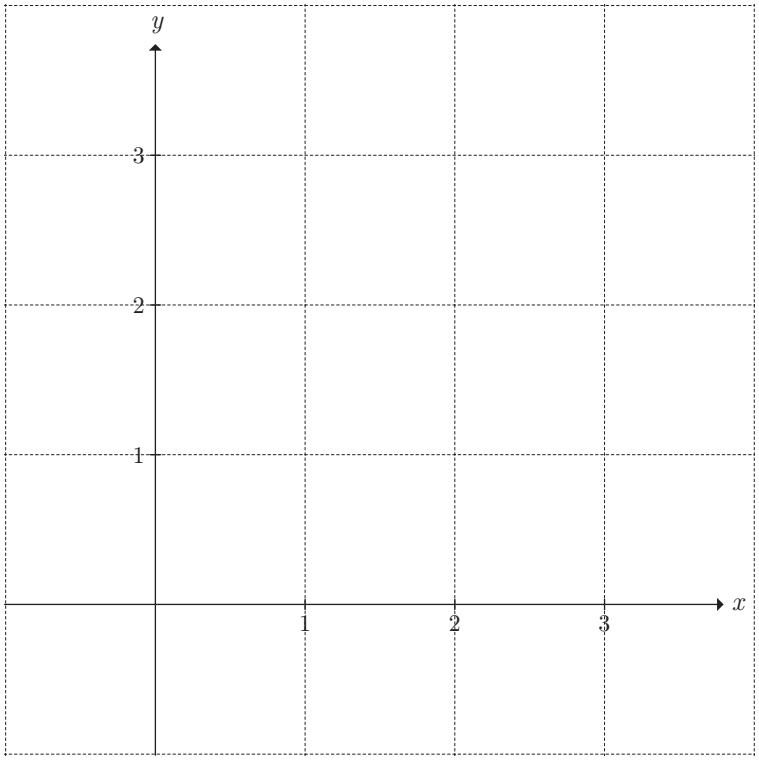
46. In the same coordinate system for $\mathbb{X} = \mathbb{R}_0^+$ draw the graphs of the functions $y = x^3$ and $y = \sqrt[3]{x}$



51. In the first quadrant of the coordinate system, mark the entire area in which the graphs of the functions $y = x^a$ lie if $0 < a < 1$

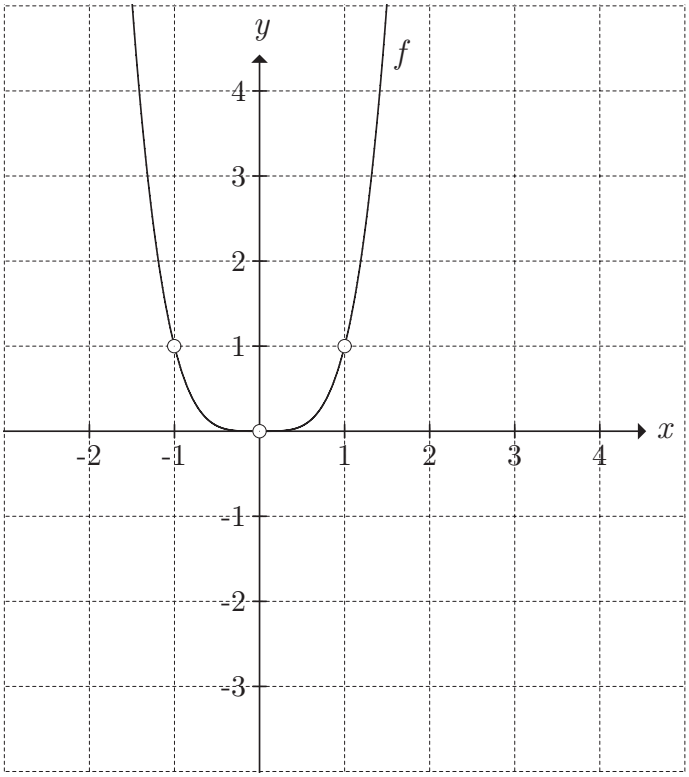


52. In the first quadrant of the coordinate system, mark the entire area in which the graphs of the functions$y = x^a$ lie if $a > 1$.

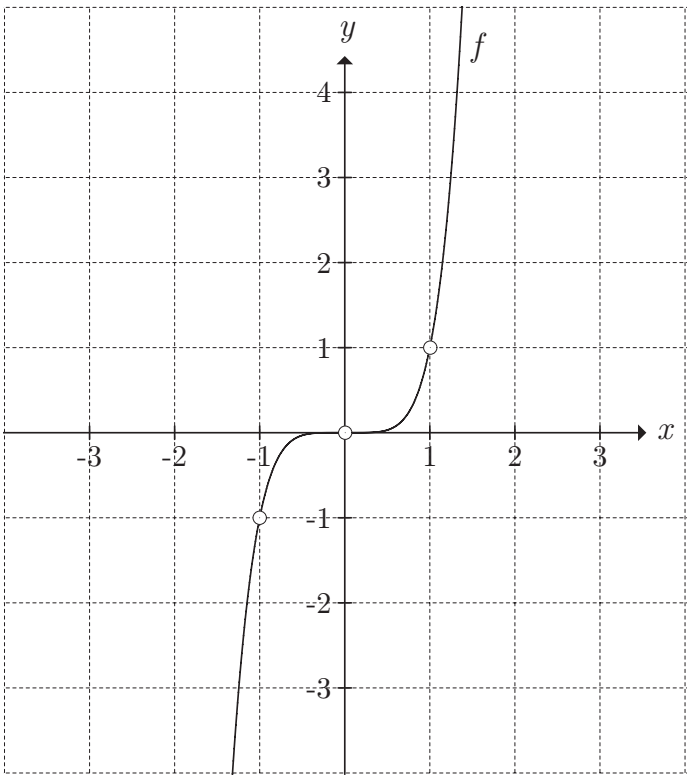


Given the function f draw the graphs of the functions f_1 to f_4.

67. $f: y = x^4$ $f_1: y = x^4 - 3$ $f_2: y = -x^4$ $f_3: y = (x-3)^4$ $f_4: y = 2x^4$

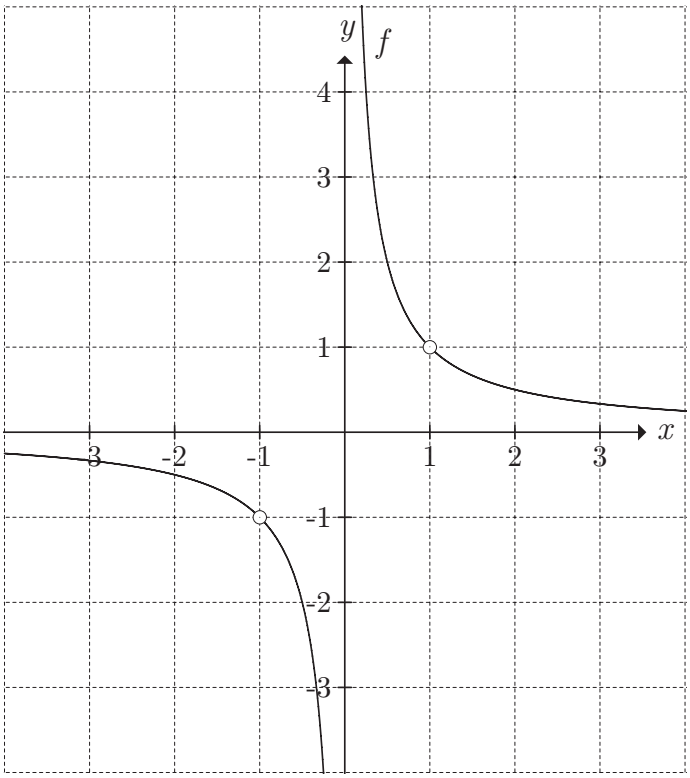


68. $f: y = x^5$ $f_1: y = (x-2)^5$ $f_2: y = x^5 + 2$ $f_3: y = 4x^5$ $f_4: y = -x^5 - 3$

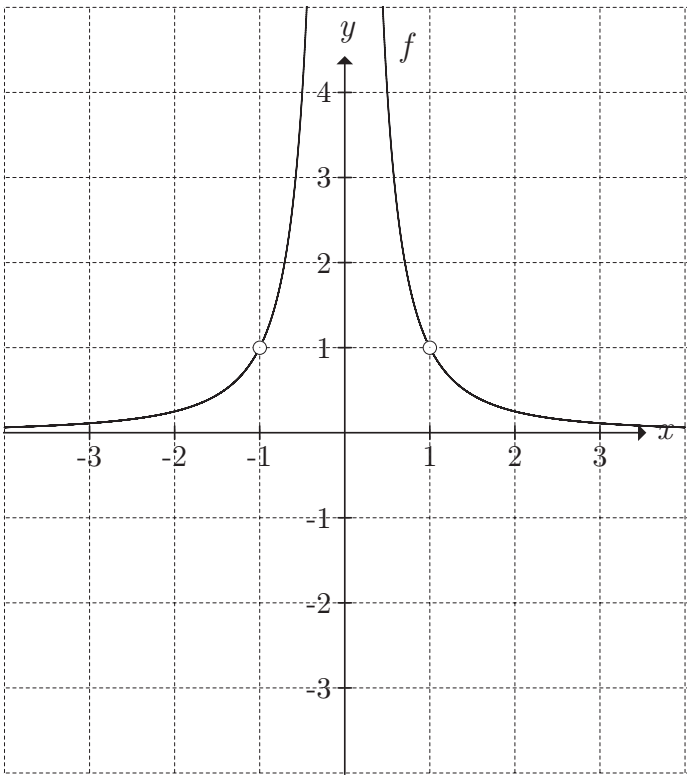


Given the function f draw the graphs of the functions f_1 to f_4.

69. $f: y = x^{-1}$ $f_1: y = -x^{-1}$ $f_2: y = -x^{-1} + 2$ $f_3: y = 3x^{-1}$ $f_4: y = (x+1)^{-1}$

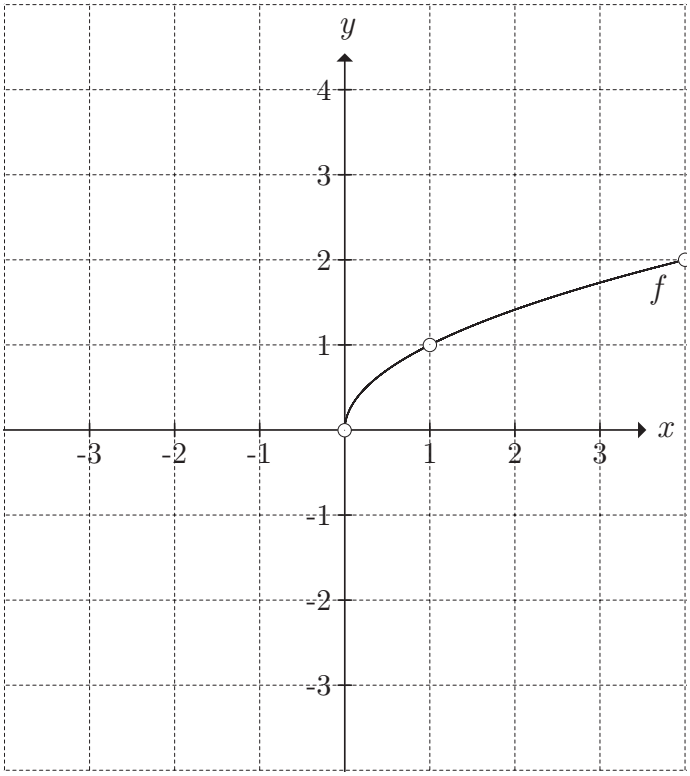


70. $f: y = x^{-2}$ $f_1: y = x^{-2} - 3$ $f_2: y = (x+2)^{-2}$ $f_3: y = -x^{-2}$ $f_4: y = -(x-3)^{-2} + 4$

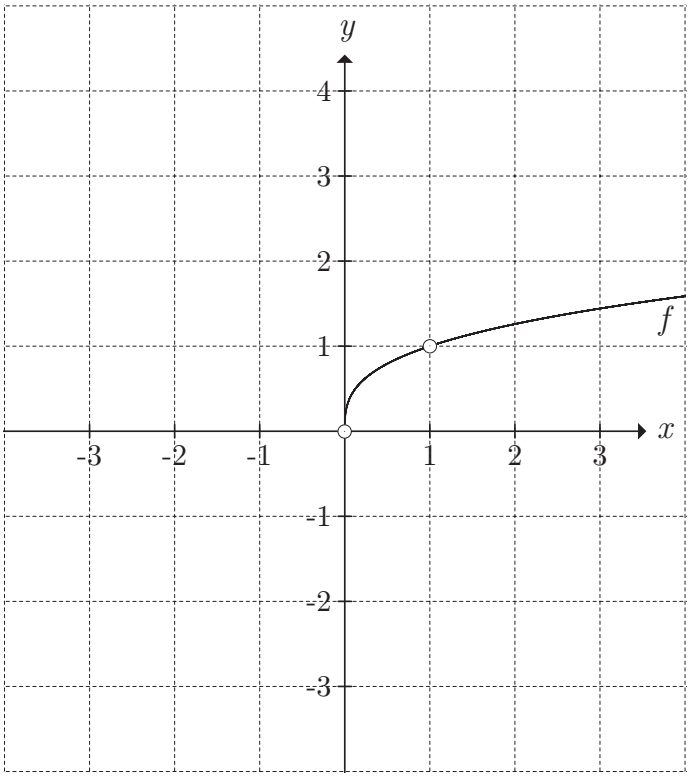


Given the function f draw the graphs of the functions f_1 to f_4.

71. $f: y = \sqrt{x}$ $f_1: y = \sqrt{x} - 3$ $f_2: y = 2\sqrt{x}$ $f_3: y = \sqrt{-x}$ $f_4: y = \sqrt{x+3}$

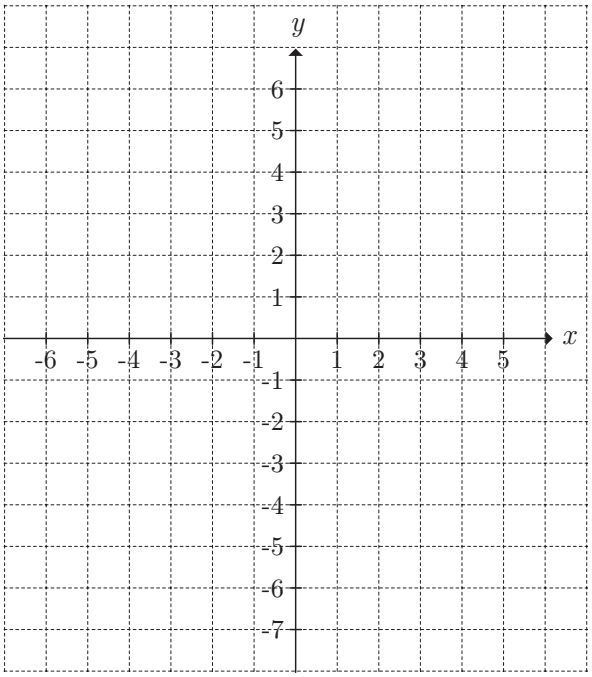


72. $f: y = \sqrt[3]{x}$ $f_1: y = \sqrt[3]{x+2}$ $f_2: y = 3\sqrt[3]{x}$ $f_3: y = -\sqrt[3]{-x}$ $f_4: y = \sqrt[3]{x-1} - 3$

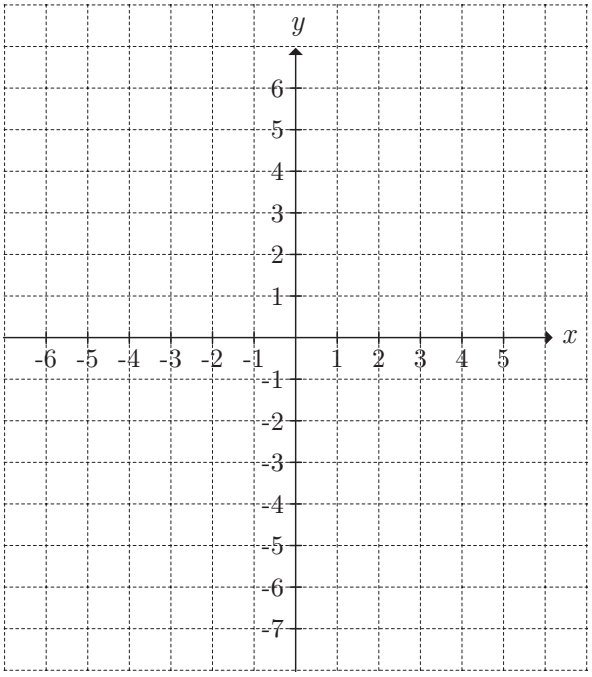


Draw the graphs of the following power functions with natural exponents.

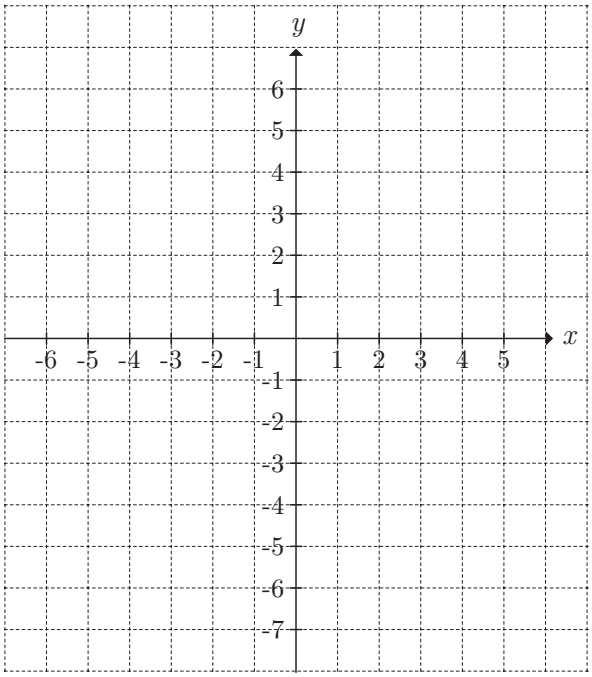
79. a) $f: y = -\frac{1}{2}(x-3)^4 + 7$



b) $g: y = (x+4)^3 - 6$

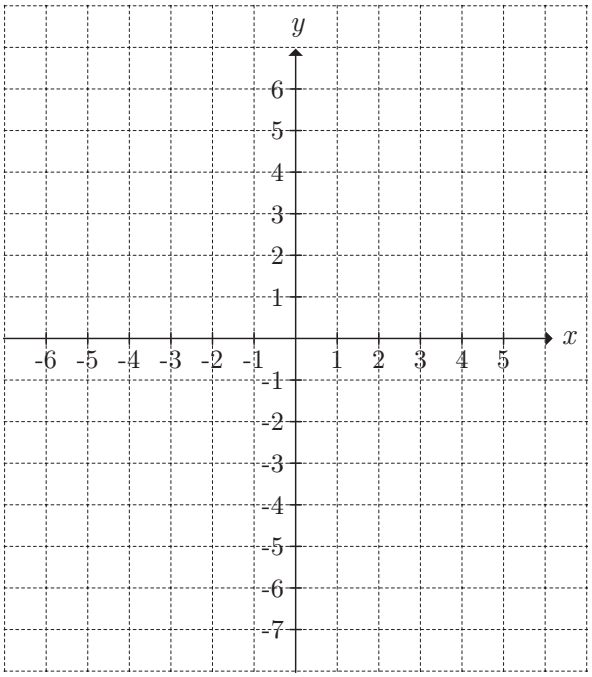


c) $h: y = -\frac{1}{4}(x+3)^5 + 5$

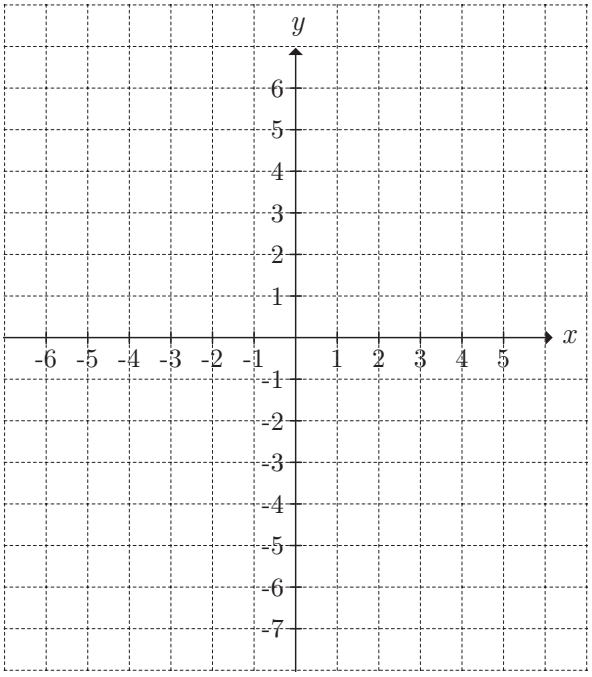


Draw the graphs of the following power functions with natural exponents.

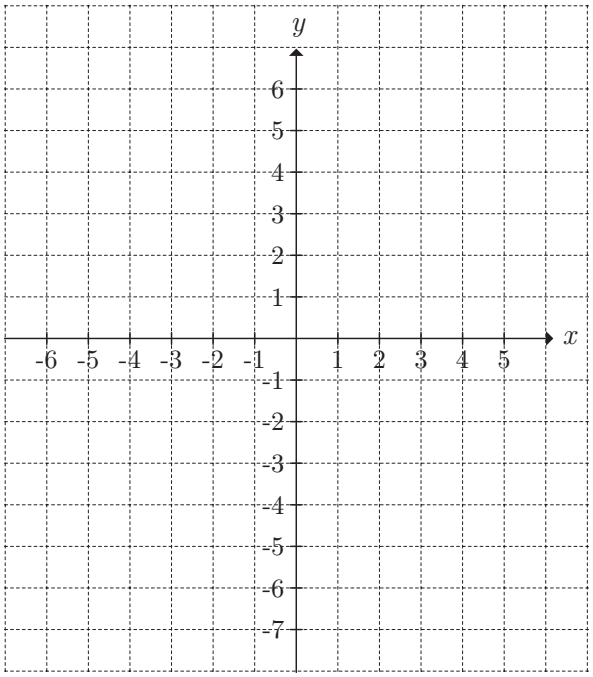
80. a) $f: y = \frac{1}{8}(x+3)^6 - 4$



b) $g: y = -\frac{3}{2}(x-5)^3 - 6$

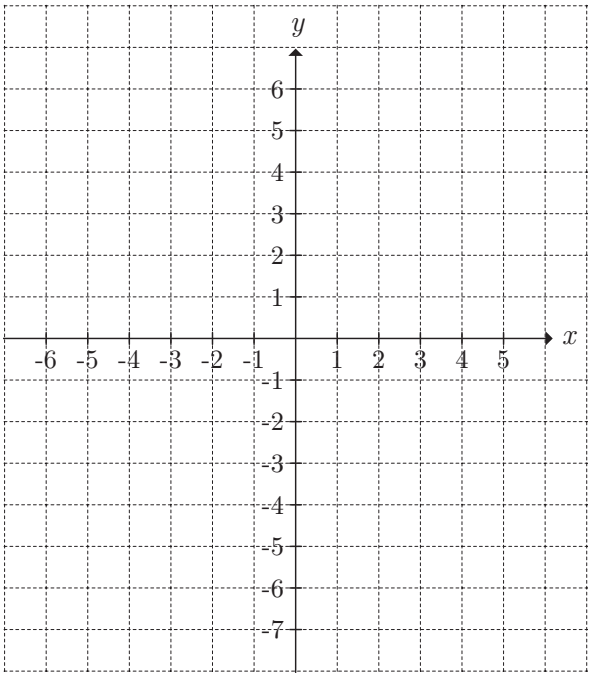


c) $h: y = -2(x-5)^2 + 4$

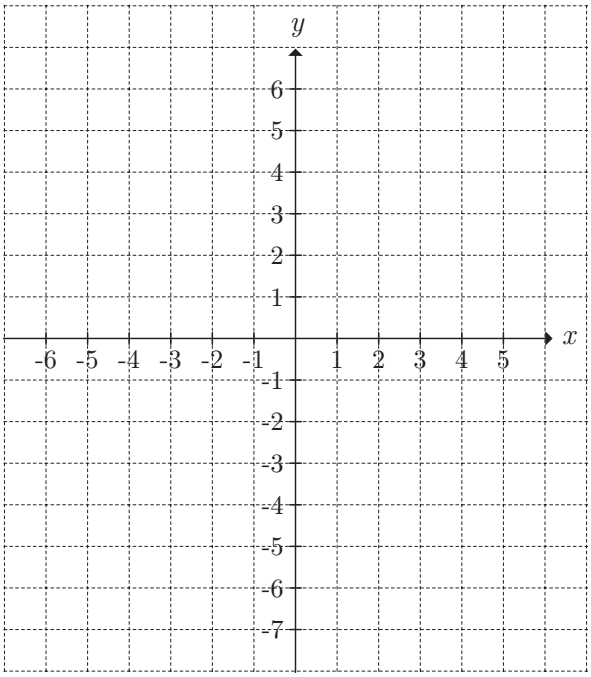


Draw the graphs of the following power functions with integer exponents and their asymptotes.

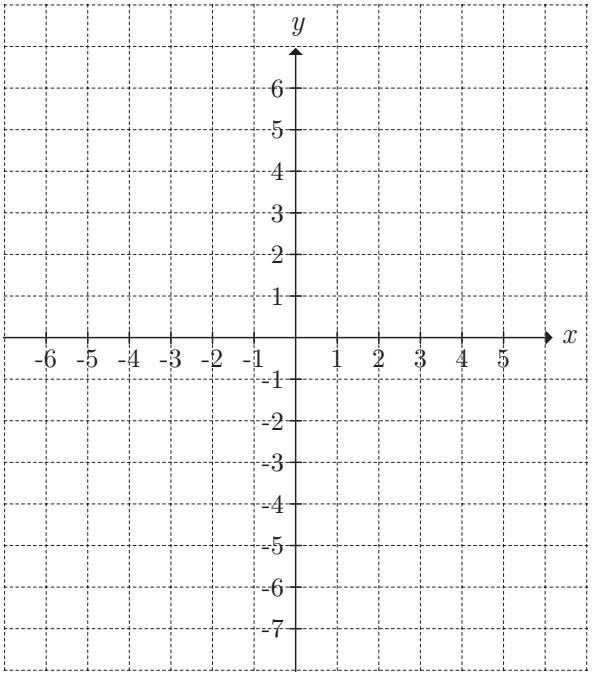
81. a) $f: y = 2(x+3)^{-3} + 2$



c) $h: y = -3(x-1)^{-1} - 4$

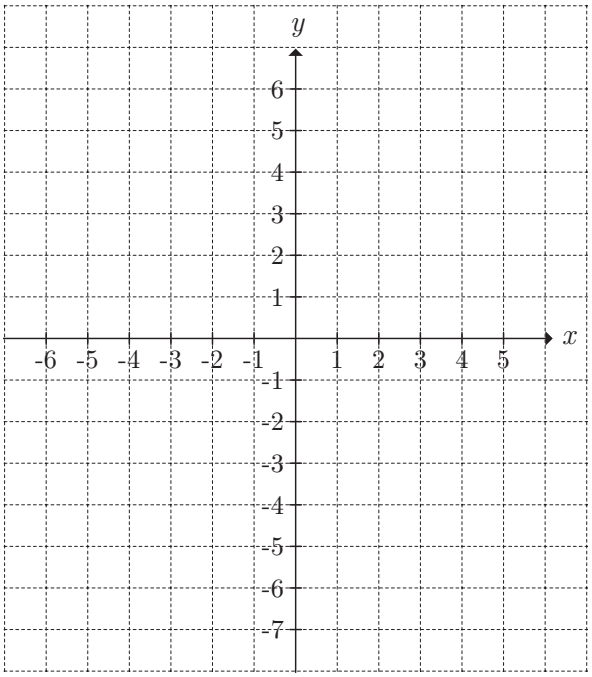


b) $g: y = -(x-4)^{-2} + 5$

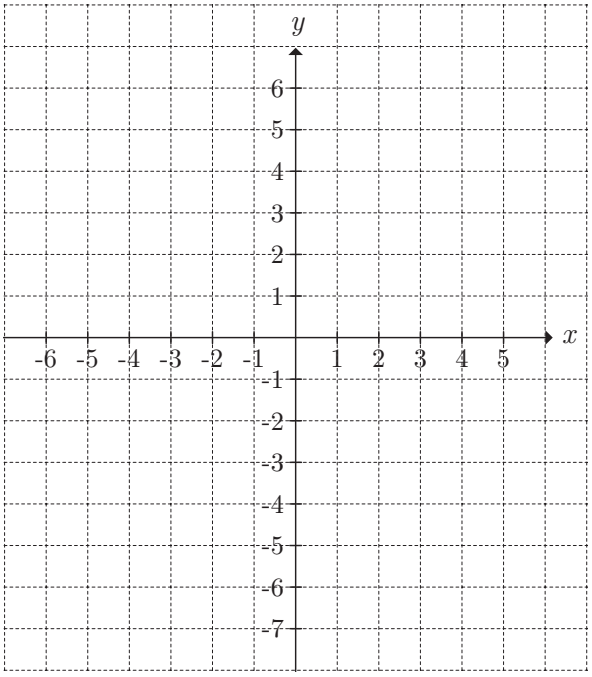


Draw the graphs of the following power functions with integer exponents and their asymptotes.

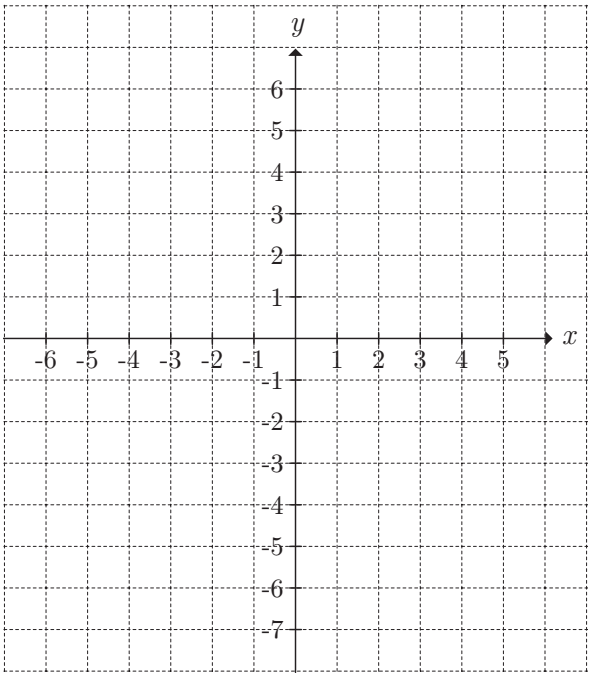
82. a) $f: y = 3(x+5)^{-4} - 6$



b) $g: y = -(x-4)^{-3} + 4$

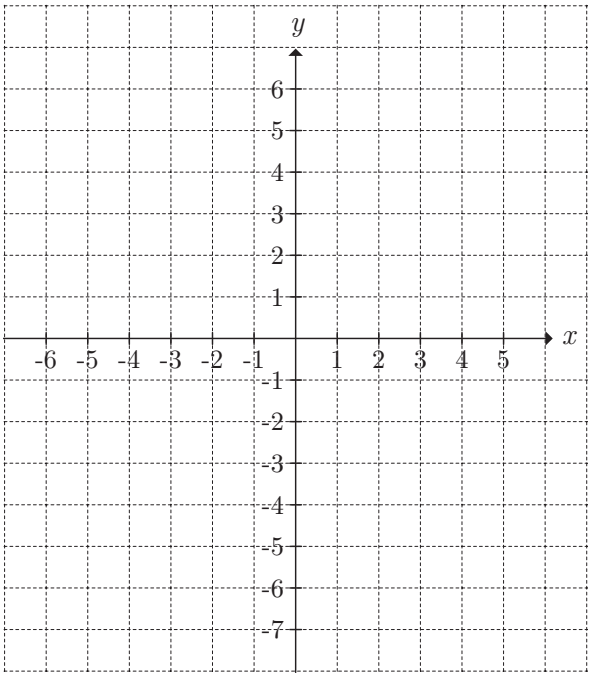


c) $h: y = 2(x+1)^{-1} + 1$

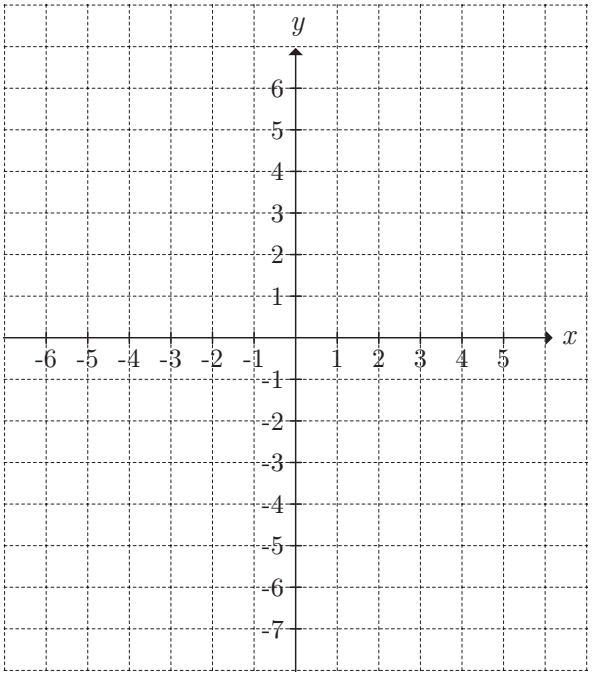


Draw the graphs of the following power functions with rational exponents.

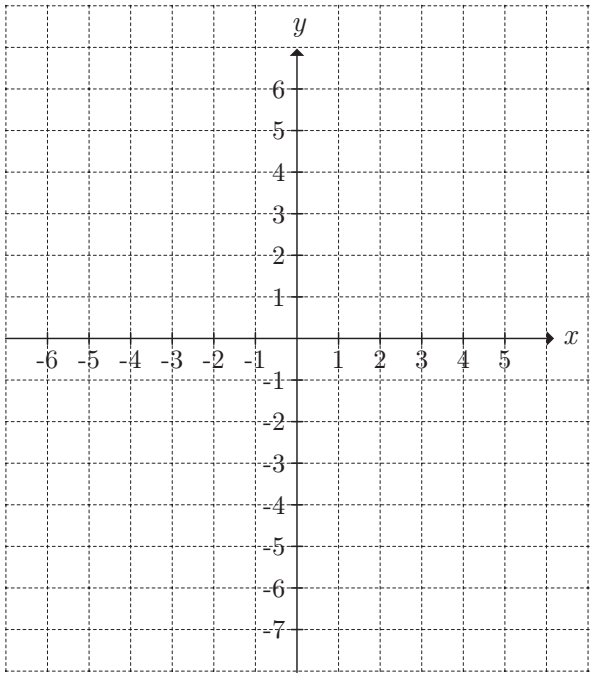
83. a) $f: y = 2(x+4)^{\frac{1}{2}} - 5$



b) $g: y = -(x+5)^{\frac{1}{4}} + 6$

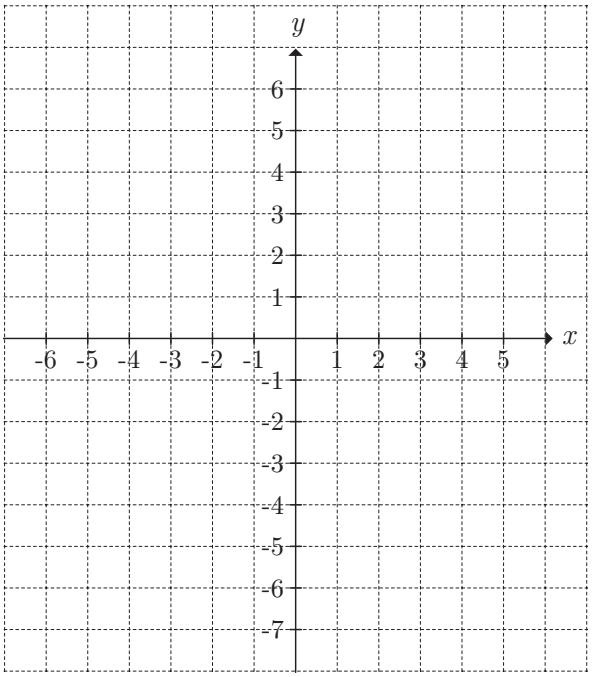


c) $h: y = (6-x)^{\frac{1}{3}} - 3$

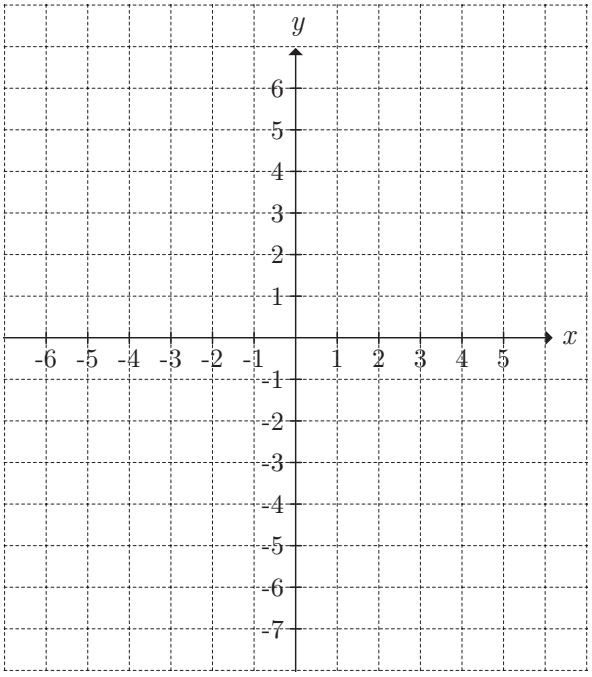


Draw the graphs of the following power functions with rational exponents.

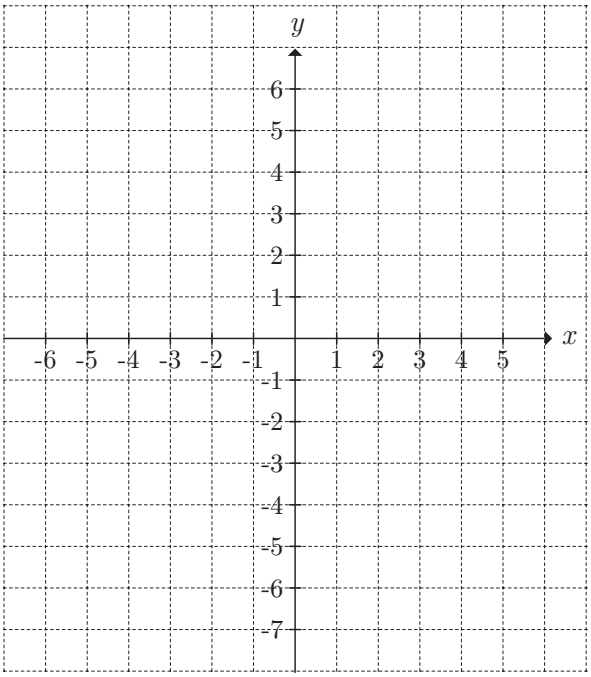
84. a) $f: y = \frac{1}{2}(x+2)^{\frac{1}{3}} - 3$



b) $g: y = -2(x+6)^{\frac{1}{4}} + 5$

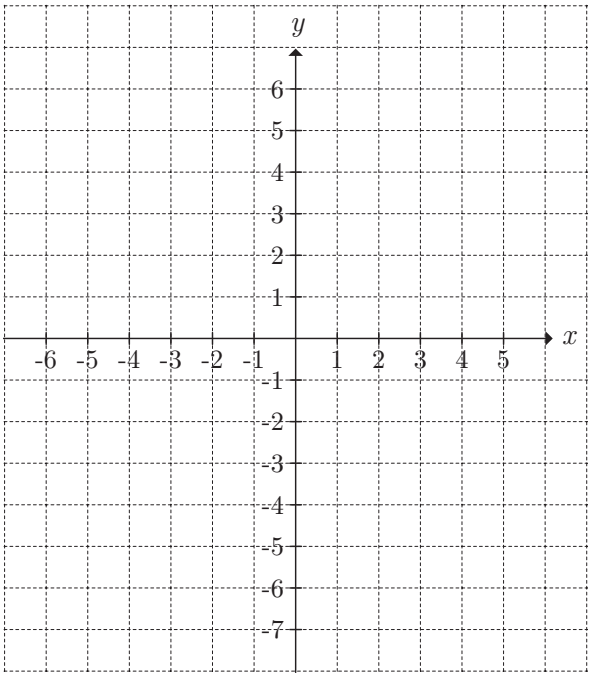


c) $h: y = -(4-x)^{\frac{1}{2}} - 4$

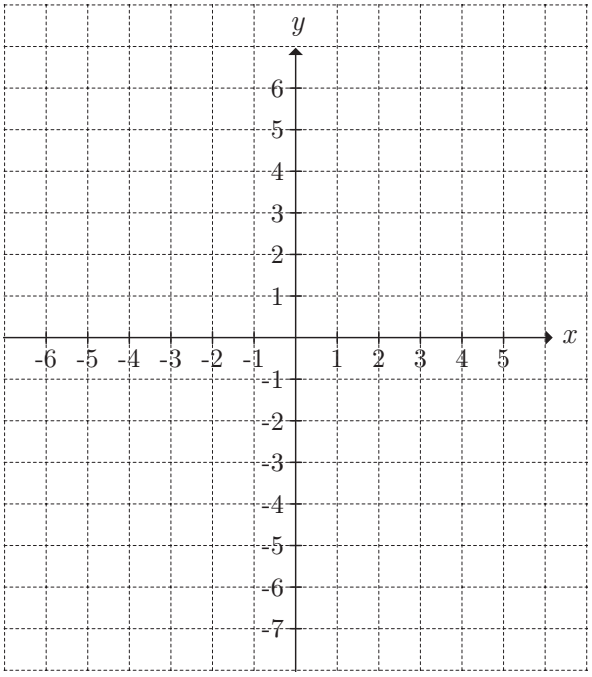


Draw the graphs of the following power functions, with asymptotes if necessary.

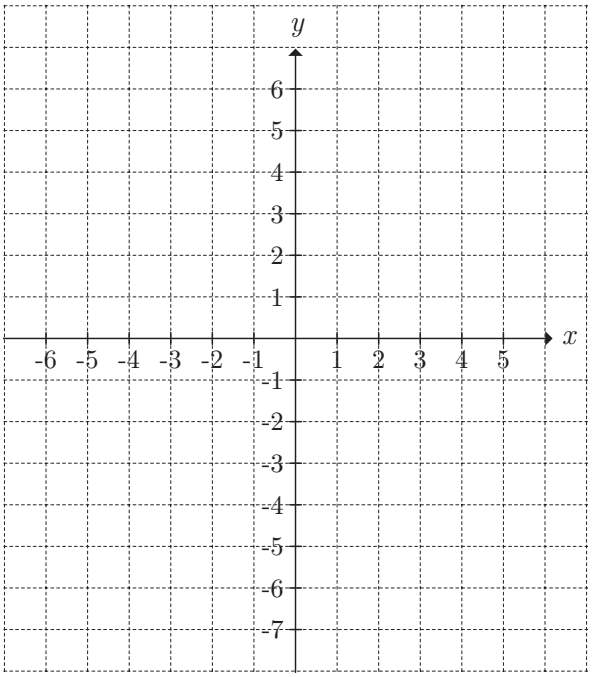
85. a) $f: y = 2(x+4)^{-2} - 6$



b) $g: y = -\frac{1}{2}(x-4)^3 - 3$

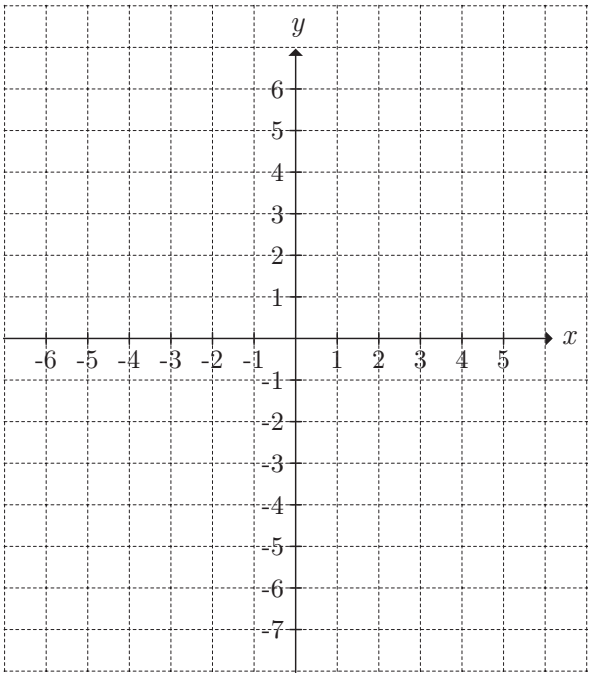


c) $h: y = 3(x+6)^{\frac{1}{3}} + 1$

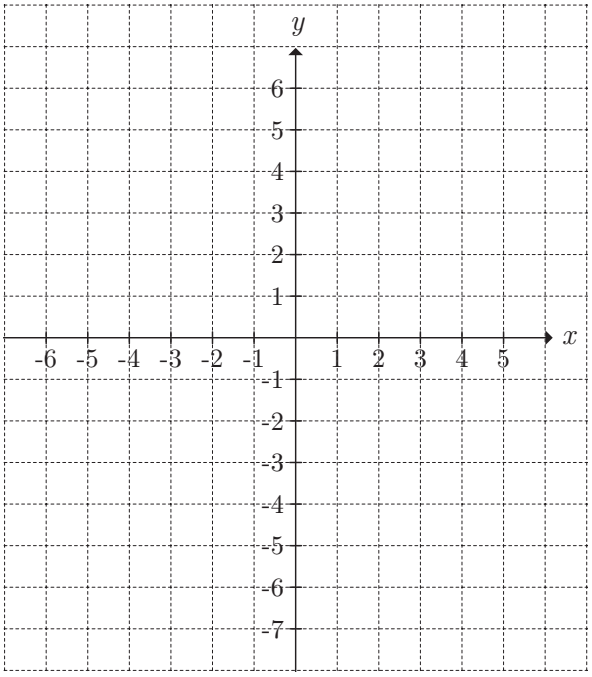


Draw the graphs of the following power functions, with asymptotes if necessary.

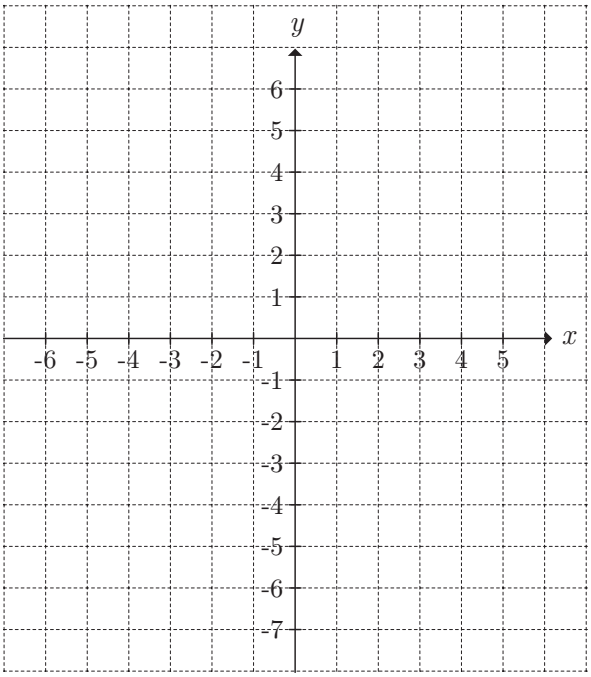
86. a) $f: y = -\frac{1}{4}(x-3)^4 + 6$



b) $g: y = -2(x+6)^{\frac{1}{2}} + 5$

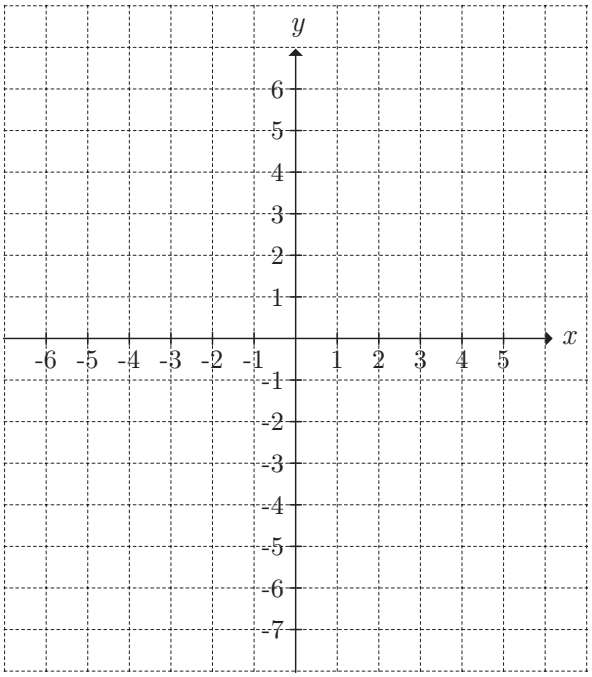


c) $h: y = 3(x+4)^{-3} - 4$

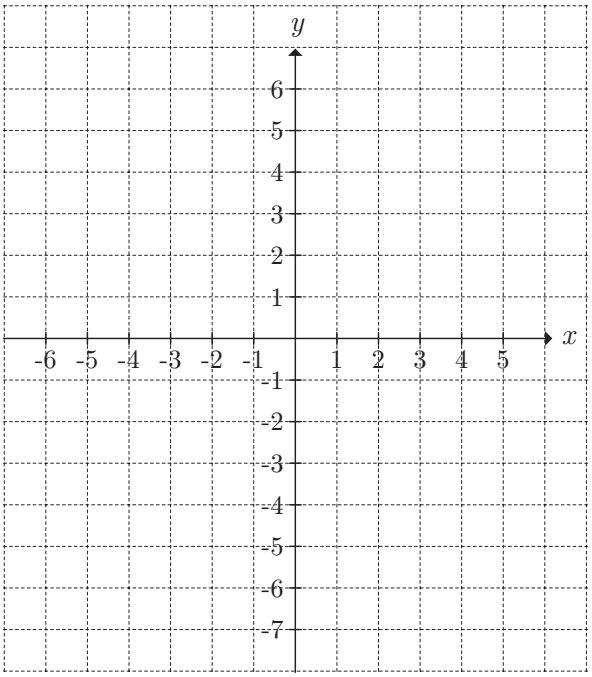


Draw the graphs of the following power functions, with asymptotes if necessary.

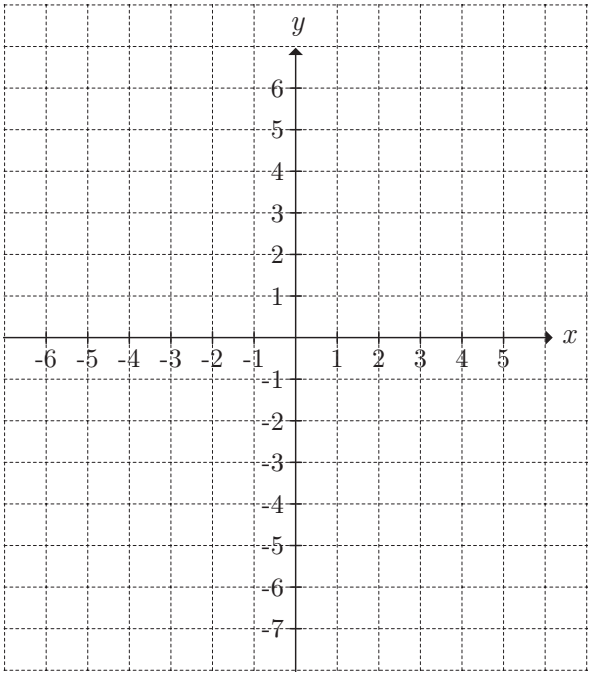
87. a) $f: y = -2(x-2)^{3.5} + 5$



c) $h: y = (x+3)^{0.\overline{6}} - 2$

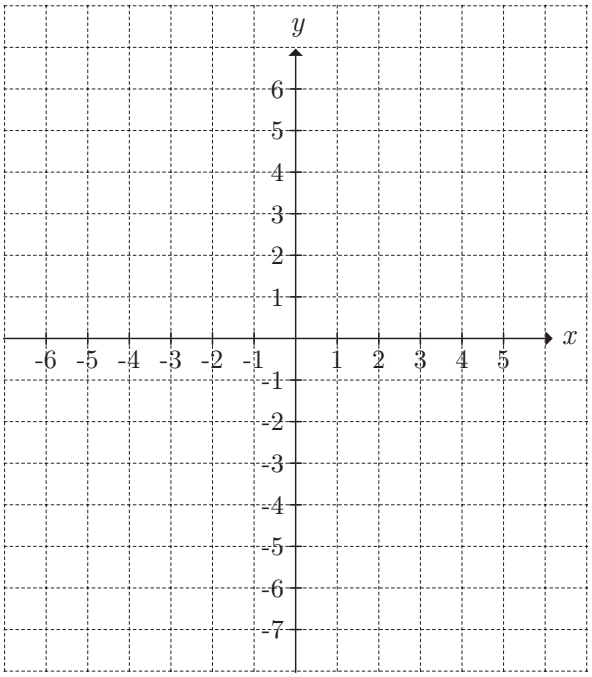


b) $g: y = (x+5)^{-2.5} - 5$

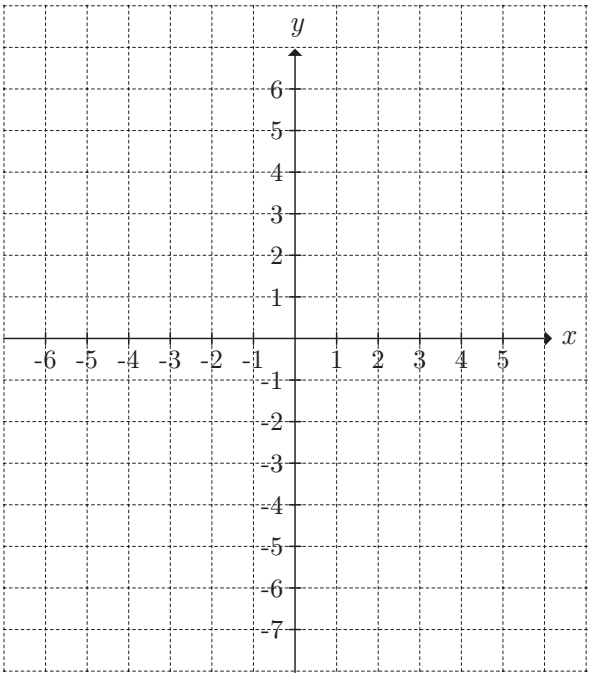


Draw the graphs of the following power functions, with asymptotes if necessary.

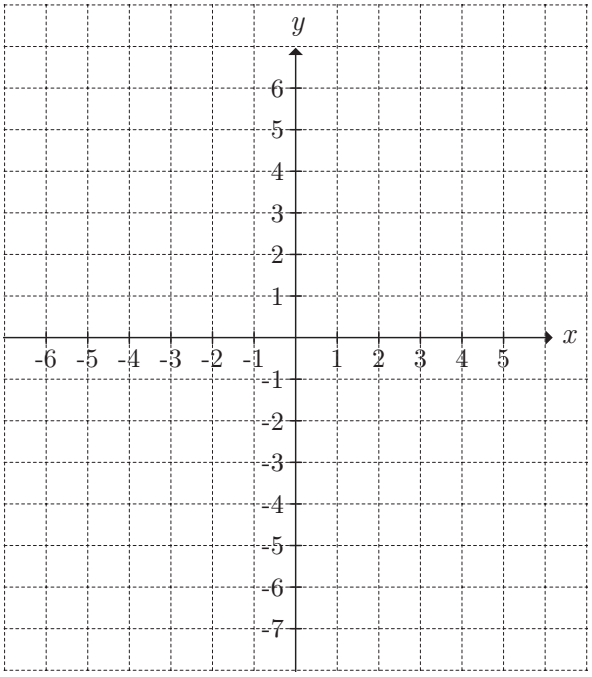
88. a) $f: y = 2(x+5)^{-3.5} - 6$



c) $h: y = -(x+2)^{0.75} + 3$

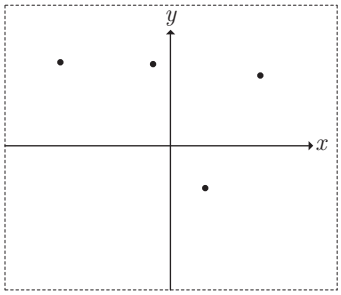


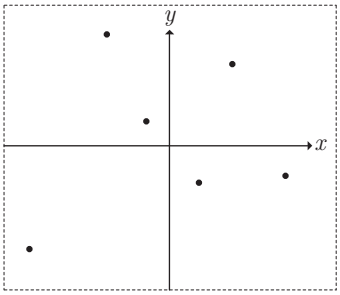
b) $g: y = (x+1)^{1.5} - 5$

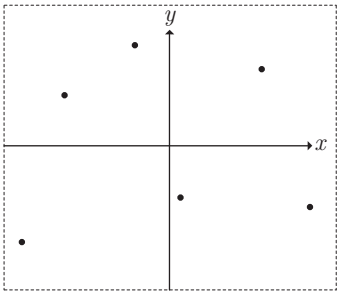


Data was collected using a measurement and then transferred to a coordinate system. Now we are looking for a polynomial function with the smallest possible degree whose graph (approximately) passes through these points. Sketch a possible curve and specify the degree of the function.

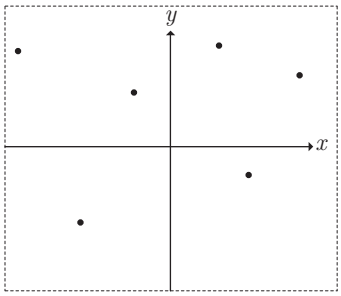
123.

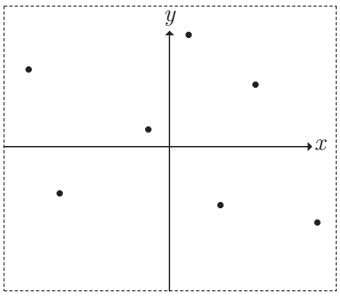


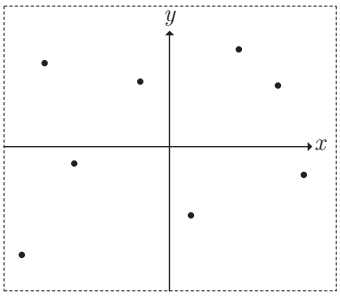




124.





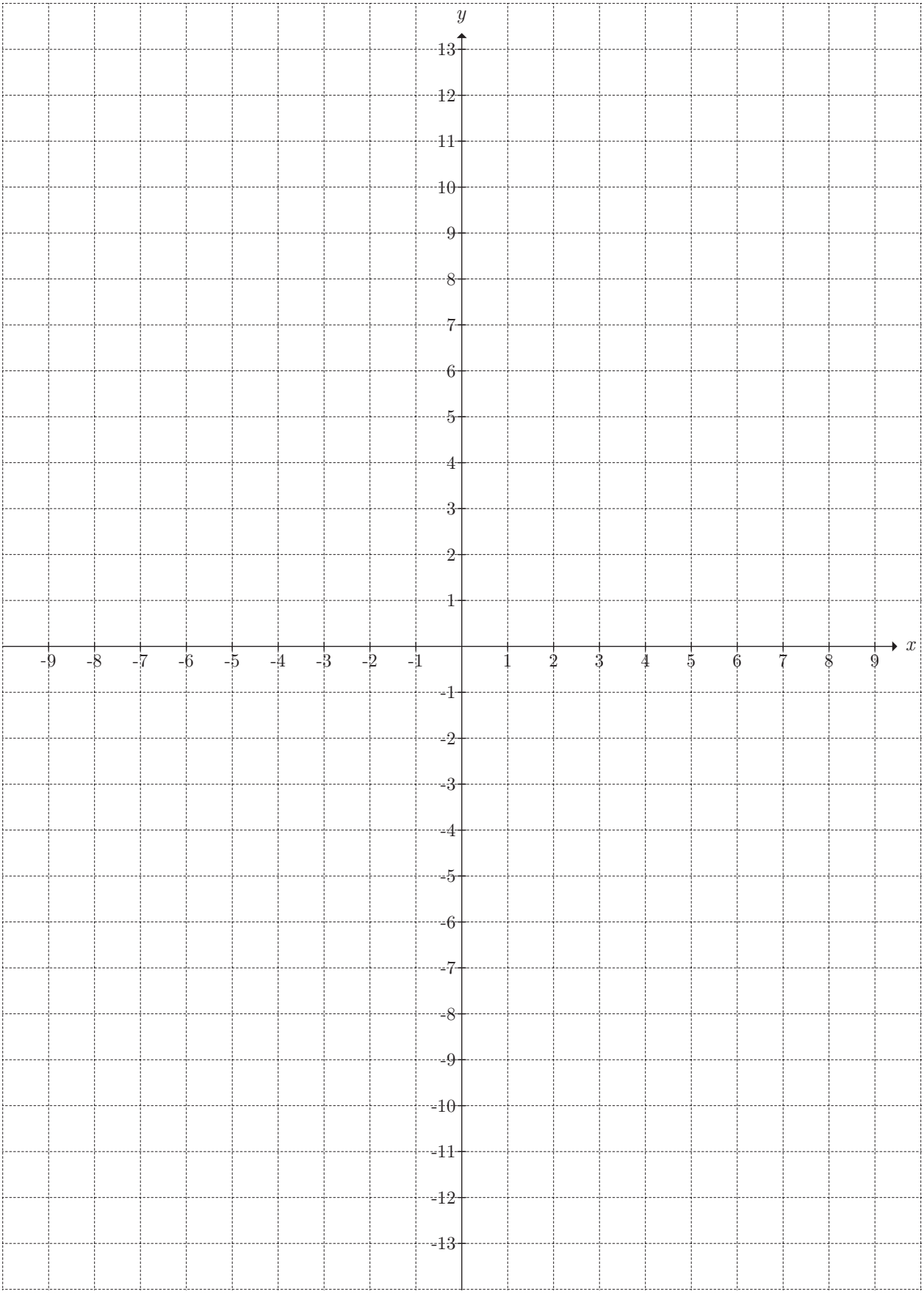


10 Exponential and Logarithmic Functions

9. Draw the graphs of the functions in the same coordinate system.

a) $f(x) = 2^x$ c) $h(x) = \left(\frac{1}{2}\right)^x$

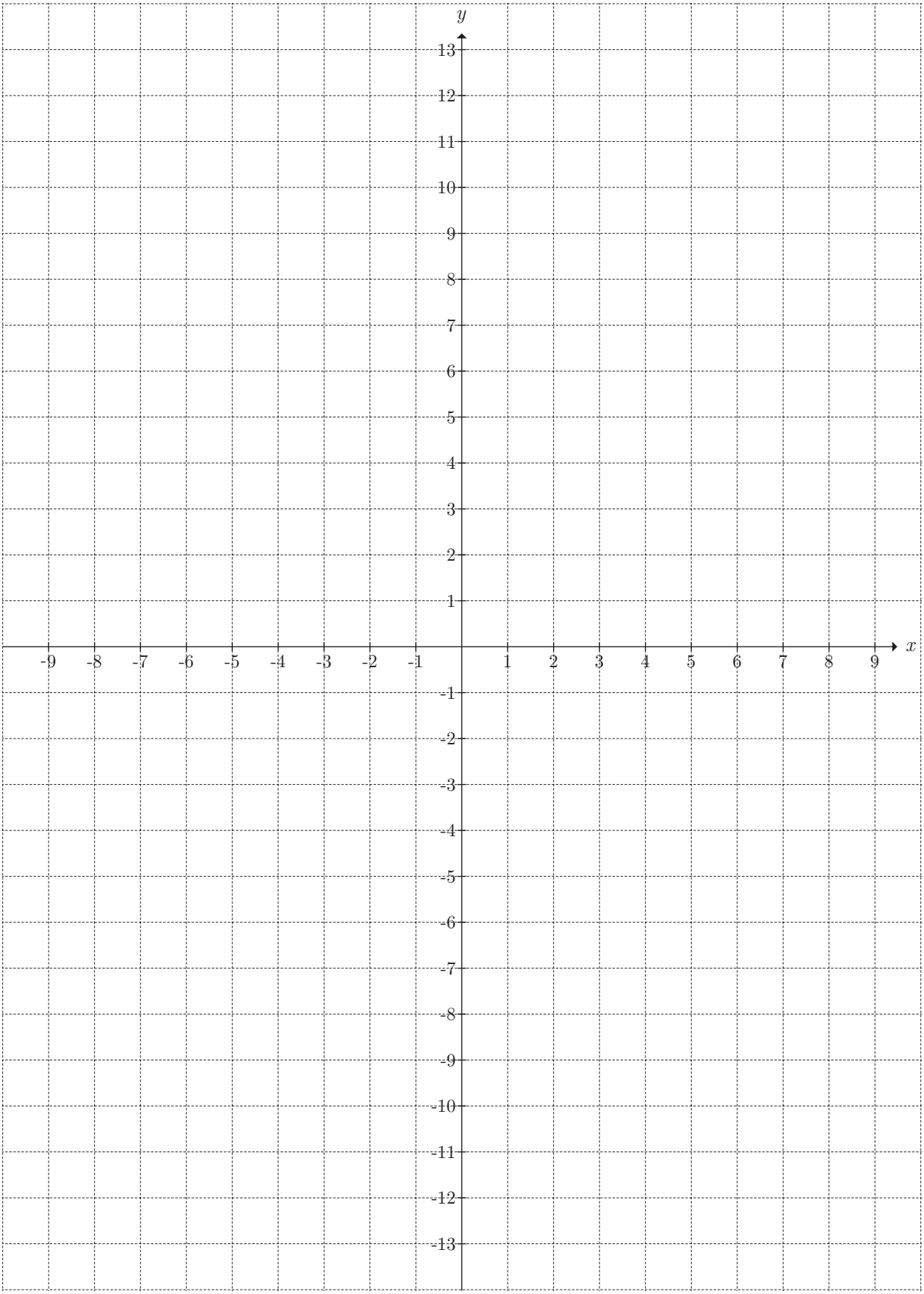
b) $g(x) = 3^x$ d) $k(x) = \left(\frac{1}{3}\right)^x$



10. Draw the graphs of the functions in the same coordinate system.

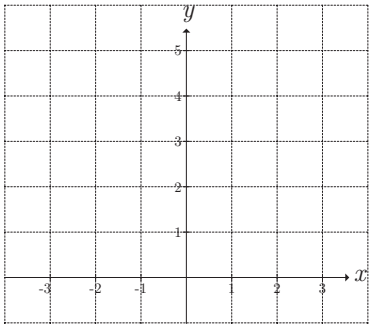
a) $f(x) = 4^x$ c) $h(x) = \left(\frac{1}{4}\right)^x$

b) $g(x) = 5^x$ d) $k(x) = \left(\frac{1}{5}\right)^x$

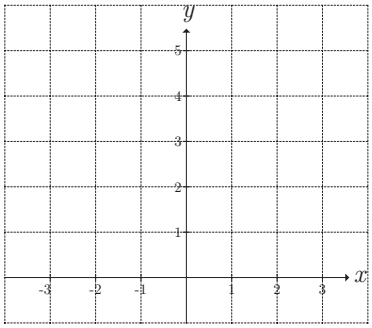


In which region do the graphs of $f(x) = a^x$ lie if a must satisfy the given condition?

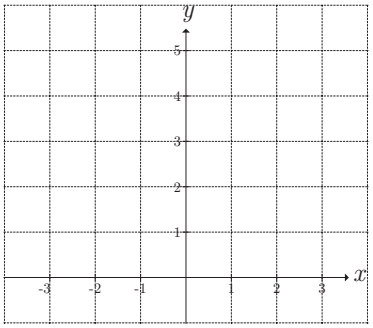
13. a) $1 < a < 3$



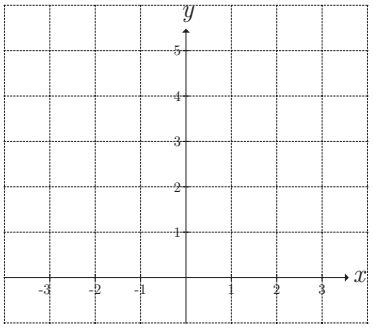
b) $0 < a < 1$



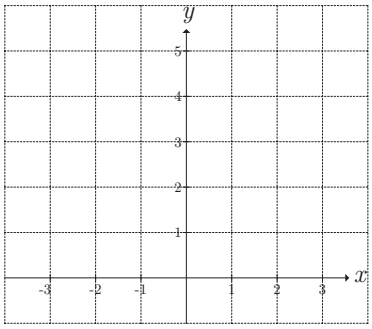
c) $\frac{1}{2} < a < 2$



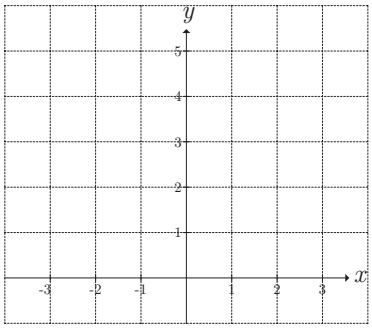
d) $0 < a < \frac{1}{2} \quad \vee \quad a > 2$



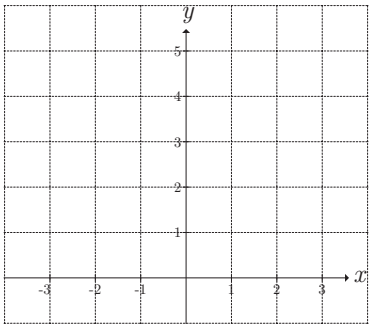
14. a) $0 < a < \frac{1}{2}$



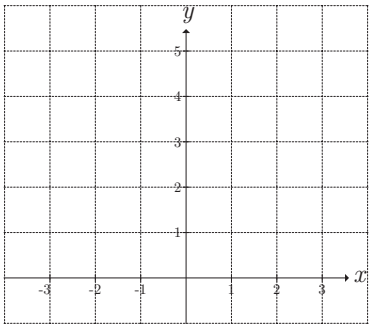
b) $1 < a < 5$



c) $\frac{1}{5} < a < \frac{1}{2}$

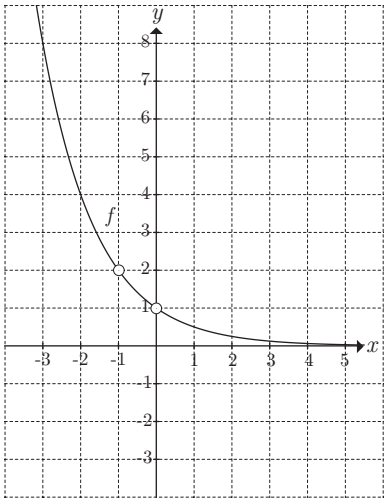


d) $0 < a < \frac{1}{4} \quad \vee \quad a > \frac{1}{2}$

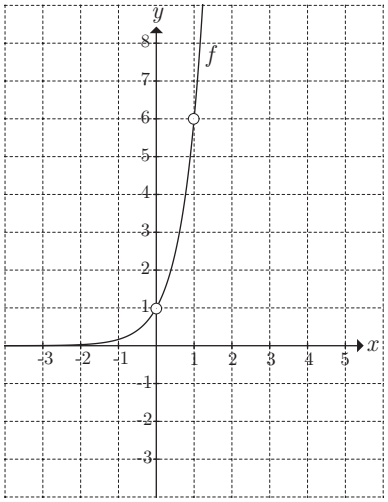


The graph of f can be seen in the sketch. Sketch the graphs of the other functions.

55. $f(x) = \left(\frac{1}{2}\right)^x$
$g(x) = 3 \cdot f(x), \quad i(x) = f(2x),$
$h(x) = f(x+2), \quad j(x) = f(x) + 3$

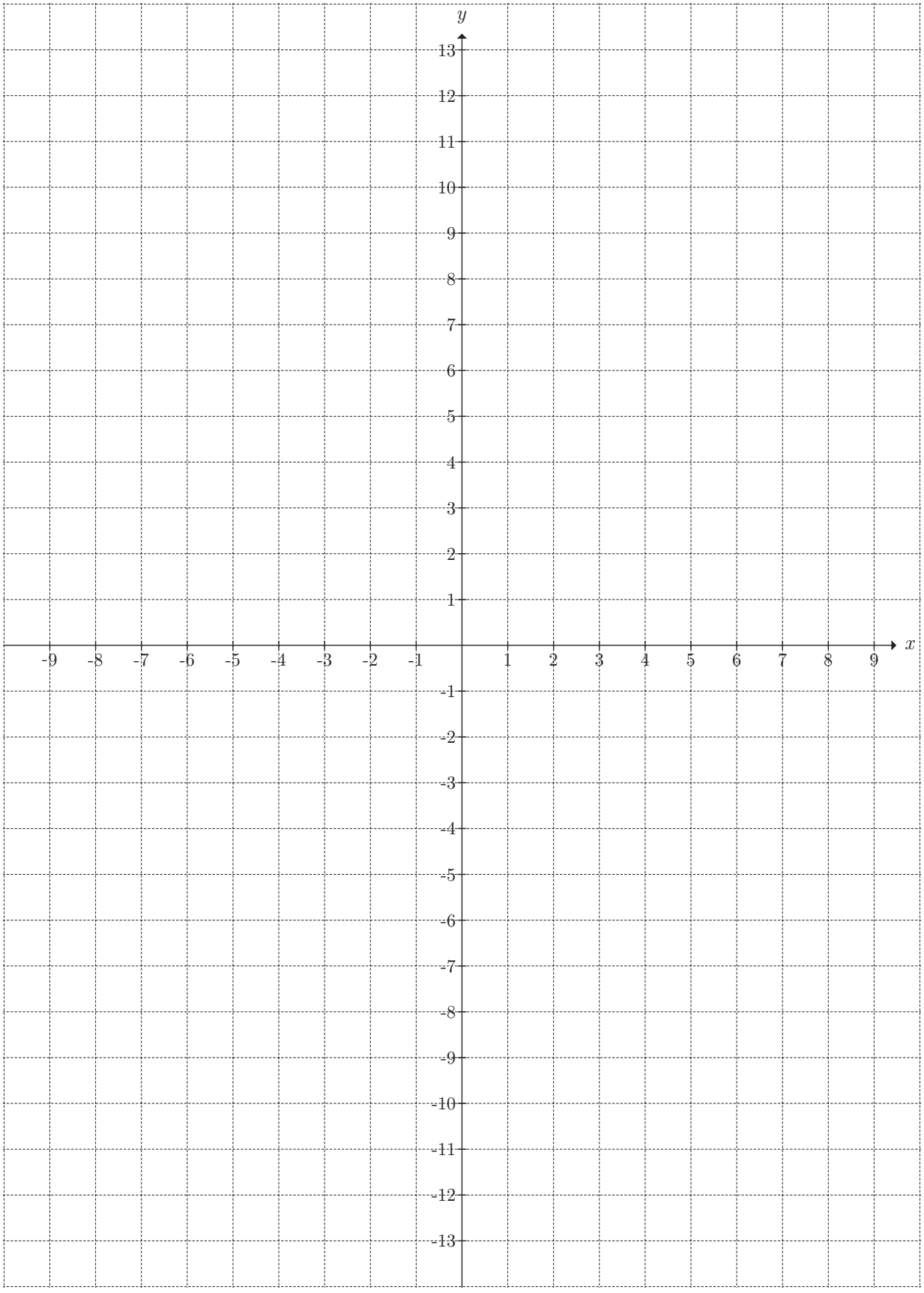


56. $f(x) = 6^x$
$g(x) = f(x-3), \quad i(x) = f(x+1) - 2,$
$h(x) = f\left(\frac{1}{2}x\right), \quad j(x) = -2f(x)$



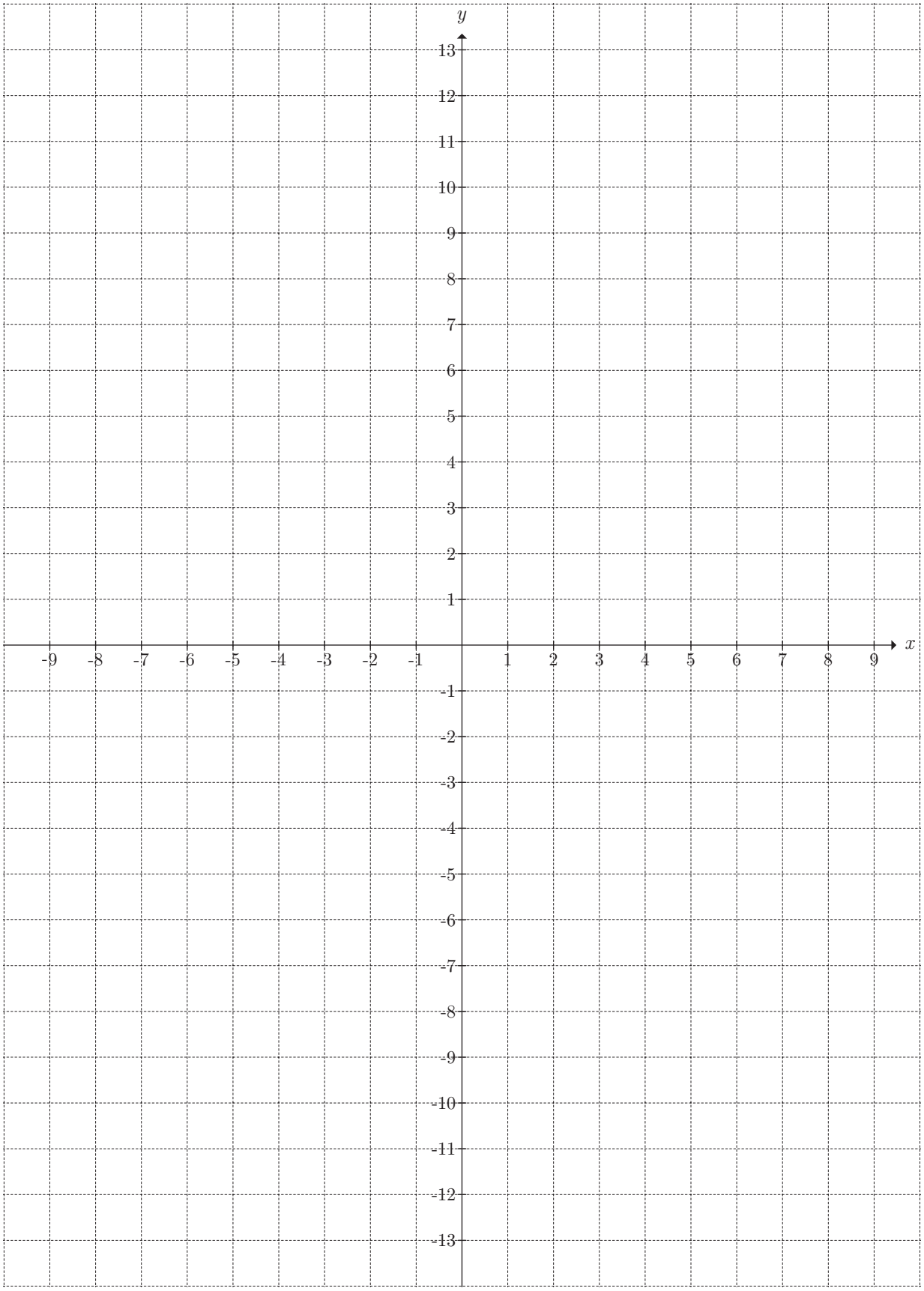
139. Sketch the graphs of the functions based on the graph of $f(x) = \mathrm{e}^x$.

$g(x) = \mathrm{e}^{x+2}$, $\quad i(x) = 2\mathrm{e}^x$,
$h(x) = -\mathrm{e}^{0.5x}$, $\quad j(x) = \mathrm{e}^x - 1$



140. Sketch the graphs of the functions based on the graph of $f(x) = \mathrm{e}^x$.

$g(x) = \mathrm{e}^{2x}$, $\quad i(x) = -\frac{1}{2}\mathrm{e}^x$,
$h(x) = -\mathrm{e}^{1-x}$, $\quad j(x) = \mathrm{e}^{x-3} + 1$

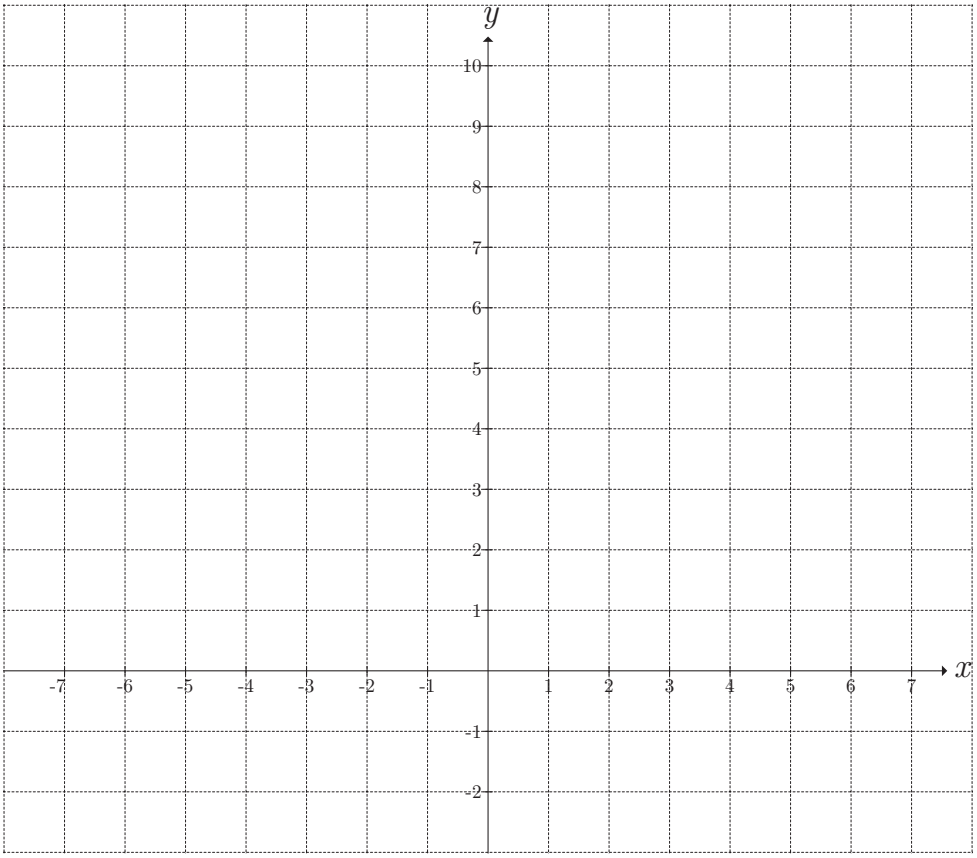


141.

a) Sketch the graphs of $f(x) = \mathrm{e}^x$, $g(x) = \mathrm{e}^{-x}$ and $h(x) = \cosh(x)$.

b) Show that cosh is an even function.

c) Examine the monotonocity behaviour of cosh.

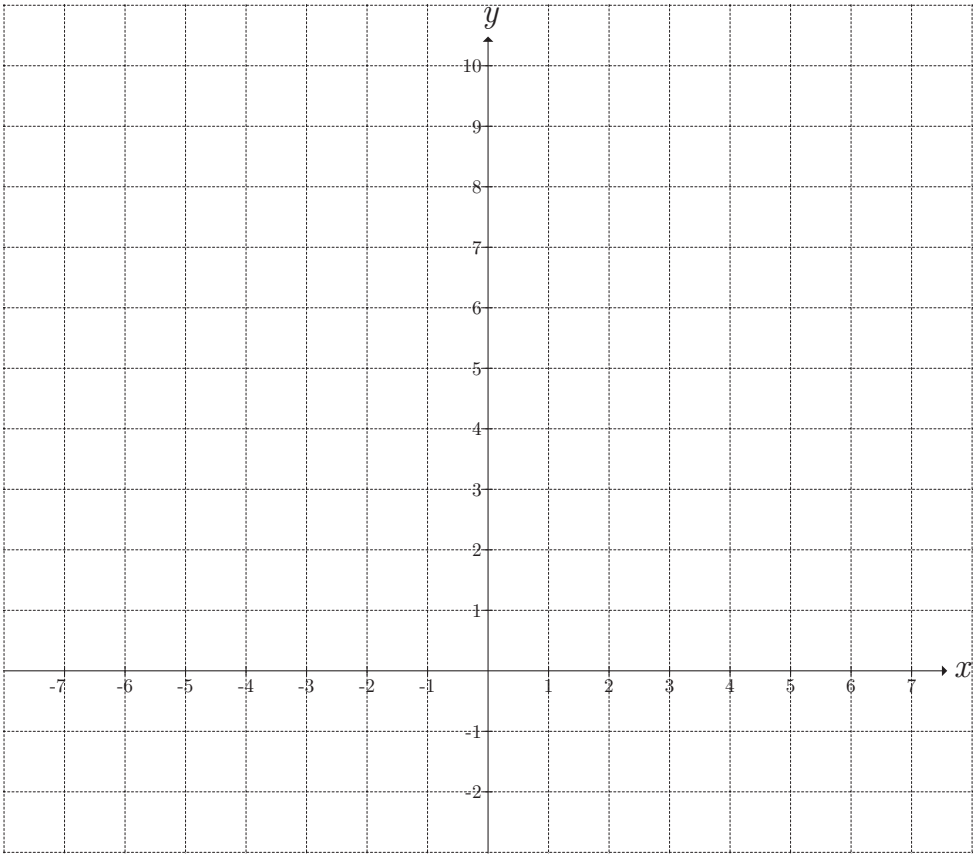


142.

a) Sketch the graphs of $f(x) = \mathrm{e}^x$, $g(x) = \mathrm{e}^{-x}$ and $h(x) = \sinh(x)$.

b) Shotw that sinh is an odd function.

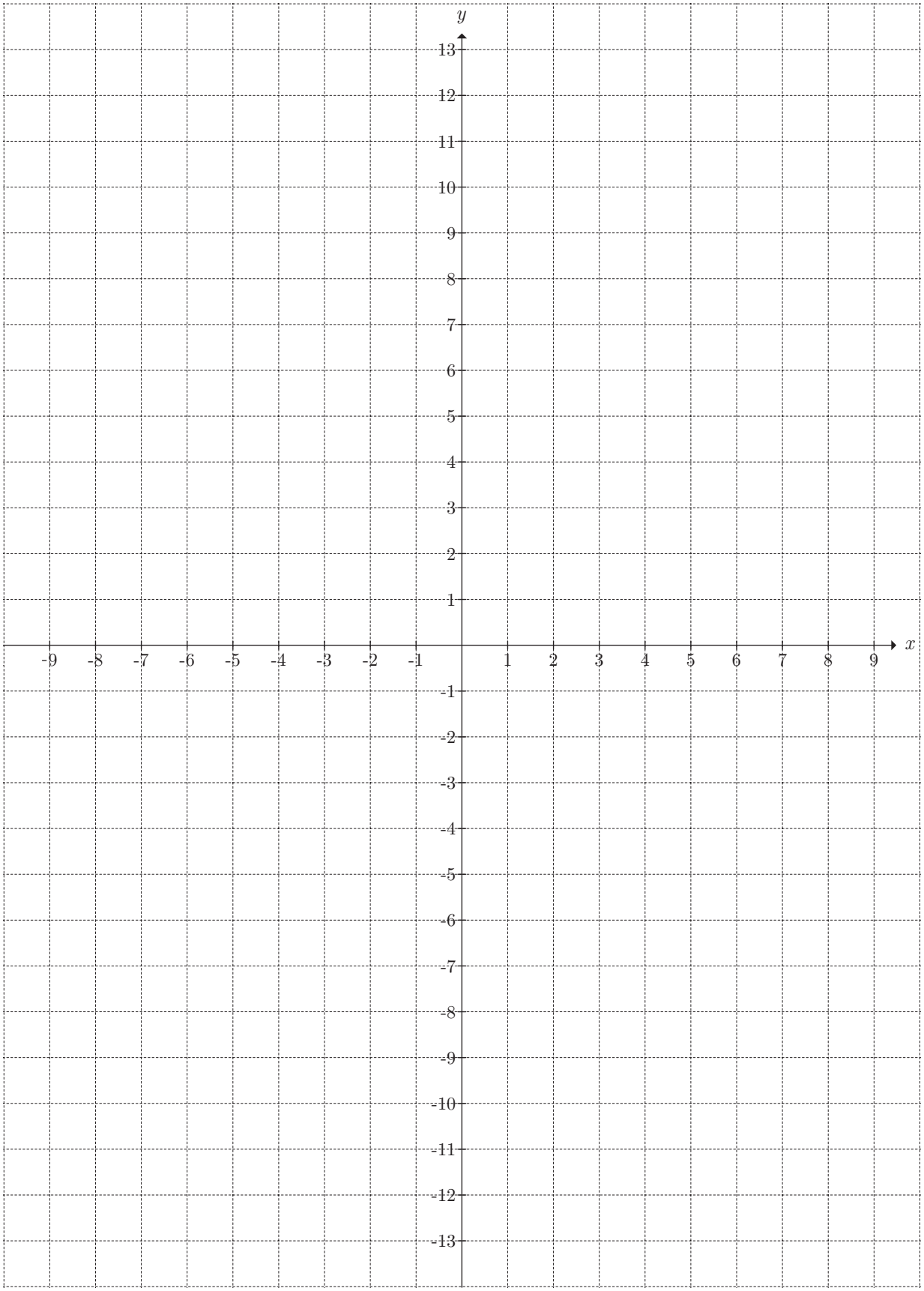
c) Examine the monotonicity behaviour of sinh.



307. Draw the graphs of the functions in the same coordinate system.

a) $f(x) = \log_2 x$ c) $h(x) = \log_{\frac{1}{2}} x$

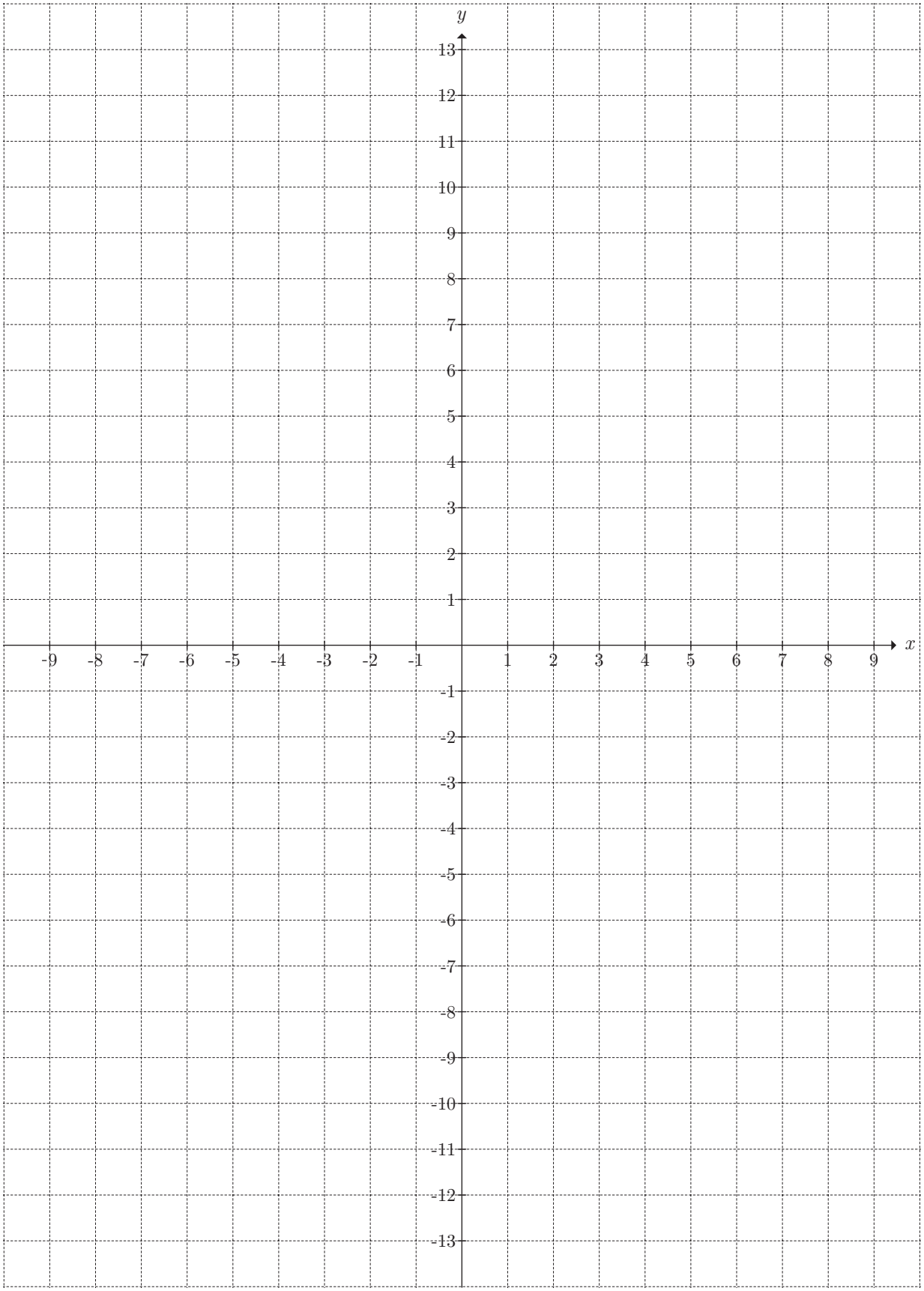
b) $g(x) = \log_3 x$ d) $k(x) = \log_{\frac{1}{3}} x$



308. Draw the graphs of the functions in the same coordinate system.

a) $f(x) = \log_4 x$ c) $h(x) = \log_{\frac{1}{4}} x$

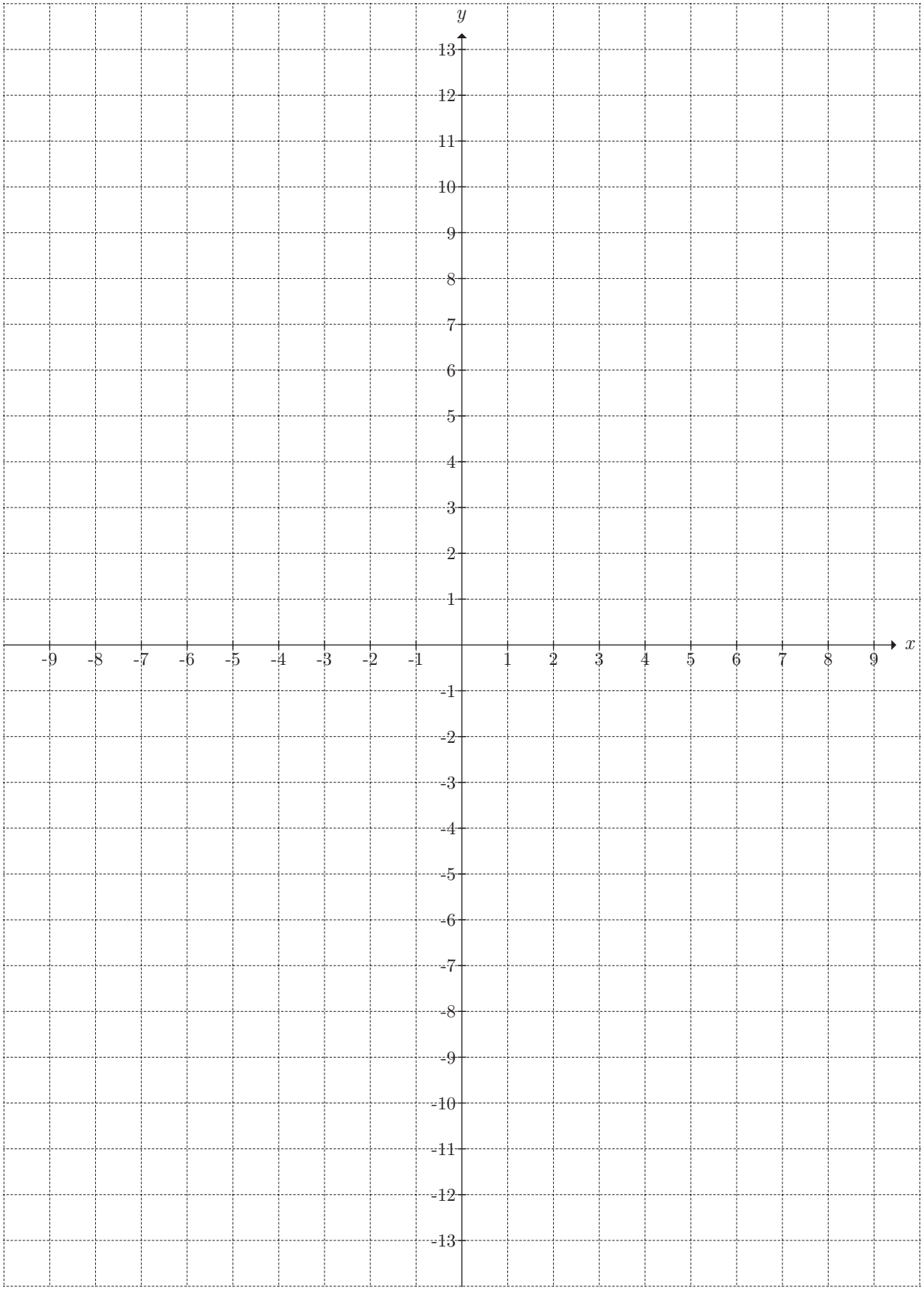
b) $g(x) = \log_5 x$ d) $k(x) = \log_{\frac{1}{5}} x$



321. Draw the graphs of the functions in the same coordinate system.

a) $f(x) = 2^x$ c) $h(x) = \mathrm{e}^x$

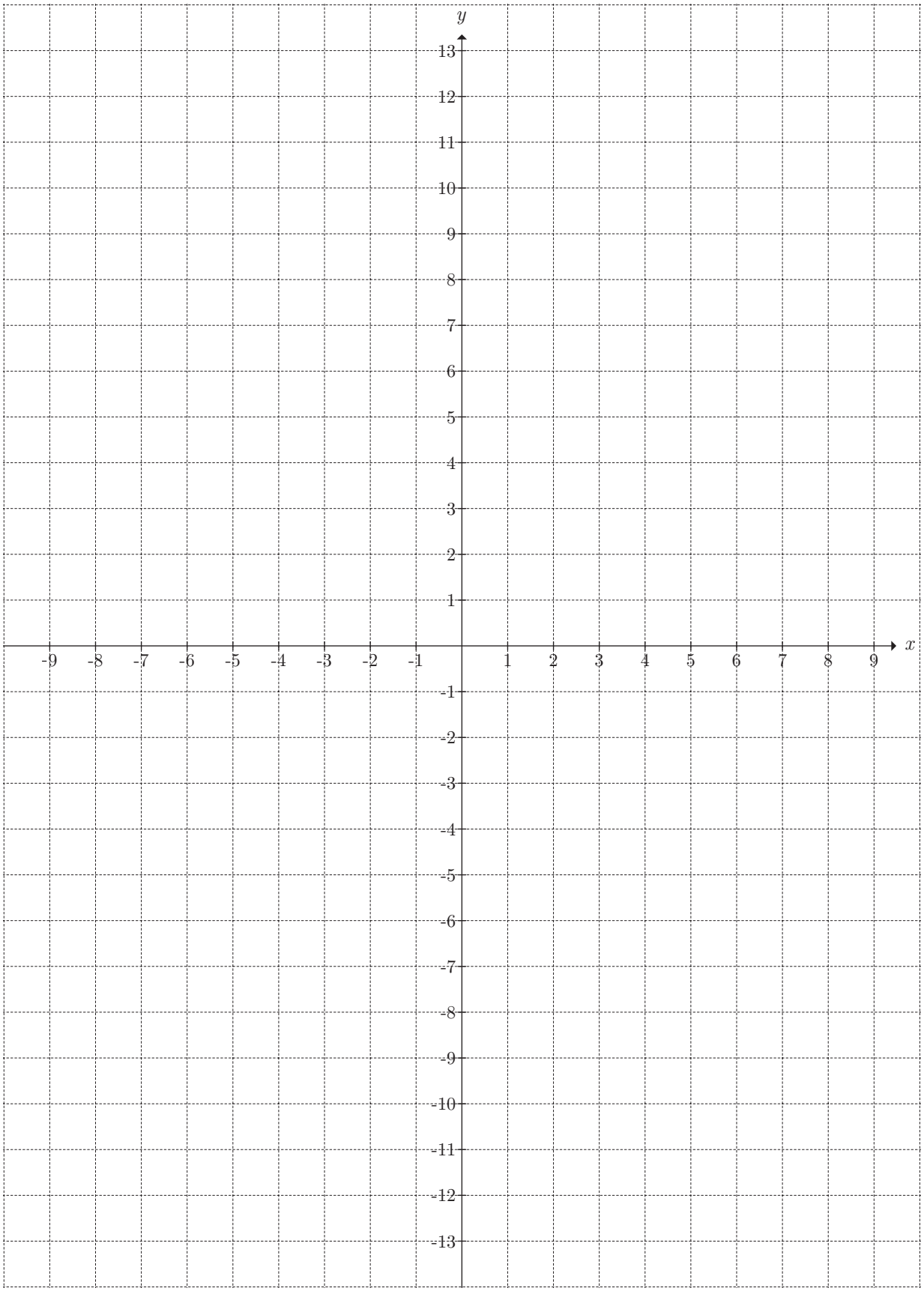
b) $g(x) = \log_2 x$ d) $k(x) = \ln x$



322. Draw the graphs of the functions in the same coordinate system.

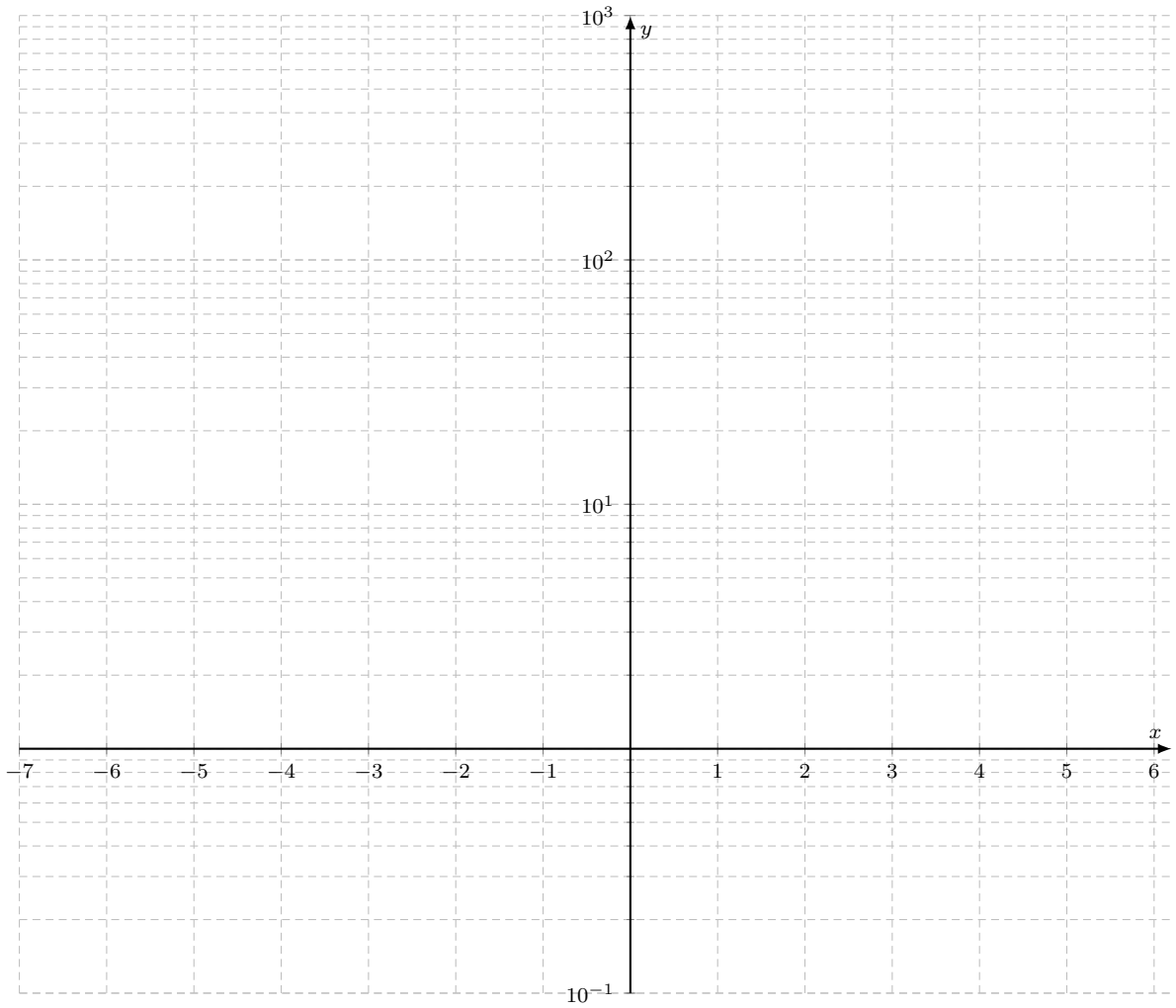
a) $f(x) = 4^x$ c) $h(x) = \left(\frac{1}{4}\right)^x$

b) $g(x) = \log_4 x$ d) $k(x) = \log_{\frac{1}{4}} x$



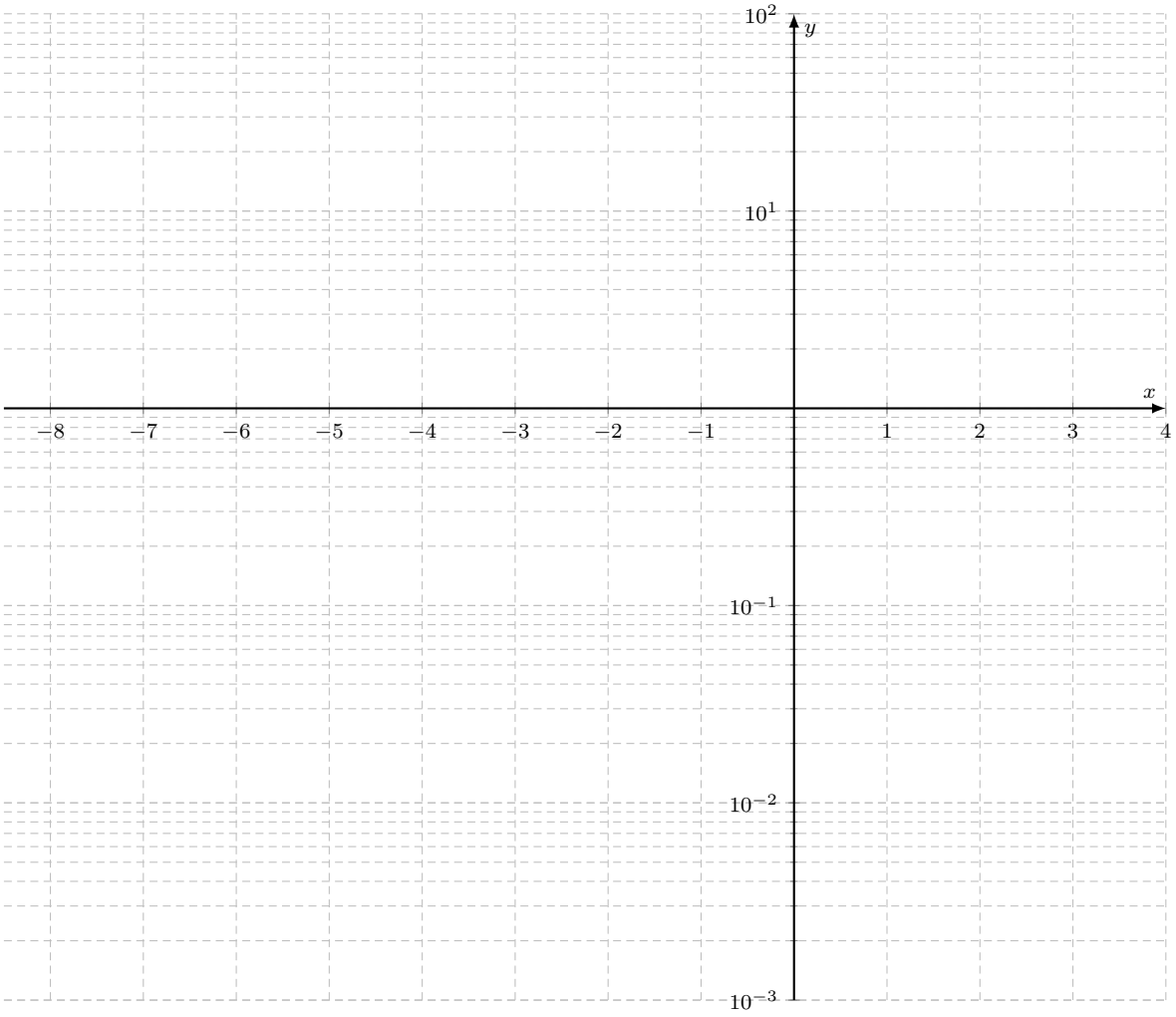
411. Draw the graphs of the functions in the ordinate-logarithmic coordinate system.
The base of the logarithm is 10.

$$f: \quad x \mapsto 2^x, \quad h: \quad x \mapsto 3 \cdot 2^x$$
$$g: \quad x \mapsto 3^x, \quad k: \quad x \mapsto x^2$$



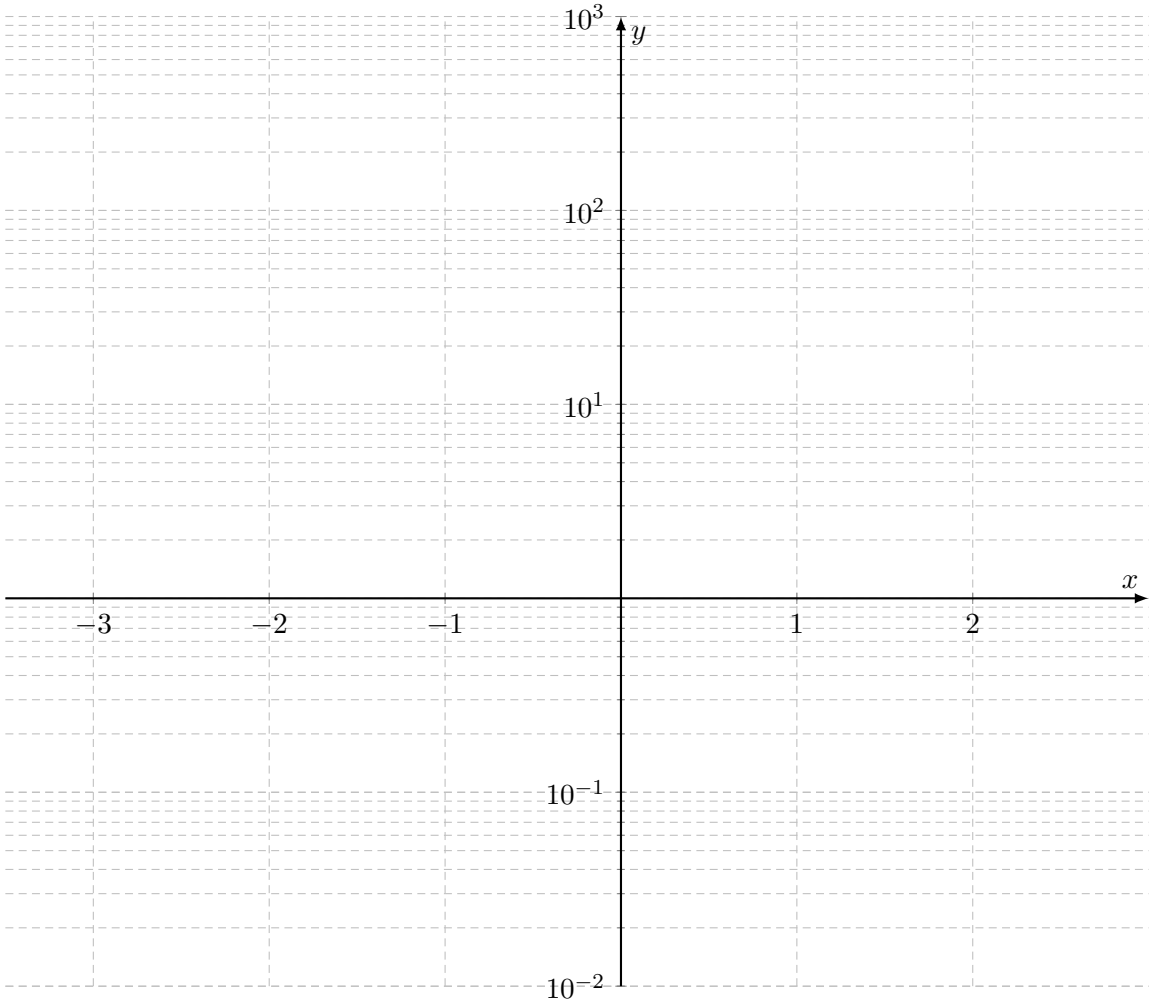
412. Draw the graphs of the functions in the ordinate-logarithmic coordinate system.
The base of the logarithm is 10.

$$f: \quad x \mapsto \left(\tfrac{1}{2}\right)^x, \quad h: \quad x \mapsto 3 \cdot \left(\tfrac{1}{2}\right)^x$$
$$g: \quad x \mapsto \left(\tfrac{1}{4}\right)^x, \quad k: \quad x \mapsto \tfrac{4}{x}$$



415. By plotting on a logarithmic coordinate system with base 10, investigate whether the following points could lie on the graph of an exponential function.

x	-3	-2	-1	0	1	2
y	0.032	0.16	0.6	4	23	100



416. By plotting on a logarithmic coordinate system with base 10, investigate whether the following points could lie on the graph of an exponential function.

x	-2	-1	0	1	2	3
y	160	40	8	1.6	0.2	0.064