

Algebra

Answers

Verein zur Förderung der Mathematik

Answers

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This PDF contains all the answers to the exercises in the book «Algebra» published by the Verein zur Förderung der Mathematik.

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1 Polynomials

1.1 Factorisation

1. ①: all except $\mathcal{C}$, ②: $\mathcal{A}, \mathcal{B}, \mathcal{E}, \mathcal{F}, \mathcal{G}$
 ③: $\mathcal{A}$ ④: $\mathcal{E}, \mathcal{G}$
3. a) $2(2x^2 + 6x) = 2x(2x + 6) = 4x(x + 3)$
 b) $3x(2x^2y^2 - 4xy^4) =$
 $6xy(x^2y - 2xy^3) = 2x^2y^2(3x - 6y^2)$
 c) $2(4a - b) = 4(2a - \frac{b}{2}) = \frac{1}{4}(32a - 8b)$
 d) $2(2x + y) = \frac{1}{2}(8x + 4y) = \frac{1}{5}(20x + 10y)$
 e) $4c(d - \frac{5}{2}) = 8(\frac{1}{2}cd - \frac{5}{4}c) = \frac{c}{3}(12d - 30)$
 f) $\frac{1}{3}(3a + 18b) = \frac{2}{3}(\frac{3}{2}a + 9b) = \frac{3}{4}(\frac{4}{3}a + 8b)$
5. ①- $\mathcal{G}$, ②- $\mathcal{E}$, ③- $\mathcal{B}$, ④- $\mathcal{C}$, ⑤- $\mathcal{H}$, ⑥- $\mathcal{A}$, ⑦- $\mathcal{F}$, ⑧- $\mathcal{D}$
7. a) $2(3x + 2y)$ d) $2a(a - 3)$
 b) $a(x - y)$ e) $3b(2b + 3a)$
 c) $7(5a - 12)$ f) $4x^2(3x - 2)$
9. a) $a(a - 1)$ d) $a^4b^2(a^2 + b^2)$
 b) $a^2b^3(a^2b^3 + 1)$ e) $8ab(2ab + 1)$
 c) $27yz(3x - 2)$ f) $9a^2b^2(3ab - 4)$
11. a) $11(11a - 8b + 6)$
 b) $a^3(a^6 - a^3 + 1)$
 c) $7abc(5a - 2b + 3c^2)$
13. a) $-1(a + b - c)$ c) $-1(x^3 + x^2 + 1)$
 b) $-1(-a - 2b + 5)$ d) $-1(-a^2 - a - 1)$
15. a) $\frac{1}{6}(a + 2b + 6)$
 b) $\frac{1}{5}(2a^2 + 5a - 15)$
 c) $\frac{1}{12}(3x^2 - 8x + 12)$
 d) $\frac{1}{24}(9a^2 - 8a + 48)$
17. a) $2(x + y)(3x + 4)$
 b) $v(t - 7)$
 c) $f^2(3g - 12h + 2k)$
19. a) $(5 - 2h)(9g + 11)$
 b) $(2d - 1)(c^3d + 2f^2)$
 c) $(4 - 3m)(6k - 7u) = (3m - 4)(-6k + 7u)$
 d) $(2b + 5)(3 - a + 6c)$
 e) $(h - 4j)(15 + j) = (4j - h)(-15 - j)$
 f) $(r + 9)(q - s) = (-r - 9)(-q + s)$
2. ①: all except $\mathcal{F}$, ②: $\mathcal{B}, \mathcal{C}, \mathcal{E}, \mathcal{G}, \mathcal{H}$
 ③: $\mathcal{E}$ ④: $\mathcal{C}, \mathcal{H}$
4. a) $3(2x - 4x^2) = 3x(2 - 4x) = 6x(1 - 2x)$
 b) $4(3xy^2 - 2x^2y^3) =$
 $2xy(6y - 4xy^2) = 4xy^2(3 - 2xy)$
 c) $2(4x + 2y) = 8(x + \frac{y}{2}) = \frac{1}{2}(16x + 8y)$
 d) $2(a - \frac{3}{2}b) = \frac{1}{2}(4a - 6b) = \frac{1}{3}(6a - 9b)$
 e) $2p(\frac{q}{2} + 1) = \frac{1}{2}(2pq + 4p) = \frac{p}{3}(3q + 6)$
 f) $\frac{1}{4}(4x + 8) = \frac{3}{4}(\frac{4}{3}x + \frac{8}{3}) = \frac{2}{5}(\frac{5}{2}x + 5)$
6. ①- $\mathcal{F}$, ②- $\mathcal{D}$, ③- $\mathcal{B}$, ④- $\mathcal{G}$, ⑤- $\mathcal{A}$, ⑥- $\mathcal{H}$, ⑦- $\mathcal{E}$, ⑧- $\mathcal{C}$
8. a) $5(3a + 2b)$ d) $5b^2(b - 2)$
 b) $x(a - b)$ e) $21e(e - 2f)$
 c) $8(7x + 12y)$ f) $6a^2(a - 7)$
10. a) $x^2(x - 1)$ d) $a^6b^4(a^2 + b^2)$
 b) $y^4z^2(y^4 + 1)$ e) $27xy(3xy - 1)$
 c) $21ac(3b - 2)$ f) $8x^2z^2(xz - 2)$
12. a) $12(12x - 6y + 5)$
 b) $b^4(b^4 + b^2 + 1)$
 c) $8xyz(5x^2 - 8y^2 + 3z^2)$
14. a) $-1(x - 2y - z)$ c) $-1(a^2 + a - 1)$
 b) $-1(-2x - y - 8)$ d) $-1(-x^2 + 155)$
16. a) $\frac{1}{12}(3x + 4y + 24)$
 b) $\frac{1}{4}(3x^2 + 4x - 20)$
 c) $\frac{1}{15}(3a^2 - 10a - 30)$
 d) $\frac{1}{12}(10a^2 - 3a + 12)$
18. a) $3(5 - 2a)(1 + 2a)$
 b) $q(-3u + 4v)$
 c) $m^2(8n - 15)$
20. a) $(u + v)(4 - 5a)$
 b) $(4y + 3)(8y + 7z)$
 c) $(6p - 1)(13p - 4) = (1 - 6p)(4 - 13p)$
 d) $(10k - 11v)(k + v - m)$
 e) $(6p - 5)(r^2 - 5) = (5 - 6p)(5 - r^2)$
 f) $(8 - x)(t - u) = (x - 8)(u - t)$

21. a) $8(a^2 - 1)$
 b) $6xy(2x - 3y)$
 c) correct
 d) correct
22. a) $15r(-2s - 3r) = -15r(2s + 3r)$
 b) correct
 c) $(3 - h)(2 + 4h)$
 d) $\frac{1}{3}(15m - 12n - 2)$
23. a) $(p + 2q)(6 - 11p)$
 b) $(3x - 5y)(1 - r)$
 c) $(k - m)(k + m)$
 d) $(6a + 5)(4b - 3c)$
24. a) $(f + 3g)(7f - 2)$
 b) $(12b + 13c)(1 + d)$
 c) $(j - m)(j + 4)$
 d) $(4x - y)(2x + y)$
25. a) $(7u + 5s)(4s - 3w)$
 b) $(-9x - 1)(4x + w)$
26. a) $(3a^2 - b)(4a + 3b^2)$
 b) $(8ef + 7g)(5f + 6)$
27. a) $(a + b - c)(2m + n)$
 b) $(3 - 4s + a)(c + s)$
 c) $(-p - q + r)(3 - x)$
28. a) $(t + v + 2)(x + 3y)$
 b) $(5 - 2b + 11c)(4 - 9a)$
 c) $(u - 5v + 3)(t + w^2)$
29. a) $(x - 5)^2$ c) $(x + 6)(x - 6)$
 b) $(x + 12)^2$ d) $(x + 2)^2$
30. a) $(x + 7)^2$ c) $(x - 1)^2$
 b) $(x + 15)(x - 15)$ d) $(x - 4)^2$
31. a) $(3 - 2x)(3 + 2x)$ c) $(a + 14)(a - 14)$
 b) $(2m + 3)^2$ d) $(b - 2x)^2$
32. a) $(n - 2)^2$ c) $(ax + 1)(ax - 1)$
 b) $(3p + 2q)^2$ d) $(5g + 2)(5g - 2)$
33. a) $(f + 1)(f - 1)(f^2 + 1)$
 b) $(3c + 5d)^2$
 c) $(3 - x)(3 + x)(9 + x^2)$
 d) $(7a - 5b)^2$
34. a) $(5p - 6q)^2$
 b) $(13 + 2v^2)^2$
 c) $(j^2 + k^2)(j - k)(j + k)$
 d) $(10m + 11n)^2$
35. a) $x^2 - 16x + 64 = (x - 8)^2$
 b) $x^2 \pm 6x + 9 = (x \pm 3)^2$,
 $x^2 - 8x + 16 = (x - 4)^2$
 c) $y^2 - z^2 = (y + z)(y - z)$
 d) $25x^2 - 20x + 4 = (5x - 2)^2$
36. a) $x^2 + 26x + 169 = (x + 13)^2$
 b) $x^2 - 20.25 = (x + 4.5)(x - 4.5)$
 c) $9x^2 - 24x + 16 = (3x - 4)^2$
 d) $25a^2 - 40ab + 16b^2 = (5a - 4b)^2$,
 $25a^2 - 20ab + 4b^2 = (5a - 2b)^2$
37. a) $2(x - 4)^2$
 b) $-25(2a + 1)^2$
 c) $3a(9r - 2t)(9r + 2t)$
 d) $4w(w + 2z)(w - 2z)$
38. a) $-5(3 + a)^2$
 b) $2(4k - 7m)^2$
 c) $hj(h + 8)^2$
 d) $k(17 - 3n)(17 + 3n)$
39. a) $b(3b + 1)(3b - 1)(9b^2 + 1)$
 b) $a^2(a + 5)^2$
 c) $(k + m)(k - m)(p - q)$
 d) $3(r + 2)(r - 2)(p + k)$
40. a) $(1 + x)(1 - x)(2 - a)$
 b) $4(2 - a)(2 + a)(3 - w)$
 c) $(2 + v)(2 - v)(4 + v^2)$
 d) $-2(2 - 5y)^2$

41. a) $(c - 4 + 7a)(c - 4 - 7a)$
 b) $(6 + g + 3h)(6 - g - 3h)$
 c) $(x + y + a)(x + y - a)$
 d) $(d - f + g)(d - f - g)$
 e) $(5 - a + b)(5 + a - b)$
 f) $(r - 3 + s)(r - 3 - s)$
42. a) $(x + y + 2)(x - y + 2)$
 b) $(9b + c - 1)(9b - c + 1)$
 c) $(s - 2t + u)(s - 2t - u)$
 d) $(j + k + m)(j - k - m)$
 e) $(1 - m + n)(1 - m - n)$
 f) $(2 + x + y)(2 + x - y)$
43. a) $(3t - 7)(4t + 12) = 12t^2 + 8t - 84$
 b) $(-a + 9)(3a + 2) = -3a^2 + 25a + 18$
 c) impossible
 d) impossible
 e) $(2z + 9)(-z + 1) = -2z^2 - 7z + 9$
44. a) impossible
 b) $(x + 28)(x + 11) = x^2 + 39x + 308$
 c) $(2y - 6)(y + 7) = 2y^2 + 8y - 42$
 d) impossible
 e) $(m - 16)(x + (-11)) = m^2 - 27m + 176$
45. a) $2t^2 - 3t + (-35) = (t - 5)(2t + 7)$
 b) $-c^2 - 17c - 72 = (-c + (-9))(c + 8)$
46. a) $-3a^2 + 11a + 20 = (3a + 4)(-a + 5)$
 b) $-2x^2 - 13x + 7 = (x + 7)(-2x + 1)$
47. a) $(m + 3)(m + 4)$
 b) $(k - 5)(k - 6)$
 c) $(x + 7)(x + 9)$
 d) $(y - 3)(y - 8)$
48. a) $(a + 8)(a + 2)$
 b) $(b - 3)(b - 4)$
 c) $(d + 5)(d + 12)$
 d) $(j - 8)(j - 12)$
49. a) $(b - 6)(b + 4)$
 b) $(q - 28)(q - 1)$
 c) $(u - 9)(u - 10)$
 d) $(p + 13)(p - 2)$
50. a) $(d - 5)(d + 11)$
 b) $(w - 4)(w + 11)$
 c) $(z + 3)(z - 2)$
 d) $(x - 1)(x + 6)$
51. a) $(x + 11)(x - 9)$
 b) $(t^2 - 2)(t^2 + 5)$
 c) $(v + 18)(v - 3)$
 d) $(z - 14)(z - 7)$
52. a) $(m + 4)(m - 15)$
 b) $(s^2 - 5)(s^2 + 3)$
 c) $(w + 7)(w + 12)$
 d) $(u - 10)(u + 1)$
53. a) $a^2 + 2a - 3 = (a - 1)(a + 3)$
 b) $c^2 + 8c + 16 = (c + 4)^2$
 c) $t^2 - 36 = (t + 6)(t - 6)$
54. a) $y^2 - 11y + 28 = (y - 4)(y - 7)$
 b) $u^2 - 121 = (u + 11)(u - 11)$
 c) $r^2 + 5r - 50 = (r + 10)(r - 5)$
55. a) $x^2 + 12x - 13 = (x - 1)(x + 13)$
 b) $x^2 - 2x - 15 = (x - 5)(x + 3)$
 c) $x^2 + 14x - 15 = (x + 15)(x - 1)$
 d) $x^2 - 14x + 49 = (x - 7)^2$
56. a) $x^2 - 14x + 33 = (x - 11)(x - 3)$
 b) $x^2 + 18x + 81 = (x + 9)^2$
 c) $x^2 - 2x - 8 = (x - 4)(x + 2)$
 d) $x^2 - 7x - 8 = (x + 1)(x - 8)$
57. a) irreducible
 b) irreducible
 c) $(b + 5)(b - 5)$
 d) $(y + 3)(y + 9)$
 e) $(x - 1)(x + 8)$
 f) irreducible
58. a) irreducible
 b) $(t - 4)^2$
 c) irreducible
 d) irreducible
 e) $(g + 1)(g + 12)$
 f) irreducible

59. a) $3(g-4)(g-2)$
b) $-2(h+5)(h-10)$
c) $10(j-8)(j-7)$
d) $-a(z+1)(z-17)$
61. a) $(3b+1)(b-1)$
b) $(-5u+2)(u+1)$
c) $(2s-3)(s+1)$
d) $(3t+1)(2t+1)$
63. a) $2(x+2)(3y-5)$
b) $25(d-2)^2$
c) $8ab(1-2a)(3b-4)$
d) $80m(3+5m)(n-1)^2$
e) $22(2b+1)(2b-1)(b+5)$
65. a) $8(b+1)(b-1)$
b) $(x-9)(x-6)$
c) $6r(s+3t)^2$
d) $d(c-d)$
67. a) $(b+2)(b+3)$
b) $(9-4d)(x-3)$
c) $-q(m+q)^2$
d) $(2y+1)^2$
69. a) $(m+8)(m-11)$
b) $(x-9)^2$
c) $(15-m)(-14+m)$
d) $2(b-2)(b-4)$
71. a) $6(c-1)(c+2)$
b) $-2(d-8)^2$
c) $(j-k)(2m+n)$
d) $f^2(f^2+2)$
73. a) $(c-2)(c+3)(c-3)$
b) $4(k+3)(k-6)$
c) $3(x+2)(x+10)$
d) $(5-3a)(v+2w)$
75. a) $(x^2+1)(x-1)(x+1)$
b) $x^2(x^2+4y^2)$
60. a) $2(b+9)(b-12)$
b) $-19(x+1)(x+3)$
c) $6(x-6)(x-3)$
d) $2c(m-11)(m-1)$
62. a) $(-a+4)(a+5)$
b) $(z-7)(2z+1)$
c) $(-7t+1)(2t+1)$
d) $(6c+1)(-c-2)$
64. a) $12(y+3)(a+2)$
b) $18(s+1)(s-1)(s+3)$
c) $16(t+5)^2$
d) $49k^3(2k-3)^2(k+1)$
e) $24d(3c-1)(d^2+1)$
66. a) $(f+8)(f+4)$
b) $(3-4a)(9-h)$
c) $(y+1)(-y+3)$
d) $(4t^2+1)(2t+1)(2t-1)$
68. a) $(12+x)(12-x)$
b) $c(c+2)(c-2)$
c) $(f-3g)(a+2m)$
d) $(j+17)(j+3)$
70. a) $-3(k^2+1)(a+b)$
b) $(8+z)(7r+2s)$
c) $(q-1)(q+4)$
d) $(x-16)^2$
72. a) $5(z-5)(z+5)$
b) $(a+b)(a-1)$
c) $(y-3)(y+14)$
d) $2(x-6)^2$
74. a) $8(v+2)^2$
b) $a(a^2+1)$
c) $(3j-4k)(3j-4k-1)$
d) $(z+6)(z-12)$
76. a) $(x+5)(x+4)$
b) $x^2(x+y)^2$

77. The number can be expressed as $2k$ and $(2k)^2 = 4k^2$
78. The number can be expressed as $10x$ and $(10x)^2 = 100x^2$
79. a) -
b) $a^3 - a = a(a+1)(a-1)$ The product of three consecutive numbers contains a multiple of 2 and 3 and is therefore also divisible by 6.
80. a) -
b) $b^3 - b = b(b+1)(b-1)$ is the product of three consecutive numbers, of which $b-1$ and $b+1$ are even. One of these two numbers is divisible by 4, the other at least by 2. One of the three numbers is divisible by 3. Divisibility by 3 and by $2 \cdot 4 = 8$ implies divisibility by 24.
81. a) -
b) $c^3 - 3c^2 + 2c = c(c-1)(c-2)$ The product of three consecutive numbers contains the multiples of 2 and 3 and is therefore also divisible by 6.
82. a) -
b) $d^3 - 3d^2 + 2d = d(d-1)(d-2)$ The product of three consecutive numbers, of which d and $d-2$ are even. One of these two numbers is divisible by 4, the other at least by 2. One of the three numbers is divisible by 3. Divisibility by 3 and by $2 \times 4 = 8$ implies divisibility by 24.
83. a) $(k^2 + s^2 + ks)(k^2 + s^2 - ks)$
b) $(r^2 - 3s^2 + 4rs)(r^2 - 3s^2 - 4rs)$
c) $(r^2 - 3s^2 + 4rs)(r^2 - 3s^2 - 4rs)$
d) $(x+1)(x-1)(x+2)(x-2)$
84. a) $(b+c)(b-c)(b-2c)(b+2c)$
b) $(a^2 - 2 + 9a)(a^2 - 2 - 9a)$
c) $(a^2 - 2 + 9a)(a^2 - 2 - 9a)$
d) $(y^2 - 7 + 4y)(y^2 - 7 - 4y)$
85. a) $(4x+5)(x+3)$
b) $(y^2 - 2y - 1)(y^2 + 2y - 1)$
c) $(2g+3)(3g+2)$
d) $(n+2)(n-2)(n+4)$
e) $(-3d+1)(2d+1)$
86. a) $2(4a+b+5)(4a+b-5)$
b) $20(s+4)(s+1)(s-1)$
c) $4(x-1)(x+1)(x+3)$
d) $(7a+b+7)(7a-b-7)$
e) $(3z-2)(3z-4)$
87. a) $(5x-6)(17x+6)$
b) $(4a+7)(-a+2)$
c) $18(v-2)^2(v+1)^2$
d) $(x+11)(-3x-7)$
88. a) $(5m-3)(2m-3)$
b) $3(2a-c+6)(2a-c-6)$
c) $(a^2+ab+b^2)(a^2-ab+b^2)$
d) $2(x+3)(x-3)(x+1)$

1.2 Polynomial Long Division

89. a) $x^2 - x - 1$ 90. a) $y^2 + y - 1$ 91. a) $r^2 - r - 3$ 92. a) $d^2 + 3d - 1$
b) $a^2 + 2a + 4$ b) $b^2 + 3b + 9$ b) $c^2 + 3c - 1$ b) $x^2 - 3$
c) $m^2 - m - 1$ c) $s^2 - 2s - 6$ c) $f^2 - 2f + 3$ c) $n^2 - n - 2$
d) $p^2 - 2p + 1$ d) $d^2 - 5d + 2$ d) $y + 9$ d) $q^2 - 4q + 4$
93. a) $m^6 + m^4 + m^2 + 2$ 94. a) $k^4 + k^3 + k^2 + k$
b) $z^2 + 3$ b) $-a - 7$

95. a) $\square = -30, 2x^2 - 3x + 5$
 b) $\square = 29, 3m^3 + 8m^2 + 16m + 3$
96. a) $\square = 7, n^3 - 2n^2 - n - 4$
 b) $\square = 24, -a^3 + a^2 - 4$
97. a) $-k - 2$ remainder $3k$
 b) $3y - 2$ remainder $-21y + 10$
 c) $c^2 + c + 1$ remainder 1
 d) $q^2 - 1$ remainder 1
98. a) $x - 2$ remainder $3x$
 b) $2b - 3$ remainder $-16b + 15$
 c) $d^2 - d + 1$ remainder -1
 d) $m^2 + 1$ remainder 1
99. a) $x^3 - x^2 + 2x - 3$ remainder $5x - 3$
 b) $g^2 + g - 1$
 c) $n^2 + n + 1$
 d) $z^2 + 0.5z - 0.75$ remainder $-0.875z + 0.75$
100. a) $z^2 - z + 1$
 b) $u^3 + 1$ remainder 1
 c) $m^2 - m + 1$
 d) $t^6 - t^4 + t^2 - 1$ remainder 1

1.3 Higher Powers of Binomials

101. a) $c^3 + 3c^2d + 3cd^2 + d^3$
 b) $r^5 + 5r^4s + 10r^3s^2 + 10r^2s^3 + 5rs^4 + s^5$
102. a) $u^4 + 4u^3v + 6u^2v^2 + 4uv^3 + v^4$
 b) $m^6 + 6m^5n + 15m^4n^2 + 20m^3n^3 + 15m^2n^4 + 6mn^5 + n^6$
103. a) $-a^3 + 3a^2b - 3ab^2 + b^3$
 b) $16x^4 + 32x^3 + 24x^2 + 8x + 1$
 c) $0.125 - 0.75g + 1.5g^2 - g^3$
 d) $\frac{1}{8} + \frac{3}{4}a + \frac{3}{2}a^2 + a^3$
104. a) $x^4 - 4x^3y + 6x^2y^2 - 4xy^3 + y^4$
 b) $27x^3 - 54x^2 + 36x - 8$
 c) $\frac{1}{16} - \frac{1}{2}t + \frac{3}{2}t^2 - 2t^3 + t^4$
 d) $0.512 + 1.92a + 2.4a^2 + a^3$
105. a) $625n^8 + 500n^6 + 150n^4 + 20n^2 + 1$
 b) $x^3 + 3x + \frac{3}{x} + \frac{1}{x^3}$
 c) $r^{12} - 4r^{11} + 6r^{10} - 4r^9 + r^8$
 d) $\frac{k^8}{81} - \frac{4k^6}{27} + \frac{2k^4}{3} - \frac{4k^2}{3} + 1$
106. a) $\frac{m^4}{16} - m^3 + 6m^2 - 16m + 16$
 b) $216n^3 - 540n^2s + 450ns^2 - 125s^3$
 c) $y^3 - 3y + \frac{3}{y} - \frac{1}{y^3}$
 d) $u^6 + 9u^5 + 27u^4 + 27u^3$
107. a) $a^5 + 5a^4 + 10a^3 + 10a^2 + 5a + 1$
 b) $-\frac{1}{8y^3} + \frac{3}{2y^2} - \frac{6}{y} + 8$
 c) $64m^6 - 144m^4n + 108m^2n^2 - 27n^3$
 d) $6r^2 + 2$
 e) $-4k^6 - 6k^4 + 12k^2 + 22$
 f) 0
108. a) $b^5 - 5b^4 + 10b^3 - 10b^2 + 5b - 1$
 b) $\frac{1}{8x^3} + \frac{3}{4x^2} + \frac{3}{2x} + 1$
 c) $r^4s^4 + 12r^3s^3t + 54r^2s^2t^2 + 108rst^3 + 81t^4$
 d) $-h^9 + 3h^6 - 7h^3 - 12h^2 - 12h - 3$
 e) $2a^3 - 6a^2 + 6a - 2$
 f) $6n^5 + 2n^3$
109. a) -36
 b) 96
 c) $\frac{3}{2}$
 d) 150
110. a) 80
 b) -2
 c) 54
 d) 300
111. a) $(4t - 3)^3 = 64t^3 - 144t^2 + 108t - 27$
 b) $(-x + 10)^3 = -x^3 + 30x^2 - 300x + 1000$
112. a) $(j^2 - 1)^4 = j^8 - 4j^6 + 6j^4 - 4j^2 + 1$
 b) $\left(\frac{1}{5}k - 1\right)^3 + \left(\frac{1}{5}k + 1\right)^3 = \frac{2k^3}{125} + \frac{6k}{5}$
113. a) 70
 b) $\frac{105}{2}$
114. a) 70
 b) 63

2 Rational Terms and Rational Equations

2.1 GCD and LCM

Note:

The greatest common divisor of $-4x$ and $6x$ can be given as $2x$ and $-2x$. This means that for each sub-task, there are two possibilities for both the GCD and the LCM. For the sake of simplicity, only one of the two solutions is noted.

1. a) $5x^2y$; $175x^3y^2$ c) 1 ; $2m^2 + m$ 2. a) $3a$; $45ab$ c) $7uv$; $14u^2v^2$
 b) $2a$; $12a$ d) $k - k^2$; $k^4 - k^3$ b) 1 ; $a^2 - b^2$ d) $x + y$; $x^2 - y^2$
3. a) 1 ; $216x^3 + 36x^2y + 6xy^2 + y^3$ 4. a) $8m^4p^2$; $96m^5p^3$
 b) $4 + x^2$; $32 - 2x^4$ b) $3r - 4s$; $30r - 40s$
 c) $4a^2b$; $144a^4b^5c^3$ c) $2a + 3b$; $110a + 165b$
 d) $2y - 3z$; $12xy^2 - 27xz^2$ d) $9k^2p^2$; $189k^3m^2p^3$
5. a) $x + 2y$; $5(x + 2y)^2$ 6. a) $2c + d$; $4(2c + d)^2$
 b) $b + c$; $12(b + c)^2$ b) $a - b$; $5(a - b)^2$
 c) $u - v$; $(u - v)^2(u + v)$ c) 1 ; $(a - 2)(a + 2)(a - 5)(a + 5)$
 d) $x + 1$; $(x - 3)(x - 5)(x + 1)$ d) $11 + 7z$; $2x(11 - 7z)(11 + 7z)$
7. a) $2rs$; $24r^4s^3$ 8. a) u^3v ; $18u^6v^4$
 b) $a - b$; $10(a - b)$ b) 1 ; $2k(2k + 1)(2k + 2)$
 c) $m + n$; $4n(m + n)^2(m - n)$ c) $a + b$; $2a(a + b)^2(a - b)$
 d) $c - 5$; $2(c - 5)^2(c - 2)$ d) $d + 1$; $(d + 1)(d - 2)(d + 2)(d + 3)$

2.2 Reducing and Expanding Fractions

9. a) $2b$ c) $-\frac{5mp}{2}$ 10. a) $\frac{7}{3d}$ c) $\frac{1}{3m}$
 b) $\frac{3x}{5y}$ d) $\frac{3r^2}{2s^2}$ b) $\frac{5r}{2}$ d) $\frac{2k}{7m^3}$
11. a) $2c - d$ c) $\frac{8}{x-3y}$ 12. a) $-\frac{1}{x+1}$ c) $\frac{x-5}{x+5}$
 b) $\frac{2}{3}$ d) $\frac{t^2-s^2}{t^2+s^2}$ b) $\frac{3a-1}{2}$ d) $\frac{x-2y}{x}$
13. a) k^2 c) $\frac{1}{9}$ 14. a) $3(f - h)$ c) $\frac{3}{2}$
 b) $\frac{x}{1-y}$ d) $\frac{a+2}{3}$ b) $\frac{1}{y+z}$ d) $-\frac{11}{3}$
15. a) 2 c) $\frac{c+r}{2}$ 16. a) $\frac{4r-3}{5s}$ c) $\frac{2m-3n}{7}$
 b) $\frac{4}{r-3}$ d) $\frac{j}{7(j-1)}$ b) $(u - 1)^2$ d) $\frac{-6j}{4n^2+7}$
17. a) -1 c) -1 18. a) -1 c) -1
 b) $-\frac{2}{5}$ d) -1 b) $\frac{-2}{3}$ d) -2
19. a) $\frac{3x-4y}{5a+2}$ c) $\frac{f-g}{f+g+5}$ 20. a) $\frac{9x+7}{2}$ c) $\frac{3n-10}{5n+2}$
 b) $\frac{h+3j}{j+1}$ d) $\frac{-2+m}{4}$ b) $\frac{6a+9}{1-b}$ d) $\frac{1+a}{3}$

21. a) $\frac{(x^2+4)(x-2)}{2(x+2)}$ c) $\frac{h+3}{h-2}$ 22. a) $\frac{a}{a+5}$ c) $\frac{j^2+1}{j-2}$
 b) $\frac{b-9}{2b+1}$ d) $\frac{16-2s}{3s}$ b) $\frac{2y^2-2}{3y}$ d) $\frac{10a+15}{12a-18}$
23. a) $\frac{2a-b+1}{5}$ c) $-\frac{5v+4w+8}{4}$ 24. a) $\frac{a-1-b}{3}$ c) $m+n+2$
 b) $c-d+1$ d) $\frac{(p^2+q^2)(p-q)}{5}$ b) $\frac{(1+x^2)(1-x)^2}{5}$ d) $\frac{-3+2p-q}{8}$
25. a) $2t^2+3t+1$ c) $\frac{1}{2m^3-2m^2+5m}$ 26. a) $2a^2-7a+1$ c) $\frac{1}{-5d^2+3}$
 b) $4x^2+3x+2$ d) $9.5x^3+9$ b) $\frac{1}{3m^2-7}$ d) $\frac{1}{p^2+2}$
27. ①-D, ②-A, ③-C, ④-B, ⑤-F, ⑥-E 28. ①-D, ②-F, ③-A, ④-B, ⑤-C, ⑥-E
29. a) $\frac{15+3a}{15}$ c) $\frac{-24+28m}{-16-20m}$ 30. a) $\frac{-18+16b}{6b}$ c) $\frac{-11p+10p^2}{-9p+8p^2}$
 b) $\frac{7x-7x^2}{21x^2+14x}$ d) $\frac{6x+9}{8x+12+2fx+3f}$ b) $\frac{2c^2k-6ck^2}{6ck^2-2ck}$ d) $\frac{r^2-2rs}{-3rs+6s^2}$
31. a) $\frac{-8k}{4-5k}$ c) $\frac{c-ab}{a-b-c}$ 32. a) $\frac{27p}{27-p}$ c) $\frac{-5p+6q^2-4}{7q+8p+4}$
 b) $\frac{-6m-n}{-4}$ d) $\frac{(x+1)(x-2)}{3-x}$ b) $\frac{-m-n}{n-m}$ d) $\frac{(j-7)(j+1)}{(-j+1)(j+7)}$
33. a) $\frac{4m-3p}{12m^2p} = \frac{-20m^2p^2+15mp^3}{-60m^3p^3}$ 34. a) $\frac{9ab^2-9b^3}{27a^2b^6}$
 b) $\frac{25k^2m}{9m^2-4k} = \frac{125k^4mn}{45k^2m^2n-20k^3n}$ b) $\frac{9m^2+10m+1}{m^2+6m+5}$
 c) $\frac{2+x}{3+x} = \frac{4+4x+x^2}{6+5x+x^2}$ c) $\frac{4a+b}{3a-2b} = \frac{28a^2+7ab}{21a^2-14ab}$
 d) $\frac{14z^2-56}{-35z-70} = \frac{-2z+4}{5}$ d) $\frac{-15+15a}{10}$
35. a) $\frac{-35x+55}{20x-10}$ c) $\frac{-15p^2+61p-60}{3p^2+13p-30}$ 36. a) $\frac{12r-24t}{3r+6t}$ c) $\frac{4u-7v}{u+3v}$
 b) $\frac{90m+40n}{26m-8n}$ d) $\frac{84r-14s}{7r+7s}$ b) $\frac{49a+98c}{70a-175}$ d) $\frac{4y^2-49z^2}{16y-56z}$
37. a) $\frac{y^2}{4xy}; \frac{x^2}{4xy}$ 38. a) $\frac{7s+7}{35s}; \frac{15s+5s^2}{35s}$
 b) $\frac{81s}{90r^2s^2}; \frac{100r}{90r^2s^2}$ b) $\frac{12k^2}{44km}; \frac{132m^2}{44km}$
 c) $\frac{6}{2(v^2-4)}; \frac{5v^2-10v}{2(v^2-4)}$ c) $\frac{-r^2-9r-18}{3(r+3)(r-3)}; \frac{12}{3(r+3)(r-3)}$
 d) $\frac{6p+2}{10+10q}; \frac{25p}{10+10q}$ d) $\frac{4m+4n}{m(m+n)}; \frac{5m+mn}{m(m+n)}$
39. a) $\frac{4k^2+2k-2}{k(k+1)^2(k-1)}; \frac{10k^3-10k^2}{k(k+1)^2(k-1)}$ 40. a) $\frac{12+4u}{12u(u-1)}; \frac{12+3u}{12u(u-1)}$
 b) $\frac{-24a+12b}{-3a-12b}; \frac{5a+6b}{-3a-12b}$ b) $\frac{(4d+5f)^2}{(4d-5f)(4d+5f)}; -\frac{(4d-5f)^2}{(4d+5f)(4d-5f)}$
 c) $\frac{8}{12s-6t}; \frac{-15}{12s-6t}$ c) $\frac{12+4d}{4c^2+4cd}; \frac{c^2+6cd}{4c^2+4cd}$
 d) $\frac{1-x}{1-x^2}; \frac{1+x}{1-x^2}$ d) $\frac{p^2-pq}{p^2-q^2}; \frac{1-q}{p^2-q^2}$
41. a) $\frac{-42}{70a}; \frac{40}{70a}; \frac{35}{70a}$ 42. a) $\frac{6bc}{abc}; \frac{7ac}{abc}; \frac{8ab}{abc}$
 b) $\frac{2-2b}{2-2b^2}; \frac{3-3b}{2-2b^2}; \frac{4+4b}{2-2b^2}$ b) $\frac{15m+15}{-3m^2+3m+6}; \frac{m+1}{-3m^2+3m+6}; \frac{9}{-3m^2+3m+6}$
 c) $\frac{5s^2-15s}{s^2-9}; \frac{-s^2-8s+33}{s^2-9}; \frac{2s}{s^2-9}$ c) $\frac{x^2-7x+12}{x^3-5x^2-2x+24}; \frac{x^2-2x-8}{x^3-5x^2-2x+24}; \frac{x^2-x-6}{x^3-5x^2-2x+24}$
 d) $\frac{6k+2}{42k^2+26k+4}; \frac{35k+10}{42k^2+26k+4}; \frac{-42k-12}{42k^2+26k+4}$ d) $\frac{2r^2-8r+6}{r^3-4r^2+3r}; \frac{r-3}{r^3-4r^2+3r}; \frac{r^2-1}{r^3-4r^2+3r}$
43. a) 2 errors c) 2 errors 44. a) 1 error c) 1 error
 b) 3 errors d) 3 errors b) 3 errors d) 2 errors

45. a) $\frac{x}{y} < \frac{x+a}{y+a}$

b) $\frac{x}{y} = \frac{ax}{ay}$

46. a) $\frac{x}{y} > \frac{x-a}{y-a}$

b) $\frac{x}{y} = \frac{\frac{x}{a}}{\frac{y}{a}}$

47. a) —

b) —

48. a) —

b) —

	true	false		true	false
49. a)	☐	☒	c)	☐	☒
b)	☐	☒	d)	☐	☒

	true	false		true	false
50. a)	☐	☒	c)	☒	☐
b)	☒	☐	d)	☒	☐

	true	false		true	false
51. a)	☒	☐	c)	☒	☐
b)	☒	☐	d)	☒	☐

	true	false		true	false
52. a)	☒	☐	c)	☒	☐
b)	☒	☐	d)	☐	☒

2.3 Addition and Subtraction

53. a) $\frac{p}{10}$
b) -2

c) $-\frac{4t^3}{1+t}$
d) 2

55. a) $\frac{d}{35}$
b) $\frac{9}{4c}$

c) $\frac{x-2}{x^3}$
d) $\frac{3x+2}{6+2x}$

57. a) $\frac{25m-28}{7}$
b) $-\frac{r^2+36}{r}$

c) $\frac{81+x^2}{x}$
d) $\frac{1-32d^2}{7d}$

59. a) $2a$
b) $\frac{6-n}{12n}$

c) $-\frac{5}{8n}$
d) $\frac{b-a}{a^2b^2}$

61. a) $\frac{2u}{w-x}$
b) $\frac{4rs}{s^2-r^2}$

c) $\frac{1}{y-3}$
d) $\frac{7}{j-3}$

63. a) $\frac{3}{2}$
b) $\frac{1}{6+6x}$

c) $\frac{y}{2y+3z}$
d) $\frac{1}{5(m+2)}$

65.

$\frac{9}{5+5x}$		
$\frac{5}{6+6x}$	$\frac{29}{30+30x}$	
$\frac{2}{3+3x}$	$\frac{1}{6+6x}$	$\frac{4}{5+5x}$

67.

$\frac{2u^2-4u+2v^2-4v}{u^2-v^2}$		
$\frac{u+v-2}{u-v}$	$\frac{u^2-2uv-2u-2v+v^2}{u^2-v^2}$	
$\frac{u+v}{u-v}$	$\frac{2}{v-u}$	$\frac{u-v}{u+v}$

54. a) $\frac{6b}{7}$
b) 1

c) $\frac{-4}{5}$
d) 2

56. a) $-\frac{k}{30}$
b) $-\frac{3}{2m}$

c) $-\frac{a}{3t^2}$
d) $\frac{5}{3}$

58. a) $\frac{3m-5n}{m-n}$
b) $\frac{5y^2+3y-2}{y}$

c) $\frac{6r-27}{4}$
d) $-\frac{4}{c}$

60. a) $\frac{3w-4v}{8vw}$
b) $\frac{33m^2+14m-7}{60m^2}$

c) $\frac{u-v}{20}$
d) $\frac{7}{8y}$

62. a) -1
b) $\frac{1}{b+4}$

c) $\frac{1}{y+1}$
d) 1

64. a) $-\frac{17}{12b+24}$
b) $\frac{s}{3-s}$

c) $\frac{-10x-105y}{4x^2-49y^2}$
d) $\frac{-2}{9f+3}$

66.

$\frac{39+130a}{30a}$		
$\frac{1+18a}{6a}$	$\frac{17+20a}{15a}$	
$\frac{-7+8a}{6a}$	$\frac{4+5a}{3a}$	$\frac{-3-5a}{15a}$

68.

$\frac{-12y^2+30y-15}{4-y^2}$		
$\frac{-2y^2+19y-2}{4-y^2}$	$\frac{-10y^2+11y-13}{4-y^2}$	
$\frac{1+3y}{2-y}$	$\frac{-2+5y}{2+y}$	$\frac{-9-y-5y^2}{4-y^2}$

69. a) $\frac{a+b}{2ab}$ c) $-\frac{1}{2r}$ 70. a) $\frac{6}{u}$ c) $\frac{6y^2-2y+2}{y(y-4)}$
 b) $\frac{4xy-2x-y}{2x^2y}$ d) $\frac{2b}{a+b}$ b) $-\frac{1}{12h}$ d) $\frac{3c+15}{c^2-3c}$
71. a) $\frac{-5c-40d}{9c^2-9d^2}$ c) $\frac{10t+28}{t(t^2-4)}$ 72. a) $\frac{1}{x+y}$ c) $\frac{2k^2+19k+5}{8k^2+12k+4}$
 b) $\frac{m-6}{m+3}$ d) 10 b) $\frac{-2m^2-m+21}{6(1-m^2)}$ d) $\frac{16(3r-5)}{r(r-4)^2}$
73. a) $\frac{3a-2b}{7a+7b}$ c) $\frac{4c}{c-1}$ 74. a) $\frac{r+8}{r-2}$ c) 7
 b) $7u-2$ d) $\frac{8y}{y^2-4}$ b) $\frac{c}{c+1}$ d) $\frac{v^2}{1-v}$

75.

+	$\frac{1}{3x}$	$\frac{-2+x}{4}$	$\frac{3+x}{-1+x}$	$\frac{4}{2+x}$
$\frac{2}{x}$	$\frac{7}{3x}$	$\frac{x^2-2x+8}{4x}$	$\frac{x^2+5x-2}{-x+x^2}$	$\frac{6x+4}{2x+x^2}$
$\frac{1}{2x}$	$\frac{5}{6x}$	$\frac{x^2-2x+2}{4x}$	$\frac{2x^2+7x-1}{-2x+2x^2}$	$\frac{9x+2}{2x^2+4x}$
$\frac{3}{x+1}$	$\frac{10x+1}{3x^2+3x}$	$\frac{x^2-x+10}{4x+4}$	$\frac{x^2+7x}{x^2-1}$	$\frac{10+7x}{x^2+3x+2}$
$\frac{7-x}{1-x}$	$\frac{-3x^2+20x+1}{3x-3x^2}$	$\frac{26-x-x^2}{4-4x}$	$\frac{4-2x}{1-x}$	$\frac{18+x-x^2}{2-x-x^2}$
$\frac{6}{2+x}$	$\frac{19x+2}{6x+3x^2}$	$\frac{20+x^2}{8+4x}$	$\frac{x^2+11x}{-2+x+x^2}$	$\frac{10}{2+x}$

76.

+	$\frac{1}{3+y}$	$\frac{7}{1-y}$	6	$\frac{2-y}{y-1}$
$\frac{5}{2-2y}$	$\frac{-3y-17}{2y^2+4y-6}$	$\frac{19}{2-2y}$	$\frac{-12y+17}{2-2y}$	$\frac{2y+1}{2-2y}$
$\frac{4}{9+3y}$	$\frac{7}{3y+9}$	$\frac{-17y-67}{3y^2+6y-9}$	$\frac{18y+58}{9+3y}$	$\frac{-3y^2+y+14}{3y^2+6y-9}$
$\frac{2}{y-3}$	$\frac{3y+3}{y^2-9}$	$\frac{19-5y}{y^2-4y+3}$	$\frac{6y-16}{y-3}$	$\frac{-y^2+7y-8}{y^2-4y+3}$
-5	$\frac{-5y-14}{y+3}$	$\frac{5y+2}{1-y}$	1	$\frac{7-6y}{y-1}$
$\frac{9}{1+y}$	$\frac{10y+28}{y^2+4y+3}$	$\frac{2y-16}{y^2-1}$	$\frac{6y+15}{1+y}$	$\frac{-y^2+10y-7}{y^2-1}$

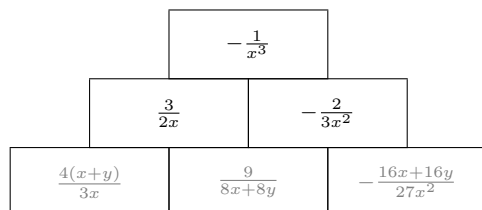
77. a) $\frac{x+4}{x-3}$ c) 0 78. a) $\frac{a-5}{a+4}$ c) $\frac{16}{4-5q}$
 b) $\frac{z-5}{z+5}$ d) $\frac{1}{k+6}$ b) $\frac{2}{p^2-p}$ d) $\frac{t}{t+1}$
79. a) $-\frac{1}{24}$ c) $-\frac{2}{f^2-4f+3}$ 80. a) $-\frac{1}{st}$ c) $\frac{12}{x-7}$
 b) $\frac{u+5}{5u}$ d) $\frac{6}{v^2-1}$ b) $\frac{2}{p}$ d) $\frac{9}{u-4}$

2.4 Multiplication, Powers and Division

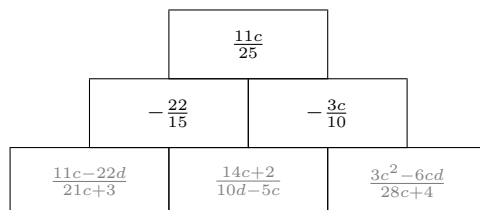
81. a) $\frac{4b}{5c}$ c) $-\frac{3xy}{2z}$ 82. a) $-\frac{rs}{t}$ c) $\frac{50}{7}$
 b) $-\frac{15m}{4}$ d) $\frac{10a}{3}$ b) $-8fz$ d) $\frac{5r^2}{2}$
83. a) $\frac{57}{2m^2}$ c) $-\frac{14b^2}{5c}$ 84. a) $-\frac{49x^2}{9}$ c) $\frac{2t^5}{3r}$
 b) $-\frac{27s}{4}$ d) $\frac{77bc}{8}$ b) $\frac{22}{a^3}$ d) $\frac{12z}{x}$
85. a) -6 c) $-\frac{3}{a+2}$ 86. a) $-\frac{10x}{3}$ c) $-2y-3z$
 b) -4 d) $\frac{5b}{2b-2}$ b) $\frac{-f+2h}{f+3h}$ d) $\frac{d+8}{d-4}$

87. a) $\frac{9b}{2}$ c) $\frac{28m}{27kn}$ 88. a) $\frac{55xy}{6z}$ c) $\frac{9pq^3+9q^4}{20p}$
 b) $-\frac{7d^2}{10c}$ d) $-\frac{8u}{5}$ b) $-\frac{3fh^3}{2g}$ d) $-\frac{15}{2v+2w}$
 89. a) $\frac{1}{2}$ c) $\frac{15k^2(k-1)}{28}$ 90. a) $-\frac{8}{9r-6s}$ c) $\frac{15}{4z-6}$
 b) $\frac{2c-2}{3}$ d) $-\frac{5g+10}{8(g+h)}$ b) $\frac{-2t-2}{t-1}$ d) $\frac{v+2}{2v+2}$

91.

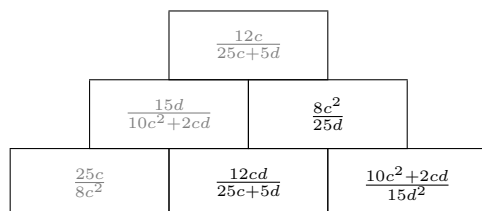


92.

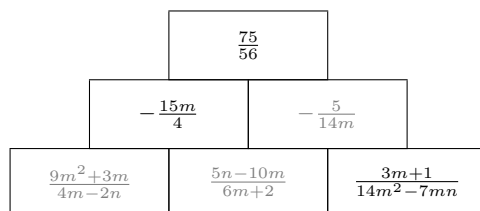


93. a) $\frac{4m-4}{5m^2+10m}$ c) $\frac{a-2b}{2a+4b}$ 94. a) $\frac{c^2+4c-5}{c^2+10c+21}$ c) $\frac{4t+16}{3t-9}$
 b) $-\frac{6}{5x+5x^2}$ d) $\frac{p^2-p-20}{p^2-5p-24}$ b) $\frac{-x^2-2x}{x^2+x-6}$ d) $-\frac{2k+3}{4}$
 95. a) $\frac{9j^4}{64m^2}$ c) $\frac{h^6}{4+4h+h^2}$ 96. a) $\frac{16c^2d^2}{25}$ c) $\frac{9+6d+d^2}{4}$
 b) $-\frac{343}{b^3}$ d) $\frac{1}{a^{24}}$ b) $\frac{512}{125x^6}$ d) $\frac{81y^4}{z^4}$
 97. a) $\frac{121p^2}{9q^2}$ c) $\frac{36-12a^2+a^4}{4a^2}$ 98. a) $\frac{169n^2}{81}$ c) $\frac{16-8x^2+x^4}{4x^2}$
 b) $\frac{144-24m+m^2}{9}$ d) $\frac{s^3+3s^2+3s+1}{s^3}$ b) $\frac{4-12c+9c^2}{c^2}$ d) $\frac{8a^3-12a^2+6a-1}{a^3}$
 99. a) $-\frac{42a}{25b}$ c) $\frac{9b^3}{20}$ 100. a) $10u$ c) $\frac{9}{m^2+4m+4}$
 b) $\frac{3s+27}{s-1}$ d) $\frac{1+x}{x}$ b) $-\frac{105x}{y}$ d) $\frac{3z+6}{32}$
 101. a) $ab+2a^3$ c) $\frac{1}{2}$ 102. a) $\frac{p^4-p^2q^2+p^2}{q^4}$ c) $\frac{6n^2+2}{n^3}$
 b) $\frac{16-49x^2}{x^2}$ d) $-\frac{12y^2+16}{27}$ b) $\frac{k+3}{4k+2}$ d) $\frac{4d}{c}$
 103. a) $\frac{x}{3}$ c) $\frac{3f}{16}$ 104. a) $\frac{3m}{5p}$ c) $\frac{1}{29}$
 b) $-\frac{5a}{4b^2c^2}$ d) $\frac{s}{19t}$ b) $\frac{3}{8q}$ d) $\frac{4v}{25z}$
 105. a) $\frac{4}{9m}$ c) $\frac{2z-3}{6}$ 106. a) $\frac{-3b}{4c}$ c) $\frac{11-f^2}{11}$
 b) $\frac{2a-b}{12a^2b}$ d) $\frac{d-f}{5df}$ b) $\frac{17n}{3p^2}$ d) $\frac{2xy-3yz}{16xz}$
 107. a) $\frac{5np}{12}$ c) $-\frac{9a}{10b^2cd}$ 108. a) $\frac{1}{m^2n^2}$ c) $-\frac{u}{3v}$
 b) $\frac{s}{3k}$ d) $-\frac{4hm^3}{f}$ b) $\frac{21}{20b}$ d) $\frac{6x^2y}{7}$

109.



110.



111. a) $\frac{5c+10}{2}$ c) $-\frac{16s}{15r}$ 112. a) $\frac{-4}{5x(x-y)}$ c) $\frac{m}{2m-10}$
 b) $\frac{u}{4v^2}$ d) $\frac{3z^2+z}{z-2}$ b) $\frac{3}{2a^2-2}$ d) $\frac{3y+3}{2y-4}$
113. a) $-s^2-3st-2t^2$ c) $\frac{x^2-81}{2x^2+10x-48}$ 114. a) $\frac{z^2+4z+3}{2z-2}$ c) $\frac{24d+24f}{7d}$
 b) $\frac{16h(h+m)}{7}$ d) $\frac{27}{p^3+2p^2}$ b) $\frac{r^2+6r+9}{r^2-6r-7}$ d) $\frac{4v^5+4v^4}{3}$
115. a) $\frac{2m^2+3m+1}{m^2+m-6}$ c) $\frac{48p^2+48pq}{p-q}$ 116. a) $\frac{x^2+x-12}{x^2+6x+9}$ c) $\frac{-25+10y-y^2}{5}$
 b) $-16k-12$ d) $\frac{42-3a}{14+2a}$ b) $-\frac{9}{5r+5s}$ d) $\frac{306+n-n^2}{306-n-n^2}$
117. a) $\frac{16a+32b}{9a+6b}$ c) $\frac{9x+3y}{8x-2y}$ 118. a) $\frac{10x^4}{x^2+6x+9}$ c) $\pm \frac{2}{3+m}$
 b) $\frac{32m+64}{27m-27}$ d) $\frac{2y+2z}{3y}$ b) $\frac{16c+16}{21c+84}$ d) $\frac{33d-33c}{5c+5d}$
119. a) 4 c) $\frac{3cd^2-2c^2d}{3}$ 120. a) $\frac{30mn^2-30n^3}{m+n}$ c) $-\frac{1}{4q^2+5pq}$
 b) $\frac{14s}{5r}$ d) $\frac{12k-2}{18k+3}$ b) $\frac{3y^2+3yz}{y-z}$ d) $\frac{3}{2}$
121. a) $\frac{5ad}{6bc}$ c) $\frac{a}{6bc}$ 122. a) $18u^2$ c) $\frac{15uw}{4v}$
 b) $\frac{8ac}{3b}$ d) $8ac$ b) $\frac{9uz}{10vw}$ d) $\frac{3u}{20vw}$
123. a) $\frac{6ac}{5b}$ c) $\frac{6dh}{5gn}$ 124. a) $\frac{72z^2}{13y}$ c) $\frac{2r^2}{45m^4}$
 b) $\frac{8uvy^2}{3}$ d) $\frac{m}{12p^2}$ b) $\frac{18ps^2}{55qr}$ d) $\frac{9w^2yz}{5ax^3}$
125. a) $\frac{8x}{15}$ c) $\frac{5a-5}{2a^2+10a+8}$ 126. a) $-\frac{5}{6}$ c) $\frac{f^2-4f-21}{f^2+3f-4}$
 b) $-\frac{6}{5}$ d) $\frac{d^2+2dk}{3dk-6k^2}$ b) $\frac{3x+3y}{4}$ d) $\frac{p^2-4}{3}$
127. a) $\frac{3}{7}$ c) $\frac{4b}{12c-ab}$ 128. a) 1 c) $\frac{x-y}{2}$
 b) 8 d) $\frac{9uv-u^2}{u^2+9uv-v^2}$ b) $\frac{35}{2v+u}$ d) $\frac{c^2-d^2}{c^2+d^2+2}$
129. a) $\frac{-2r-1}{7r+5}$ c) $\frac{16w^2}{4uwx-3u^2z^2}$ 130. a) $\frac{4n^3}{3mn^2-3m^2}$ c) $-\frac{12}{25}$
 b) $\frac{z-6}{4z+10}$ d) $\frac{3}{14}$ b) $-v-3$ d) $\frac{4t^2-12st+9s^2}{st-2t+3s}$
131. a) $\frac{16-24p}{25p+5q}$ b) $\frac{2h+2k}{4h^2-k^2}$ 132. a) $\frac{x^2+4x-5}{6x^2+12x}$ b) $\frac{9y^2-18y+9}{y^2-5y+6}$

2.5 Miscellaneous Exercises

133. a) true ☒ false ☐ c) true ☒ false ☐ 134. a) true ☒ false ☐ c) true ☒ false ☐
 b) true ☒ false ☐ d) true ☒ false ☐ b) true ☐ false ☒ d) true ☒ false ☐
135. a) true ☒ false ☐ c) true ☒ false ☐ 136. a) true ☐ false ☒ c) true ☒ false ☐
 b) true ☐ false ☒ d) true ☐ false ☒ b) true ☒ false ☐ d) true ☐ false ☒
137. a) $\frac{a^2x+2a-x}{a^2}$ c) $\frac{m-n}{m}$ 138. a) $\frac{6p^2-3p^2q+2q}{p^2}$ c) 0
 b) 0 d) $\frac{100r^2-180r+81}{r}$ b) $\frac{4s^2+t^2}{2st}$ d) $\frac{18h^2-12h+2}{3h}$

139. a) $-\frac{25}{2}$ c) $\frac{1}{x-3}$ 140. a) $\frac{16c^2+49d^2}{49d^2}$ c) 0
 b) $\frac{4n^2+9n+16}{5n^2-10n}$ d) $\frac{25-30b}{36b^2}$ b) $\frac{4}{z+2}$ d) $-\frac{w^2+z^2}{w}$
141. a) $\frac{3}{4}$ c) $-\frac{4d^2}{c+d}$ 142. a) $-\frac{8}{9k}$ c) $-\frac{10x}{3y}$
 b) $\frac{-a-b}{ab}$ d) $\frac{x+3}{x+4}$ b) $\frac{7}{9}$ d) $\frac{2a}{a-2c}$
143. a) $-$ c) $:$ 144. a) $+$ c) $\cdot$
 b) $\cdot$ d) $-$ b) $+$ d) $:$
145. a)

true	false
☐	☒
b) ☒	☐

 c)

true	false
☒	☐
d) ☐	☒

 146. a)

true	false
☒	☐
b) ☐	☒

 c)

true	false
☒	☐
d) ☒	☐
147. a) 0 b) $\frac{8}{(4-x)^3}$ 148. a) $-\frac{3}{(y-1)(y+1)^2}$ b) 0
149. a) $-p-2q$ b) $\frac{4x+4}{5}$ 150. a) $6q$ b) $\frac{2m+3}{6m^2-2m^3}$

2.6 Rational Equations

151. a) 4; -5 c) 0; -0.5 152. a) 0; 1 c) 5
 b) -1; 8 d) 3; -4 b) $\frac{6}{5}$; 9 d) 0; -8
153. a) $\mathbb{R} \setminus \{0, 2\}$ b) $\mathbb{R} \setminus \{-1, 9\}$ 154. a) $\mathbb{R} \setminus \{3, 4\}$ b) $\mathbb{R} \setminus \{-2, 0\}$
155. a) $-$ b) $-$ 156. a) $-$ b) $-$
157. a) yes b) no 158. a) no b) yes
159. a) 1 c) 1.9 160. a) 1 c) $\frac{16}{13}$
 b) 1 d) $-\frac{2}{3}$ b) $-\frac{1}{60}$ d) $\frac{39}{40}$
161. a) 4 c) 4 162. a) 0 c) $\{\}$
 b) 4 d) $\{\}$ b) 0 d) 0
163. a) 2.5 c) $-\frac{2}{13}$ 164. a) -1 c) $\{\}$
 b) $\{\}$ d) $\{\}$ b) 1 d) 0
165. a) 4 c) $\frac{4}{73}$ 166. a) $\frac{3}{4}$ c) $-\frac{15}{64}$
 b) 5 d) $\{\}$ b) 27 d) $\frac{12}{13}$
167. a) $-\frac{2}{5}$ c) $\frac{13}{3}$ 168. a) $\{\}$ c) -2
 b) 1 d) $\mathbb{R} \setminus \{-6, -4\}$ b) $\frac{1}{2}$ d) $\{\}$
169. a) -3 c) 2, 3 170. a) $\frac{1}{2}$ c) -9, 4
 b) 4 d) -7, 0, 1 b) 5 d) 0, 3, 10
171. a) $-\frac{7}{17}$ c) $\{\}$ 172. a) $\frac{35}{11}$ c) 1
 b) $\{\}$ d) $\mathbb{R} \setminus \{\pm 2\}$ b) $\{\}$ d) $\{\}$
173. a) 5 c) -5 174. a) 3 c) -1
 b) $\{\}$ d) $-\frac{2}{3}$ b) 4 d) $\frac{3}{2}$

175. a) -9 c) -4 176. a) 1 c) 7
 b) 5 d) $\frac{4}{5}$ b) -3 d) $\frac{3}{2}$
177. a) $-\frac{1}{2}, \frac{1}{4}$ c) $-3, 0$ 178. a) $-\frac{1}{6}, \frac{1}{5}$ c) $-7, 1.5$
 b) $-\frac{2}{3}$ d) $-2, 4$ b) $-\frac{1}{3}, \frac{1}{4}$ d) -3
179. a) $-\frac{1}{2}$ c) 1 180. a) -3 c) 0
 b) 0 d) 8 b) 0 d) $\{\}$
181. a) Area of a trapezium;
 $a = \frac{2A}{h} - c, c = \frac{2A}{h} - a, h = \frac{2A}{a+c}$
 b) Area of an equilateral triangle;
 $s = \sqrt{\frac{4A\sqrt{3}}{3}}$
 c) Area of a kite, a rhombus;
 $e = \frac{2A}{f}, f = \frac{2A}{e}$
 d) Area of a general triangle;
 $a = \frac{2A}{h_a}, h_a = \frac{2A}{a}$
182. a) Altitude of an equilateral triangle;
 $s = \frac{2h\sqrt{3}}{3}$
 b) Right triangle altitude theorem;
 $h = \sqrt{pq}, p = \frac{h^2}{q}, q = \frac{h^2}{p}$
 c) Leg rule in the right triangle;
 $a = \sqrt{pc}, p = \frac{a^2}{c}, c = \frac{a^2}{p}$
 d) Leg rule in the right triangle;
 $b = \sqrt{qc}, q = \frac{b^2}{c}, c = \frac{b^2}{q}$
183. a) Circumference of a circle;
 $d = \frac{u}{\pi}$
 b) Arc length of a circular sector;
 $\alpha = \frac{180^\circ \cdot b}{\pi r}, r = \frac{180^\circ \cdot b}{\alpha \pi}$
 c) Area of a circle;
 $r = \sqrt{\frac{A}{\pi}}$
 d) Area of a circular sector;
 $\alpha = \frac{360^\circ \cdot A}{\pi r^2}, r = \sqrt{\frac{360^\circ \cdot A}{\pi \alpha}}$
184. a) Volume of a circular cylinder;
 $r = \sqrt{\frac{V}{\pi h}}, h = \frac{V}{\pi r^2}$
 b) Lateral surface area of a circ. cylinder;
 $r = \frac{M}{2\pi h}, h = \frac{M}{2\pi r}$
 c) Circumference of a circle;
 $r = \frac{u}{2\pi}$
 d) Area of a circle;
 $d = \sqrt{\frac{4A}{\pi}}$
185. a) $\frac{6}{5a}$ c) $\frac{a^2-a}{2-a}$ 186. a) $\frac{2}{3}b^2$ c) $\frac{b}{1+b}$
 b) $\frac{5-5a}{4}$ d) $\frac{-a-4b}{3}$ b) $\frac{4b^2-6}{3}$ d) 1
187. a) $\{\}$ c) $\frac{5m}{k^2+1}$ 188. a) $-\frac{3}{4s-r}$ c) $-\frac{3m+4n}{2n+1}$
 b) $\frac{m}{1-3m}$ d) $\frac{4d}{c}$ b) $\frac{7}{20}$ d) $-r$
189. a) $\frac{b}{1-10b}$ c) $\frac{s-s^2}{3s+5}$ 190. a) $-\frac{18f}{11}$ c) $\frac{b^2+2b}{b+4}$
 b) $-\frac{4n+n^2}{2n-4}$ d) $\frac{1}{d}$ b) $-\frac{5a+1}{8a+2}$ d) $\frac{vw}{3v+2w}$
191. a) $-t-r$ c) tr 192. a) $5a$ c) $\frac{1}{d-4}$
 b) $m-n$ d) $\frac{1}{m-4}$ b) $\frac{1}{u+1}$ d) c
193. a) if $a \neq 0 : \frac{10a+15}{a}$,
 if $a = 0 : \{\}$
 b) if $b \neq -3 : 3$,
 if $b = -3 : \mathbb{R} \setminus \{0\}$
 c) if $s \neq -1 : \frac{1-s}{5}$,
 if $s = -1 : \mathbb{R} \setminus \{-1, \frac{1}{6}\}$
 d) if $c \neq -1 : -\frac{1}{c+1}$,
 if $c = -1 : \{\}$
194. a) if $a \neq 5 : \frac{a}{5-a}$,
 if $a = 5 : \{\}$
 b) if $a \neq 1 : \frac{5a+5}{a-1}$,
 if $a = 1 : \{\}$
 c) if $m \neq 0 : -m$,
 if $m = 0 : \mathbb{R} \setminus \{0\}$
 d) if $c \neq 1 : c+1$,
 if $c = 1 : \mathbb{R} \setminus \{1\}$

195. 12.5 196. 16 197. 12 198. -6
199. $\frac{4}{9}$ 200. $\frac{21}{30}$ 201. $\frac{-6}{-20}$ 202. $\frac{39}{14}$
203. 11 204. 2 205. $\frac{320}{480}$ 206. $\frac{729}{810}$
207. -7 208. -1 209. $\frac{18}{16}$ 210. $\frac{225}{270}$
211. 17 212. 29 213. Fr. 36.- 214. 28
215. 119 h 216. 4000 217. 9 h 10 min 218. 9 h
219. 12:24 220. 3.6 h 221. 8.75 h 222. $110 \frac{\text{km}}{\text{h}}$
223. a) $] - \infty, 0[\cup] \frac{1}{4}, \infty[$
b) $] 0, 4[$
c) $] - \infty, -9[\cup] 0, \infty[$
d) $] \infty, 0[\cup] 1, \infty[$
225. a) $] - 4, -\frac{15}{4}[$ c) $] - 4, 0[$
b) $] - 8, -4[$ d) $] - \frac{17}{4}, -4[$
227. a) $] - 3, -1[$
b) $] - \infty, -5] \cup] - 3, \infty[$
c) $] - \infty, -7] \cup] - 3, \infty[$
d) $] - 4, -3[$
229. a) $] - \infty, \frac{1}{7}[\cup] 1, \infty[$
b) $] - \infty, -\frac{1}{3}] \cup] 1, \infty[$
c) $] -\frac{1}{8}, 1[$
d) $] - \infty, 1[\cup] \frac{11}{5}, \infty[$
231. a) $] - \infty, -3[$ c) $] - 2, \infty[$
b) $] - 2, \infty[$ d) $] - \infty, -5[$
233. a) $] - \infty, -3[\cup] 2, \infty[$
b) $] - 8, -6[$
c) $] - 1, 8[$
d) $] - \infty, -2.5[\cup] 6, \infty[$
235. a) $] - 6, 4[$
b) $] - 8, -3[\cup] 6, \infty[$
c) $] - 10, -4[\cup] 2, \infty[$
d) $] - 9, 0[\cup] 3, \infty[$
237. a) $] - \infty, -4[\cup] 2, \infty[$
b) $] - \infty, -5[\cup] 3, 4[$
c) $] - 8, -3[\cup] 5.5, \infty[$
d) $] - \infty, -2] \cup] - 1, 0[\cup] 1, \infty[$
224. a) $] 0, \frac{1}{7}[$
b) $] - \infty, 0[\cup] 7, \infty[$
c) $] - \infty, -1[\cup] 0, \infty[$
d) $] - 25, 0[$
226. a) $] 5, \frac{26}{5}[$ c) $] 0, 5[$
b) $] 5, 10[$ d) $] \frac{24}{5}, 5[$
228. a) $] - \infty, 6[\cup] 10, \infty[$
b) $] - \infty, 0[\cup] 6, \infty[$
c) $] - \infty, -2[\cup] 6, \infty[$
d) $] - \infty, 6[\cup] 12, \infty[$
230. a) $] - \infty, -\frac{4}{3}[\cup] 0, \infty[$
b) $] - \frac{4}{3}, \frac{17}{6}[$
c) $] - \frac{4}{3}, 7[$
d) $] - \infty, -\frac{4}{3}[\cup] - \frac{13}{16}, \infty[$
232. a) $] - 8, \infty[$ c) $] - \infty, 3[$
b) $] 4, \infty[$ d) $] - \infty, -4.5[$
234. a) $] - 0.5, 7[$
b) $] - \infty, -9[\cup] - 3, \infty[$
c) $] - 2, 4[$
d) $] - 6, 1[$
236. a) $] - \infty, -3[\cup] 4, \infty[$
b) $] - \infty, 1[\cup] 2, 5[$
c) $] - 1.5, -1[\cup] 4, \infty[$
d) $] - 3, -2] \cup] 7, \infty[$
238. a) $] - 2, 5[$
b) $] - \infty, -8] \cup] - 2, 2[$
c) $] - \infty, -6[\cup] - 2, 4[$
d) $] - \infty, -5[\cup] - 3, 0] \cup] 1, \infty[$

3 Functions

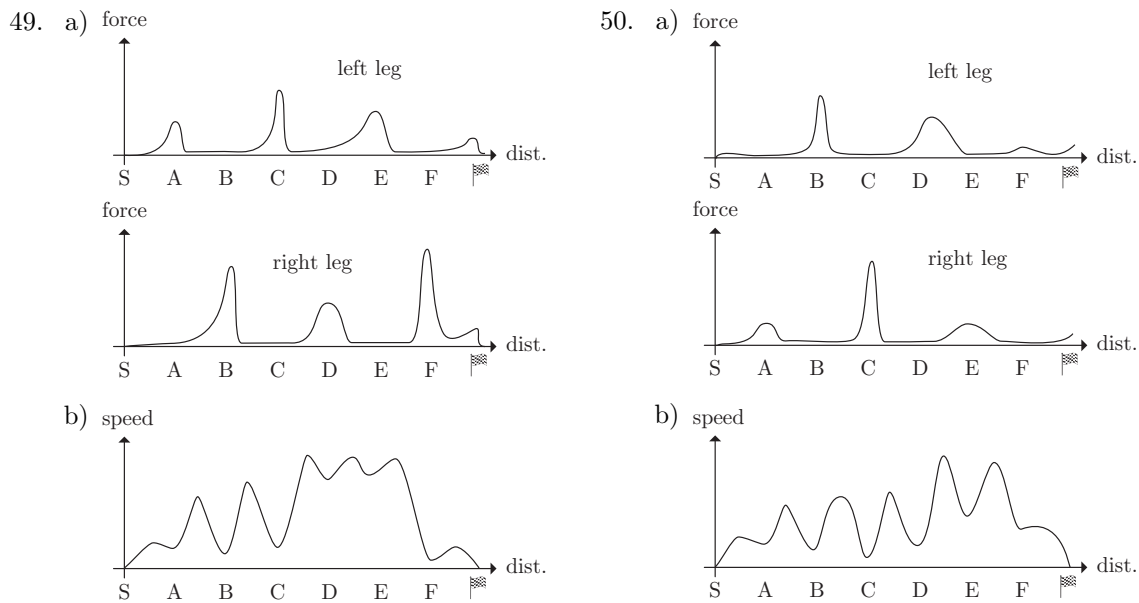
3.1 Functional Concept

1. a) yes c) yes 2. a) yes c) no
 b) no d) no b) no d) yes
3. a) $\rightarrow$ c) $\leftarrow$ 4. a) $\leftrightarrow$ c) neither
 b) $\leftrightarrow$ d) neither b) $\rightarrow$ d) $\rightarrow$
5. a) yes c) yes 6. a) no c) yes
 b) no d) no b) yes d) yes
7. a) A number is assigned its double.
 b) A number is assigned the number that is one greater than it.
 c) A number is assigned its reciprocal value.
 d) A number is assigned five times its value plus three.
8. a) A number is assigned its half.
 b) A number is assigned its square root.
 c) A number is assigned its square.
 d) A number is assigned two less than its triple.
9. $x \mapsto 90^\circ - x$ 10. $n \mapsto (n - 2) \cdot 180^\circ$
11. a) $y = 2x$ d) $y = 2x + 1$ 12. a) $y = 3x$ d) $y = x^2 + 1$
 b) $y = x^2$ e) $y = \frac{1}{x}$ b) $y = x + 7$ e) $y = |x|$
 c) $y = x + 5$ f) $y = -x^3$ c) $y = 2x - 1$ f) $y = \sqrt{x}$
13. a) $y = 4x$ c) $y = 6x^2$ 14. a) $y = \sqrt{2}x$ c) $y = \frac{1}{16}x^2$
 b) $y = \frac{\sqrt{3}}{4}x^2$ d) $y = \left(\sqrt{\frac{x}{6}}\right)^3$ b) $y = \sqrt{x}$ d) $y = \frac{2\sqrt{3}}{3}x$
15. a) $\mathbb{X} =$ All primary school pupils, $\mathbb{Y} = \{1, 2, 3, 4, 5, 6\}$
 b) $\mathbb{X} =$ All Swiss citizens, $\mathbb{Y} = \{0, 1\}$
 c) $\mathbb{X} = \mathbb{N}$, $\mathbb{Y} = \mathbb{N}$
16. a) $\mathbb{X} =$ All train passengers, $\mathbb{Y} = \{1, 2\}$
 b) $\mathbb{X} =$ All lottery players, $\mathbb{Y} = \{0, 1, 2, 3, 4, 5, 6\}$
 c) $\mathbb{X} = \mathbb{R}$, $\mathbb{Y} = \mathbb{R}_0^+$
17. a) $\mathbb{X} = \mathbb{R}$ e) $\mathbb{X} = \mathbb{R} \setminus \{-8\}$ 18. a) $\mathbb{X} = \mathbb{R}$ e) $\mathbb{X} = [-3, \infty[$
 b) $\mathbb{X} = [1, \infty[$ f) $\mathbb{X} = \mathbb{R}$ b) $\mathbb{X} = \mathbb{R} \setminus \{-5\}$ f) $\mathbb{X} = \mathbb{R}^+$
 c) $\mathbb{X} = \mathbb{R} \setminus \{-2, 2\}$ g) $\mathbb{X} = [2, \infty[$ c) $\mathbb{X} = \mathbb{R}$ g) $\mathbb{X} =] - \infty, \frac{4}{3}]$
 d) $\mathbb{X} = \mathbb{R}$ h) $\mathbb{X} =] - \infty, 7]$ d) $\mathbb{X} = \mathbb{R}$ h) $\mathbb{X} =]6, \infty[$
19. a) $\mathbb{Y} = \mathbb{R}_0^+$ c) $\mathbb{Y} = [-4, \infty[$ 20. a) $\mathbb{Y} = [1, \infty[$ c) $\mathbb{Y} = \mathbb{R}_0^-$
 b) $\mathbb{Y} = \mathbb{R}$ d) $\mathbb{Y} = \mathbb{R}_0^+$ b) $\mathbb{Y} = \mathbb{R}$ d) $\mathbb{Y} = \mathbb{R}$

21. a) $[5, 7]$ b) $[4, 8[$ c) $\mathbb{R}$ d) $[11, \infty[$
22. a) $[-6, -4]$ b) $[-9, -\frac{1}{2}[$ c) $] -\infty, -3[$ d) $[-\frac{9}{2}, \infty[$
23. a) $[0, 36]$ b) $[0, 36]$ c) $[0, 49]$ d) $[196, 324]$
24. a) $[-100, -1]$ b) $[-81, 0]$ c) $[-49, -16]$ d) $\mathbb{R}_0^-$
25. a) 10, -18, 2 26. a) 0, -21, -6
 b) 26, 1 b) 12, 4
 c) $\mathbb{X} = \mathbb{Y} = \mathbb{R}$ c) $\mathbb{X} = \mathbb{Y} = \mathbb{R}$
27. a) $6, 2\sqrt{3} + 3, 4$ 28. a) 15, 8, 0
 b) 5, 49 b) 24, ± 3
 c) 121 c) ± 2
 d) $\mathbb{X} = \mathbb{R}_0^+, \mathbb{Y} = [3, \infty[$ d) $\mathbb{X} = \mathbb{R}, \mathbb{Y} = [-1, \infty[$
29. a) -11, 22, 7 30. a) -1, 17, -55
 b) -4, 7 b) 4, -8
 c) 1, 10 c) $0, \frac{5}{6}$
 d) -17 d) -22
31. a) -95, -4, 4.75 32. a) -4, $-2\sqrt{5}$, impossible
 b) -31, ± 7 b) -10, 144
 c) 5, $y < 5$ c) $x \geq 0, y \leq 0$
 d) $\mathbb{X} = \mathbb{R}, \mathbb{Y} =] -\infty, 5]$ d) $\mathbb{X} = \mathbb{R}_0^+, \mathbb{Y} =] -\infty, 0]$
33. a) 235 kWh, 140 kWh, 250 kWh, 245 kWh 34. a) 600 m, 600 m, 300 m, 200 m
 b) 1, 105 kWh b) $\approx 0.6, 5, \approx 5.7, 6$
 c) 22, 28 c) 7, ≈ 615 m
 d) 220 kWh d) 3, 3.5
 e) 19 e) $[0, 1], \approx [1.8, 3.4], \approx [5.2, 5.8]$
 f) No, the mapping is not unique. f) No, the mapping is not unique.
35. a) 10, 2, 11 36. a) 18, 7, 38
 b) 8, 0, 7 b) 14, 3, 26
 c) 10 c) 32
 d) 10, 2, 7 d) 11, 4, 6
 e) 4, 8, 9 e) 10, 36, 7
37. a) $x \mapsto x - 2$ 38. a) $x \mapsto x + 3$
 b) $x \mapsto 2x$ b) $x \mapsto 3x + 1$
 c) yes, no c) yes, no
 d) 7, -6 d) -7, 40
 e) $2x - 7, 2x - 10$ e) $3x + 14, 3x + 22$
39. a) 15, 121 40. a) 12, 49
 b) 29, 81 b) 32, 49
 c) -2, 4 c) 1
 d) $h(x) = 8x + 12$ d) $h(x) = 2x^2 - 1$

41. a) N, V, C
b) VFKXOH, NODVVH
c) SCHREIBTISCH
43. a) H, P, W
b) AYHT, CLSV
c) KINDERWAGEN
45. a) c_8
b) c_{18}
c) c_{17} (ANANAS)
d) MATHE IST COOL
47. a) FUNKTION, c_{11}
b) $k = 13$
c) -
42. a) I, A, O
b) DOJHEUD, JHRPHWULH
c) WANDTAFEL
44. a) X, G, Q
b) GYHMU, MJCHX
c) HAUSWART
46. a) c_{10}
b) c_{16}
c) c_{19} (MISSISSIPPI)
d) PUNKT VOR STRICH
48. a) ALGEBRA, c_{23}
b) $c_{-k} = c_{26-k}$
c) You have to try at most 25 times.

3.2 Graphs

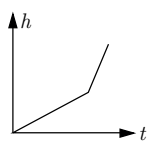


51. a) ①-B, ②-D, ③-H, ④-E, ⑤-F, ⑥-C

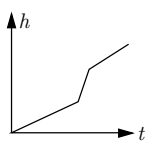
$\mathcal{A}$

$\mathcal{G}$

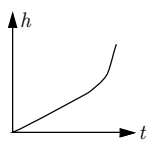
b) $\mathcal{A}$



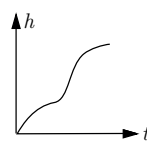
$\mathcal{B}$



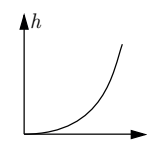
$\mathcal{C}$



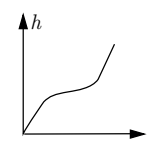
$\mathcal{D}$

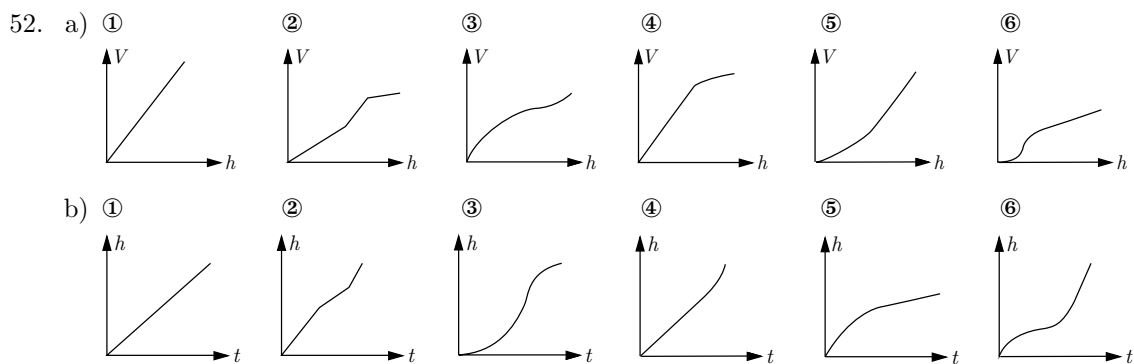


$\mathcal{E}$

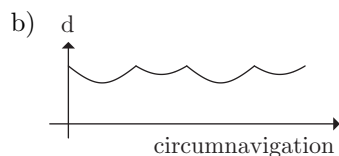


$\mathcal{F}$





53. a) $\mathcal{A}$ -④, $\mathcal{B}$ -②, $\mathcal{C}$ -⑥, $\mathcal{D}$ -⑤, $\mathcal{E}$ -⑦, $\mathcal{F}$ -①
③ has no diagram

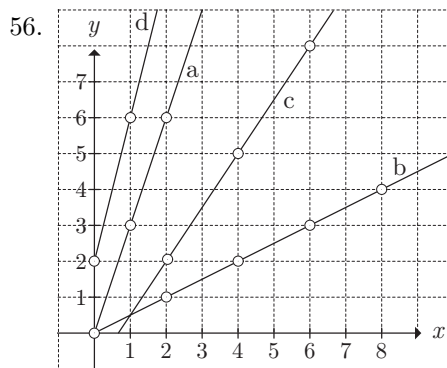
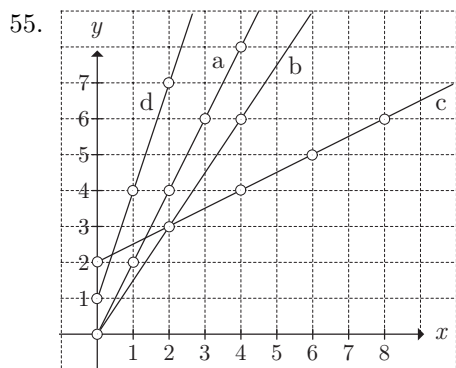
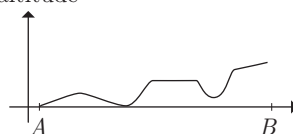


54. a) the second

b) first: $A \circ \text{---} \text{---} \text{---} \text{---} \text{---} \text{---} \text{---} \text{---} B$

third: $A \circ \text{---} \text{---} \text{---} \text{---} \text{---} \text{---} \text{---} \text{---} B$

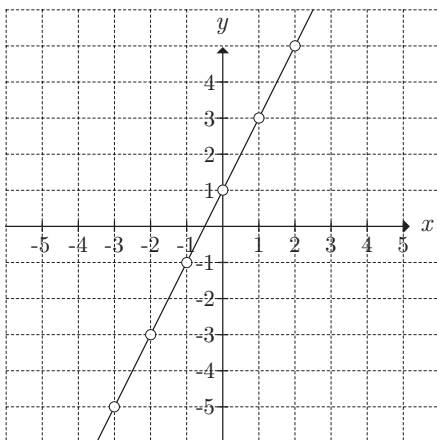
c) altitude



57. a) Both receive exactly the same amount. But I should receive more because I am older.
b) This variant is great, I get exactly 8 times as much as Sara.
c) This variant is unfair, Sara gets twice as much as I do!
d) This variant is unfair for small amounts, but it gets better for larger amounts (5 or more).
58. e) We receive almost the same amount, Lola a little more. I can just about accept that.
f) This option is great because I get five times as much as Lola.
g) With this option, I always receive 2 francs less than Lola.
h) This option is great for small amounts, but for larger amounts (7 or more) it becomes increasingly unfair.

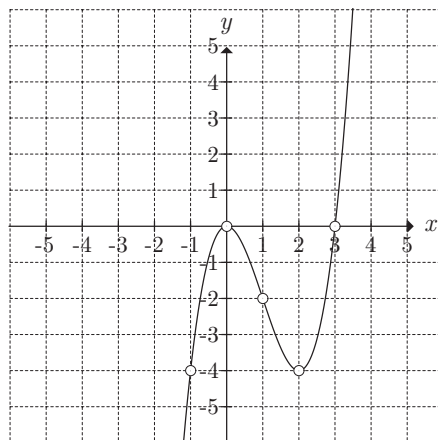
59. a)

x	-3	-2	-1	0	1	2	3
y	-5	-3	-1	1	3	5	7



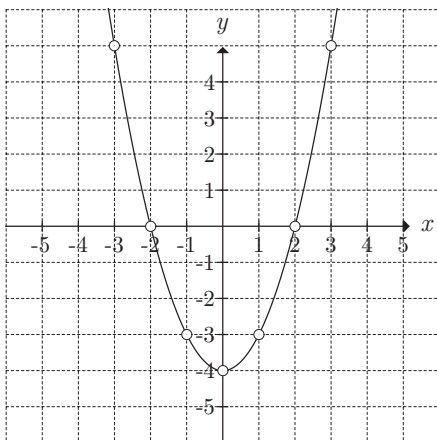
d)

x	-2	-1	0	1	2	3	4
y	-20	-4	0	-2	-4	0	16



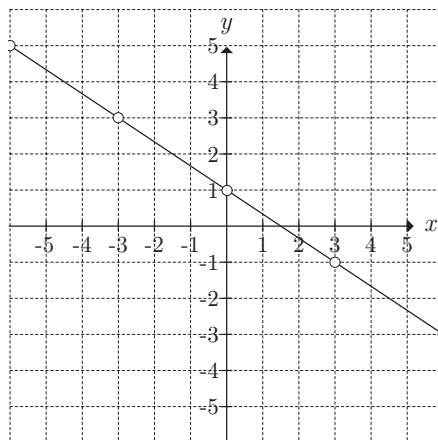
b)

x	-3	-2	-1	0	1	2	3
y	5	0	-3	-4	-3	0	5



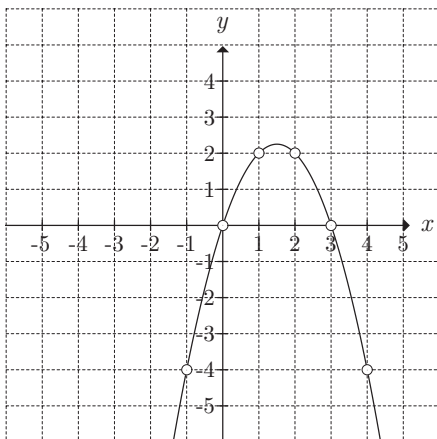
e)

x	-9	-6	-3	0	3	6	9
y	7	5	3	1	-1	-3	-5



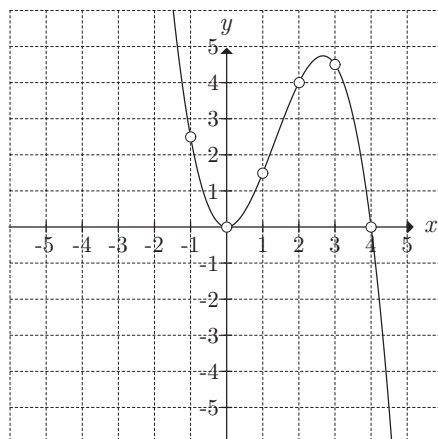
c)

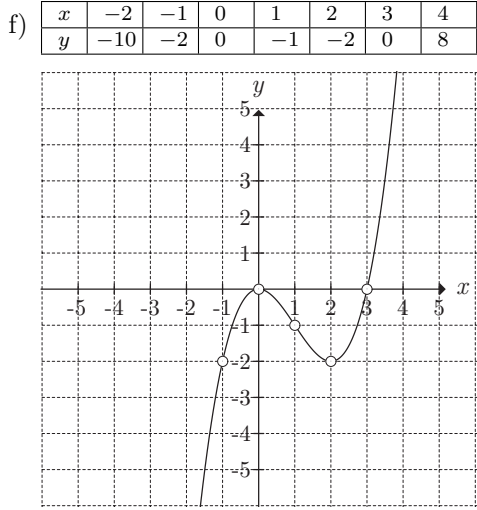
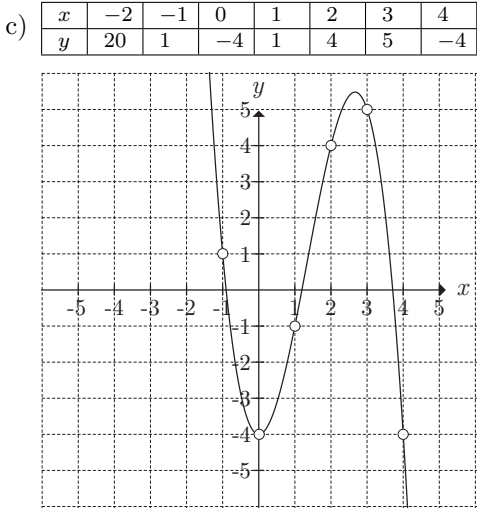
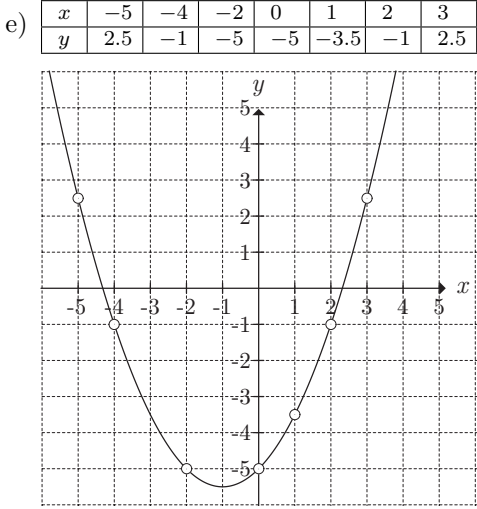
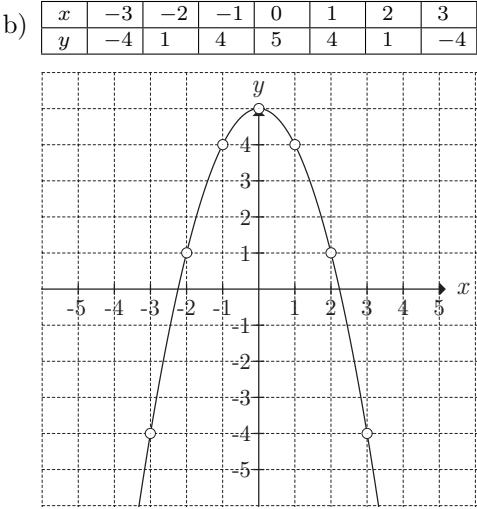
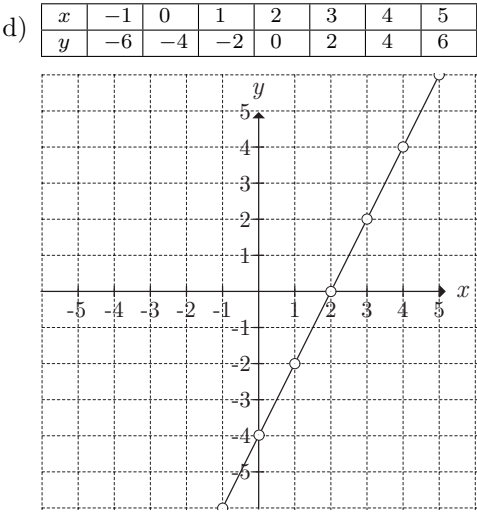
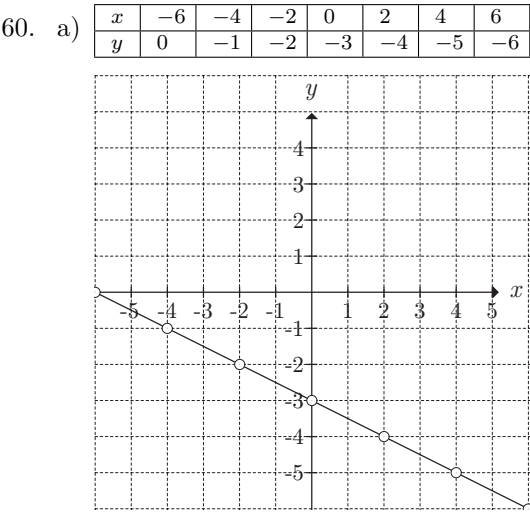
x	-2	-1	0	1	2	3	4
y	-10	-4	0	2	2	0	-4



f)

x	-2	-1	0	1	2	3	4
y	12	-1	0	1.5	4	4.5	0





61. a) no b) yes c) yes d) no e) no

		true	false		true	false
63.	a)	☒	☐	c)	☐	☒
	b)	☒	☐	d)	☐	☒

		true	false		true	false
65.	a)	☐	☒	c)	☒	☐
	b)	☒	☐	d)	☐	☒

		true	false		true	false
67.	a)	☒	☐	c)	☐	☒
	b)	☐	☒	d)	☒	☐

62. a) no b) yes c) no d) yes e) yes

		true	false		true	false
64.	a)	☒	☐	c)	☒	☐
	b)	☒	☐	d)	☐	☒

		true	false		true	false
66.	a)	☒	☐	c)	☒	☐
	b)	☐	☒	d)	☐	☒

		true	false		true	false
68.	a)	☐	☒	c)	☐	☒
	b)	☒	☐	d)	☐	☒

4 Linear Functions

4.1 Proportionality, Inverse Proportionality

1. a) proportional d) inv. prop. 2. a) inv. prop. d) neither
 b) neither e) neither b) proportional e) inv. prop.
 c) proportional f) proportional c) proportional f) neither

3. a)

x	1	2	3	5
y	4	8	12	20

d) $y = 4x$

b)

x	2	4	8	$\frac{25}{3}$
y	6	12	24	25

$y = 3x$

c)

x	-12	-3	18	54
y	16	4	-24	-72

$y = -\frac{4}{3}x$

4. a)

x	4	6	8	12
y	6	9	12	18

d) $y = \frac{3}{2}x$

b)

x	-4	6	-16	48
y	2	-3	8	-24

$y = -\frac{1}{2}x$

c)

x	4	12	28	48
y	5	15	35	60

$y = \frac{5}{4}x$

5. a)

x	1	2	3	5
y	4	2	$\frac{4}{3}$	$\frac{4}{5}$

d) $y = \frac{4}{x}$

b)

x	2	1	8	$\frac{2}{5}$
y	6	12	$\frac{3}{2}$	30

$y = \frac{12}{x}$

c)

x	-12	-3	$\frac{1}{2}$	54
y	1	4	-24	$-\frac{2}{9}$

$y = -\frac{12}{x}$

6. a)

x	4	6	8	12
y	6	4	3	2

d) $y = \frac{24}{x}$

b)

x	-4	$\frac{8}{3}$	-1	48
y	2	-3	8	$-\frac{1}{6}$

$y = -\frac{8}{x}$

c)

x	4	12	$\frac{36}{7}$	3
y	45	15	35	60

$y = \frac{180}{x}$

7. a) proportional, $y = \frac{3}{5}x$,

x	-4	5	8	14
y	-2.4	3	4.8	8.4

b) inv. prop., $y = \frac{9}{x}$,

x	6	4.5	-2.25	-0.5
y	1.5	2	-4	-18

c) proportional, $y = \frac{1}{2}x$,

x	1	2	4	8
y	$\frac{1}{2}$	1	2	4

 inv. prop., $y = \frac{32}{x}$,

x	1	2	4	8
y	32	16	8	4

8. a) inv. prop., $y = \frac{12}{x}$,

x	-3	4	5	10
y	-4	3	2.4	1.2

b) prop., $y = -x$,

x	13	1	-2	-11
y	-13	-1	2	11

 inv. prop., $y = -\frac{4}{x}$,

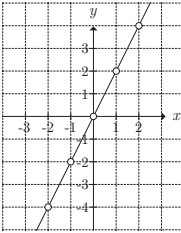
x	$\frac{4}{13}$	4	-2	$-\frac{4}{11}$
y	-13	-1	2	11

c) proportional, $y = \frac{2}{3}x$,

x	0	5	8	9
y	0	$\frac{10}{3}$	$\frac{16}{3}$	6

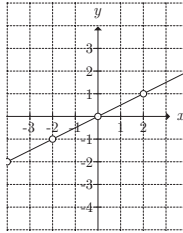
9. a)

x	-1	0	1	2
y	-2	0	2	4



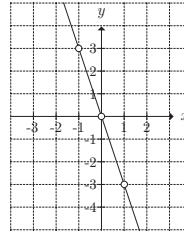
c)

x	-2	0	2	4
y	-1	0	1	2



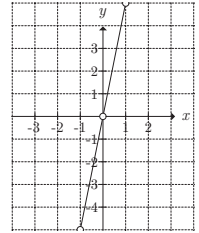
10. a)

x	-1	0	1	2
y	3	0	-3	-6



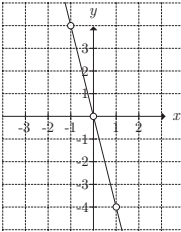
c)

x	-1	0	1	2
y	-5	0	5	10



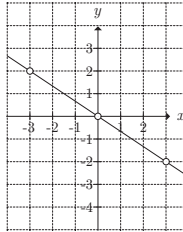
b)

x	-1	0	1	2
y	4	0	-4	-8



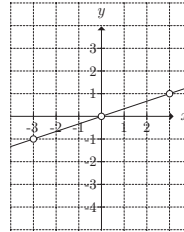
d)

x	-3	0	3	6
y	2	0	-2	-4



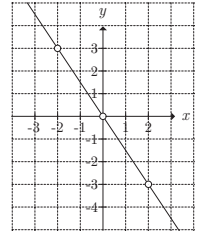
b)

x	-3	0	3	6
y	-1	0	1	2



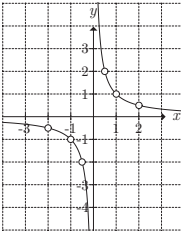
d)

x	-2	0	2	4
y	3	0	-3	-6



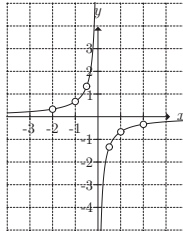
11. a)

x	$\pm \frac{1}{2}$	± 1	± 2
y	± 2	± 1	$\pm \frac{1}{2}$



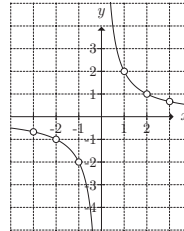
c)

x	$\pm \frac{1}{2}$	± 1	± 2
y	$\mp \frac{4}{3}$	$\mp \frac{2}{3}$	$\mp \frac{1}{3}$



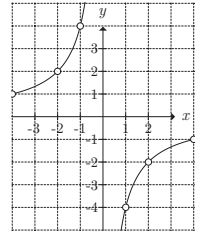
12. a)

x	± 1	± 2	± 3
y	± 2	± 1	$\pm \frac{2}{3}$



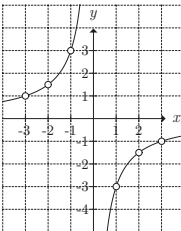
c)

x	± 1	± 2	± 4
y	∓ 4	∓ 2	∓ 1



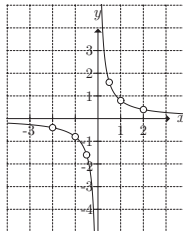
b)

x	± 1	± 2	± 3
y	∓ 3	$\mp \frac{3}{2}$	∓ 1



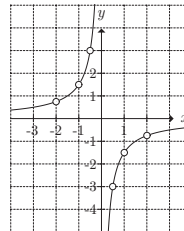
d)

x	$\pm \frac{1}{2}$	± 1	± 2
y	$\pm \frac{8}{5}$	$\pm \frac{4}{5}$	$\pm \frac{2}{5}$



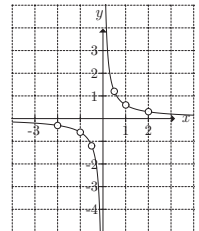
b)

x	$\pm \frac{1}{2}$	± 1	± 2
y	∓ 3	$\mp \frac{3}{2}$	$\mp \frac{3}{4}$



d)

x	$\pm \frac{1}{2}$	± 1	± 2
y	$\pm \frac{6}{5}$	$\pm \frac{3}{5}$	$\pm \frac{3}{10}$



13. $f_1 : y = \frac{2}{3}x$, $f_2 : y = 2x$, $f_3 : y = -3x$,
 $f_4 : y = -\frac{5}{3}x$, $f_5 : y = -\frac{1}{4}x$

14. $f_1 : y = \frac{1}{2}x$, $f_2 : y = \frac{4}{3}x$, $f_3 : y = 4x$,
 $f_4 : y = -\frac{3}{2}x$, $f_5 : y = -\frac{1}{3}x$

15. a) $g : y = -2x$

c) $g : y = -\frac{1}{4}x$

16. a) $g : y = -\frac{2}{3}x$

c) $g : y = 0.1x$

b) $g : y = x$

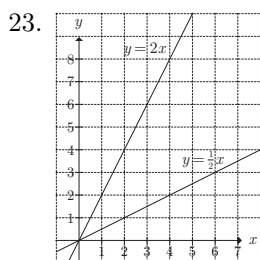
d) $g : y = 7x$

b) $g : y = 3x$

d) $g : y = -x$

17. a) e. g. $(1, 4), (2, 8), (3, 12)$
 b) e. g. $(3, 2), (6, 4), (9, 6)$
 c) e. g. $(-1, 7), (-2, 14), (-3, 21)$
 d) e. g. $(10, 9), (20, 18), (30, 27)$
 e) e. g. $(-5, 8), (-10, 16), (-15, 24)$
 f) e. g. $(9, 5), (18, 10), (27, 15)$

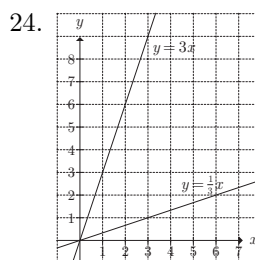
19. a) $y = \frac{1}{7}x$ c) $y = \frac{5}{4}x$
 b) $y = -\frac{2}{3}x$ d) $y = \frac{b}{a}x$
 21. a) $(0, 2)$ d) $y = \frac{2}{3}x$
 b) $(1, 3)$ e) $y = -\frac{1}{2}x$
 c) $y = 3x$ f) $y = \frac{b}{a}x$



The equation $y = \frac{1}{2}x$ is correct. For all points, such as $(4, 2)$, on the graph of this function, the x -coordinate is twice as large as the y -coordinate.

18. a) e. g. $(-1, 6), (-2, 12), (-3, 18)$
 b) e. g. $(7, 4), (14, 8), (21, 12)$
 c) e. g. $(-3, 4), (-6, 8), (-9, 12)$
 d) e. g. $(1, 8), (2, 16), (3, 24)$
 e) e. g. $(-9, 2), (-18, 4), (-27, 6)$
 f) e. g. $(5, 3), (10, 6), (15, 9)$

20. a) $y = -2x$ c) $y = -\frac{5}{4}x$
 b) $y = \frac{1}{5}x$ d) $y = \frac{3b}{a}x$
 22. a) $(-3, 0)$ d) $y = -\frac{1}{4}x$
 b) $(-4, 2)$ e) $y = 3x$
 c) $y = -\frac{1}{2}x$ f) $y = \frac{b}{a}x$

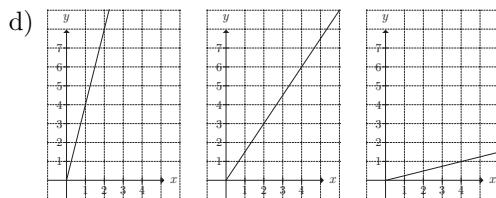


The equation $y = \frac{1}{3}x$ is correct. For all points, such as $(6, 2)$, on the graph of this function, the y -coordinate is one third as large as the x -coordinate.

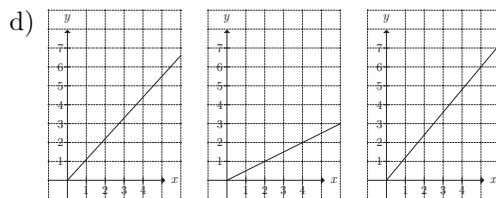
25. a) steeper
 b) I, III
 c) $(0, 0)$
 d) x -axis, y -axis
 e) a

26. a) straight lines
 b) II, IV
 c) flatter
 d) a
 e) rises, falls

27. a) $y = 4x$
 b) $y = 1.5x$
 c) $y = \frac{1}{4}x$



28. a) $y = 1.1x$
 b) $y = 0.5x$
 c) $y = 1.2x$



29. a) $y = \frac{5}{4}x$
 b) 25 g, 187.5 ml, 3.75 eggs, i. e. ≈ 4 eggs

30. a) $y = 1.054x$
 b) $\approx \$25.82, \approx \$8.43, \approx \$246.64$

31. a) $y = 1.81x$
 b) Fr. 54.30
 c) $\approx 11.05 \ell$

32. a) $y = 1.414x$
 b) ≈ 17 cm
 c) ≈ 29.7 cm

33. a) $f : y = \frac{110}{21}x \approx 5.24x$
 b) $f(16) \approx \text{Fr. } 84.-$
 c) $\mathbb{X} = \{0, 1, 2, \dots, 42\}, \mathbb{Y} = [0, 220]$

34. a) $y = 0.09x$
 b) Fr. 11.70
 c) no negative numbers

35. a) $(-1, 3)$, $y = -3x$
 b) $y = -2x$
 c) $y = -\frac{3}{2}x$
 d) $y = -\frac{b}{a}x$

- | | | | | | |
|--------|-------------------------------------|-------------------------------------|----|--------------------------|-------------------------------------|
| | true | false | | true | false |
| 37. a) | ☐ | ☒ | c) | ☐ | ☒ |
| b) | ☒ | ☐ | d) | ☐ | ☒ |

36. a) $x - b$, $a - x$
 b) 45° , 45° , 90°
 c) $x - b = a - x \Rightarrow x = (a + b) : 2$
 d) $Q_1(7, 7)$, $Q_2(8.5, 8.5)$, $Q_3(-1, -1)$

- | | | | | | |
|--------|-------------------------------------|--------------------------|----|--------------------------|-------------------------------------|
| | true | false | | true | false |
| 38. a) | ☒ | ☐ | c) | ☐ | ☒ |
| b) | ☒ | ☐ | d) | ☐ | ☒ |

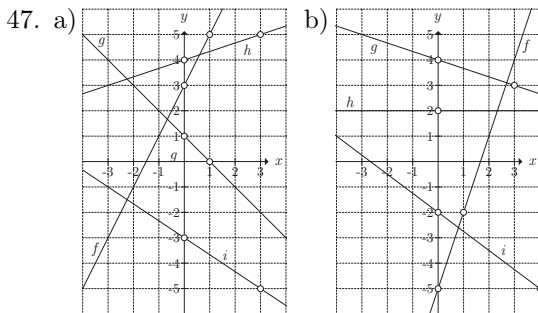
4.2 The Linear Function

39. a) yes, $y = \frac{3}{2}x - \frac{1}{2}$ e) no
 b) yes, $y = (0x +) 5$ f) yes, $y = \frac{9}{2}x + \frac{1}{2}$
 c) no g) yes, $y = 9x (+0)$
 d) yes, $y = \frac{1}{4}x + 3$ h) no

41. ①-C, ②-F, ③-D, ④-A, ⑤-E, ⑥-B

43. a) $f_1 : y = 2x + 4$, $f_2 : y = -2x + 4$,
 $f_3 : y = 2x - 4$, $f_4 : y = -2x - 4$
 b) $f_1 : y = -\frac{1}{3}x + 3$, $f_2 : y = \frac{1}{3}x + 3$,
 $f_3 : y = \frac{1}{3}x - 3$, $f_4 : y = -\frac{1}{3}x - 3$

45. $f : y = 4x + 3$ $h : y = -\frac{3}{4}x + 2$
 $g : y = -3x - 2$ $i : y = \frac{4}{3}x - 7$



49. $f : y = 2x$ $i : y = \frac{1}{4}x + 3$
 $g : y = 6$ $j : y = -\frac{2}{3}x - 5$
 $h : y = -\frac{3}{5}x + 4$ $k : y = \frac{1}{2}x - 4$

51. a) $y = 3x - 2$ c) $y = -x + 2$
 b) $y = -8x - 5$ d) $y = 7$

53. a) yes, $y = \frac{1}{2}x + 4$ c) no
 b) no d) yes, $y = -4x - 9$

55. yes; $y = 2x$, $\mathbb{X} = [0, 350]$, $\mathbb{Y} = [0, 700]$

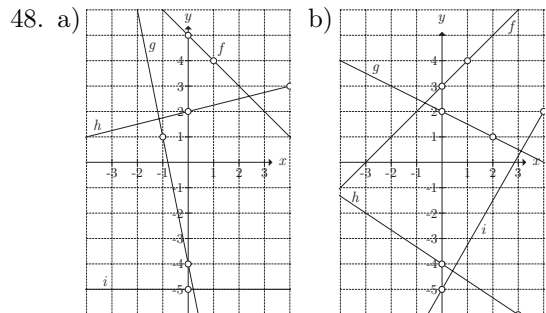
57. yes, $y = 3x - 12$, $\mathbb{X} = \{0, 1, 2, \dots, 24\}$,
 $\mathbb{Y} = \{-12, -9, -6, \dots, 60\}$

40. a) yes $y = 2x - 4$ e) no
 b) no f) yes, $y = -x (+0)$
 c) no g) no
 d) yes, $y = \frac{1}{6}x - \frac{5}{6}$ h) no

42. ①-D, ②-F, ③-E, ④-A, ⑤-C, ⑥-B

44. a) $f_1 : y = \frac{2}{3}x + 2$, $f_2 : y = -\frac{2}{3}x + 2$,
 $f_3 : y = \frac{2}{3}x - 2$, $f_4 : y = -\frac{2}{3}x - 2$
 b) $f_1 : y = 3x - 4$, $f_2 : y = -3x - 4$,
 $f_3 : y = 3x + 4$, $f_4 : y = -3x + 4$

46. $f : y = -\frac{2}{3}x + 6$ $h : y = \frac{1}{4}x + 2$
 $g : y = -4x - 1$ $i : y = \frac{3}{5}x - 7$



50. $f : y = \frac{5}{3}x + 1$ $i : y = -\frac{1}{5}x - 2$
 $g : y = -3x + 3$ $j : y = -4$
 $h : y = -\frac{3}{2}x - 5$ $k : y = \frac{1}{3}x - 7$

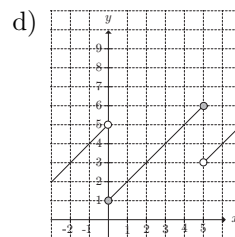
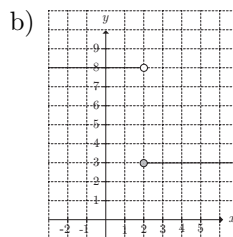
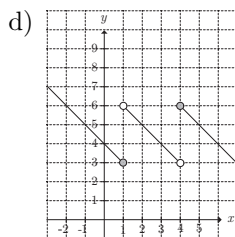
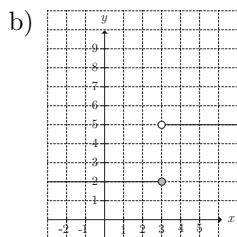
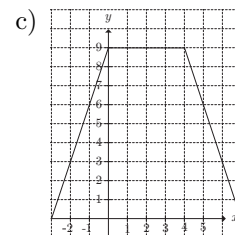
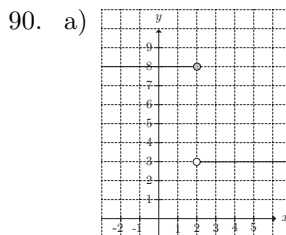
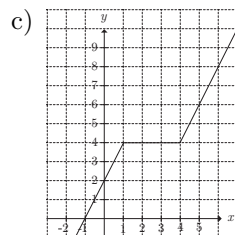
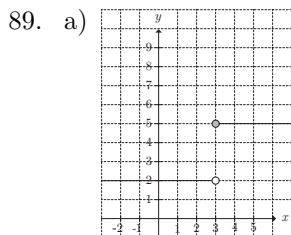
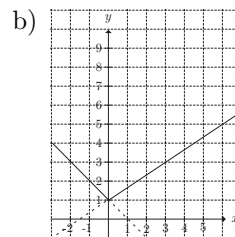
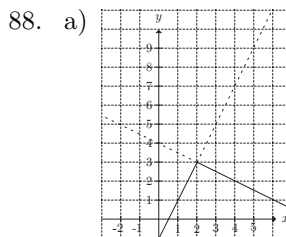
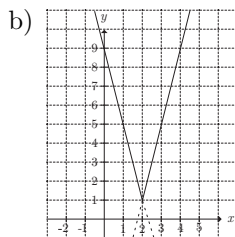
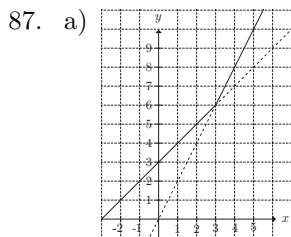
52. a) $y = 9$ c) $y = -\frac{2}{3}x$
 b) $y = 3x + 6$ d) $y = \frac{5}{4}x + 4$

54. a) yes, $y = -\frac{1}{3}x - 3$ c) yes, $y = \frac{5}{3}x + 1$
 b) no d) no

56. yes, $y = 60x + 10$, $\mathbb{X} = [0, ?]$, $\mathbb{Y} = [0, ?]$

58. yes, $y = -16x + 960$, $\mathbb{X} = \{0, 0.2, 0.4, \dots, 60\}$
 $\mathbb{Y} = \{0, 3.20, 6.40, 9.60, \dots, 960\}$

59. not linear, $y = 200 \cdot 1.08^x$
60. yes, $y = 2x + 2$, $\mathbb{X} = \{1, 2, 3, \dots, ?\}$,
 $\mathbb{Y} = \{4, 6, 8, \dots, ?\}$
61. yes, $y = 0.5x - 4$, $\mathbb{X} = \{0, 1, 2, \dots, 30\}$,
 $\mathbb{Y} = \{-4, -3.5, -3, \dots, 11\}$
62. not linear, $y = x(x + 1) : 2$
63. a) $A(1, 2)$, $B(-5, 14)$, $C(6, -8)$
 b) $A(0, -6)$, $B(12, -15)$, $C(-8, 0)$
 c) $A(-\frac{1}{20}, 0)$, $B(-1, -\frac{19}{5})$, $C(-\frac{9}{5}, -7)$
 d) $A(\frac{3}{2}, -1)$, $B(-\frac{9}{2}, 2)$, $C(0, -\frac{1}{4})$
64. a) $A(4, 7)$, $B(-2, -11)$, $C(6, 13)$
 b) $A(12, 18)$, $B(1, \frac{10}{3})$, $C(-9, -10)$
 c) $A(5, -17)$, $B(-3, 11)$, $C(2, -\frac{13}{2})$
 d) $A(-\frac{1}{15}, 0)$, $B(\frac{4}{3}, -7)$, $C(\frac{1}{5}, -\frac{4}{3})$
65. a) -6 c) $\frac{1}{2}$
 b) $\frac{9}{5}$ d) $\frac{50}{3}$
66. a) 5 c) $-\frac{35}{2}$
 b) $\frac{7}{3}$ d) $\frac{1}{4}$
67. a) $y = \frac{1}{2}x + 3$ d) $y = \frac{1}{3}x + 5$
 b) $y = \frac{1}{2}x + 5$ e) $y = 4x + 11$
 c) $y = \frac{1}{3}x + 1$ f) $y = -\frac{3}{5}x + \frac{47}{5}$
68. a) $y = -2x - 4$ d) $y = -\frac{1}{2}x - 4$
 b) $y = -2x - 7$ e) $y = -3x + 12$
 c) $y = \frac{1}{2}x - 8$ f) $y = -\frac{5}{7}x + \frac{31}{7}$
69. a) $f(x) = 2x - 3$ c) $f(x) = \frac{1}{4}x + 6$
 b) $f(x) = -\frac{1}{3}x + 3$ d) $f(x) = -2x + 8$
70. a) $f(x) = \frac{2}{3}x + 4$ c) $f(x) = -\frac{1}{2}x - 3$
 b) $f(x) = -1$ d) $f(x) = \frac{1}{5}x - \frac{17}{5}$
71. a) $f(x) = 2x + 3$ c) $f(x) = 5x - 1$
 b) $f(x) = -\frac{1}{4}x + 5$ d) $f(x) = 2x - 14$
72. a) $f(x) = -\frac{2}{3}x + 4$ c) $f(x) = -\frac{1}{6}x - 9$
 b) $f(x) = -6x + 5$ d) $f(x) = -\frac{1}{3}x + 4$
73. $B(-3, 4)$, $C(-4, -3)$, $D(3, -4)$
 $f: y = 7x - 25$ $g: y = 7x + 25$
 $h: y = -\frac{1}{7}x + \frac{25}{7}$ $j: y = -\frac{1}{7}x - \frac{25}{7}$
74. $B(-12, 5)$, $C(-5, -12)$, $D(12, -5)$
 $f: y = -\frac{17}{7}x + \frac{169}{7}$ $g: y = -\frac{17}{7}x - \frac{169}{7}$
 $h: y = \frac{7}{17}x + \frac{169}{17}$ $j: y = \frac{7}{17}x - \frac{169}{17}$
75. a) $y = \frac{1}{2}x + \frac{13}{2}$ c) $y = -\frac{2}{3}x - \frac{14}{3}$
 b) $y = -\frac{3}{4}x - \frac{1}{2}$ d) $y = \frac{6}{5}x - 3$
76. a) $y = \frac{1}{5}x + \frac{3}{5}$ c) $y = -\frac{3}{4}x + \frac{1}{2}$
 b) $y = -\frac{5}{9}x + \frac{4}{3}$ d) $y = \frac{4}{9}x - \frac{13}{3}$
77. $f: y = -\frac{1}{4}x + 5$ $i: y = \frac{1}{3}x - \frac{4}{3}$
 $g: y = \frac{1}{4}x + \frac{9}{4}$ $j: y = -3$
 $h: y = -\frac{2}{3}x - \frac{10}{3}$ $k: x = 5$
78. $f: y = 2x + 1$ $i: x = -4$
 $g: y = -\frac{4}{3}x + \frac{5}{3}$ $j: y = -\frac{1}{6}x - \frac{17}{6}$
 $h: y = 5$ $k: y = \frac{2}{7}x - \frac{38}{7}$
79. a) $f(-4) = \frac{25}{2}$, $f(\frac{3}{2}) = -4$
 b) $q = 14$, $f(4) = 42$
 c) $m = 3$, $f(-4) = -15$
 d) $m = -\frac{1}{4}$, $q = 1$, $f(15) = -\frac{11}{4}$
80. a) $f(-\frac{2}{5}) = 1$ as well as $f(7) = -\frac{69}{5}$
 b) $q = -36$, $f(-1) = -43$
 c) $m = -5$, $f(2) = -14$
 d) $m = \frac{4}{3}$, $q = 6$, $f(3) = 10$
81. a) $(-3, -2)$ c) $(-2, 3)$
 b) parallel d) $(30, 15)$
82. a) $(10, 7)$ c) parallel
 b) $(2, -4)$ d) $(11, 0)$
83. a) $f: 2$, $g: -\frac{1}{3}$
 b) $f': y = 2x - 7$
 c) $g': y = -\frac{1}{3}x - \frac{14}{3}$
 d) $A(1, -5)$, like from C to D :
 At $B -6$ in x - and 2 in y -direction
84. a) $f: -\frac{2}{5}$, $g: \frac{1}{3}$
 b) $f': y = -\frac{2}{5}x + \frac{47}{5}$
 c) $g': y = \frac{1}{3}x + \frac{26}{3}$
 d) $D(1, 9)$, like from B to C :
 At $A 3$ in x - and 1 in y -direction
85. $A(\frac{5}{4}, \frac{19}{4})$, $B(\frac{14}{3}, \frac{4}{3})$, $C(\frac{37}{5}, \frac{34}{5})$
86. $A(\frac{7}{3}, \frac{13}{3})$, $B(\frac{39}{5}, \frac{8}{5})$, $C(\frac{23}{4}, \frac{31}{4})$



$$91. \quad f: y = \begin{cases} -\frac{1}{2}x + 3 & \text{if } x < 2 \\ x & \text{if } x \geq 2 \end{cases}$$

$$g: y = \begin{cases} x + 4 & \text{if } x < 3 \\ -\frac{2}{3}x + 9 & \text{if } x \geq 3 \end{cases}$$

$$92. \quad f: y = \begin{cases} x + 1 & \text{if } x < 6 \\ -2x + 19 & \text{if } x \geq 6 \end{cases}$$

$$g: y = \begin{cases} -\frac{1}{4}x + 3 & \text{if } x < 8 \\ x - 7 & \text{if } x \geq 8 \end{cases}$$

93. a) $y = \begin{cases} -x & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$

b) $y = \begin{cases} -x - 2 & \text{if } x < 0 \\ x - 2 & \text{if } x \geq 0 \end{cases}$

c) $y = \begin{cases} -x + 1 & \text{if } x < 1 \\ x - 1 & \text{if } x \geq 1 \end{cases}$

d) $y = \begin{cases} 0 & \text{if } x < 0 \\ 2x & \text{if } x \geq 0 \end{cases}$

e) $y = \begin{cases} 3x & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$

f) $y = \begin{cases} -1 & \text{if } x < -1 \\ 2x + 1 & \text{if } x \geq -1 \end{cases}$

94. a) $y = \begin{cases} -2x & \text{if } x < 0 \\ 2x & \text{if } x \geq 0 \end{cases}$

b) $y = \begin{cases} -x - 2 & \text{if } x < -2 \\ x + 2 & \text{if } x \geq -2 \end{cases}$

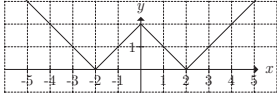
c) $y = \begin{cases} x + 1 & \text{if } x < 0 \\ -x + 1 & \text{if } x \geq 0 \end{cases}$

d) $y = \begin{cases} -2x & \text{if } x < 0 \\ 0 & \text{if } x \geq 0 \end{cases}$

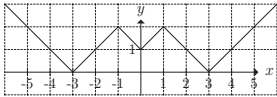
e) $y = \begin{cases} -\frac{1}{2}x & \text{if } x < 2 \\ \frac{1}{2}x - 2 & \text{if } x \geq 2 \end{cases}$

f) $y = \begin{cases} x + 1 & \text{if } x < 1 \\ 3x - 1 & \text{if } x \geq 1 \end{cases}$

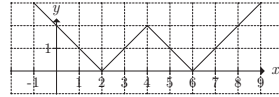
$$95. \text{ a) } y = \begin{cases} -x - 2 & \text{if } x < -2 \\ x + 2 & \text{if } -2 \leq x < 0 \\ -x + 2 & \text{if } 0 \leq x < 2 \\ x - 2 & \text{if } x \geq 2 \end{cases}$$



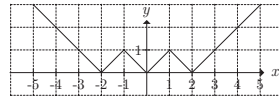
$$\text{b) } y = \begin{cases} -x - 3 & \text{if } x < -3 \\ x + 3 & \text{if } -3 \leq x < -1 \\ -x + 1 & \text{if } -1 \leq x < 0 \\ x + 1 & \text{if } 0 \leq x < 1 \\ -x + 3 & \text{if } 1 \leq x < 3 \\ x - 3 & \text{if } x \geq 3 \end{cases}$$



$$96. \text{ a) } y = \begin{cases} -x + 2 & \text{if } x < 2 \\ x - 2 & \text{if } 2 \leq x < 4 \\ -x + 6 & \text{if } 4 \leq x < 6 \\ x - 6 & \text{if } x \geq 6 \end{cases}$$



$$\text{b) } y = \begin{cases} -x - 2 & \text{if } x < -2 \\ x + 2 & \text{if } -2 \leq x < -1 \\ -x & \text{if } -1 \leq x < 0 \\ x & \text{if } 0 \leq x < 1 \\ -x + 2 & \text{if } 1 \leq x < 2 \\ x - 2 & \text{if } x \geq 2 \end{cases}$$



4.3 Applications

97. a) $y = \frac{5}{9}x - \frac{160}{9}$
 b) ca. -17.8°C , ca. 37.8°C
 c) Subtract 32° from the Fahrenheit temperature and then multiply by $\frac{5}{9}$
98. a) $y = \frac{9}{5}x + 32$
 b) 68°F , 5°F
 c) Fahrenheit temperatures are practically always positive.
99. From a print run of 2223, the profit on printing is higher.
100. From 100 hours per month, i.e. from around 25 hours per week, permanent employment is more favourable.
101. Up to 150 km, B is cheaper, then A. Zurich-Sirnach return: approx. 110 km. This means that provider B is probably the better choice.
102. Equal payouts: after 37.5 years. Life expectancy: nearly 85 years. The first offer is probably better.
103. a) $g : x \mapsto g(x) = \frac{5}{M} \cdot x + 1$
 b) 4.4, 15
 c)
104. a) $g' : x \mapsto g'(x) = \frac{5}{M'} \cdot x + 1$
 b) Nora: 5 / 5.6, Leon: 2.6 / 2.8.
 c)
- d) There is only one straight line between (0, 1) and (M, 6).
105. a) f.l.t.r. 5 %, 3.2 %, 4 %
 b) $\approx 3.9\%$
 c) f.l.t.r. $\frac{1}{20}$, $\frac{4}{125}$, $\frac{1}{25}$
 d) $\approx 2.6\text{ km}$
106. a) f.l.t.r. 312.5 %, 400 %, 500 %, 250 %
 b) $\approx 342\%$
 c) f.l.t.r. 3.125, 4, 5, 2.5
 d) $\approx 208\text{ m}$

$$107. a) f : y = \begin{cases} \text{Fr. 8.50} & \text{if } 0 \text{ kg} < x \leq 2 \text{ kg} \\ \text{Fr. 11.50} & \text{if } 2 \text{ kg} < x \leq 10 \text{ kg} \\ \text{Fr. 20.50} & \text{if } 10 \text{ kg} < x \leq 30 \text{ kg} \end{cases}$$

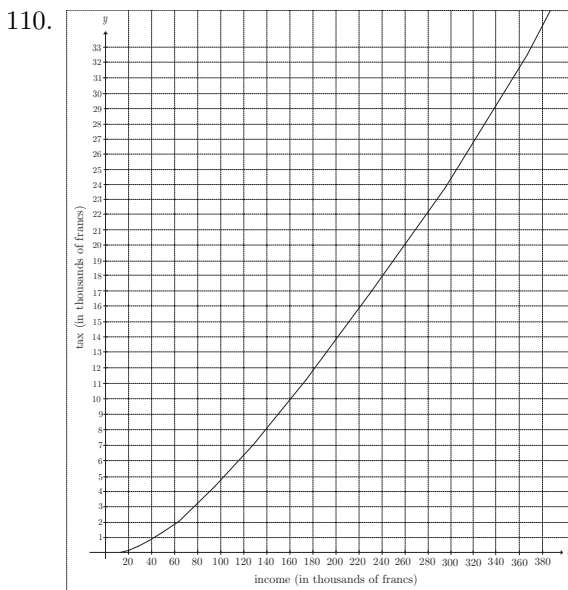
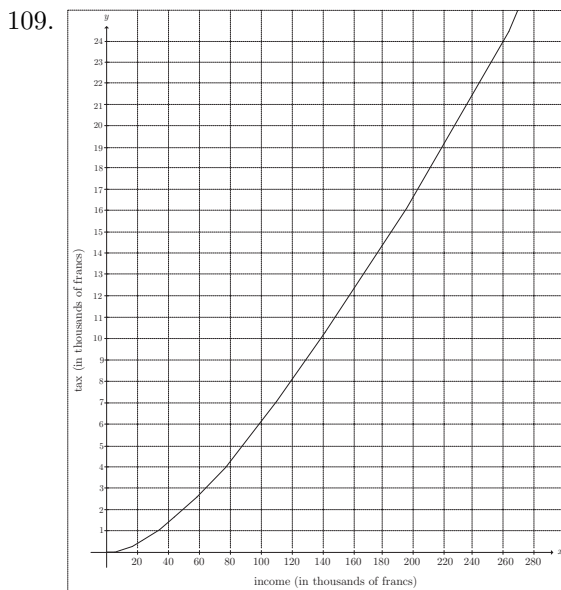
$$b) f(1.9 \text{ kg}) = \text{Fr. 8.50}, f(3.4 \text{ kg}) = \text{Fr. 11.50}, f(10 \text{ kg}) = \text{Fr. 11.50}$$

$$c) \mathbb{X} =]0, 30], \mathbb{Y} = \{8.50, 11.50, 20.50\}$$

$$108. a) f : y = \begin{cases} \text{Fr. 10.50} & \text{if } 0 \text{ kg} < x \leq 2 \text{ kg} \\ \text{Fr. 13.50} & \text{if } 2 \text{ kg} < x \leq 10 \text{ kg} \\ \text{Fr. 22.50} & \text{if } 10 \text{ kg} < x \leq 30 \text{ kg} \end{cases}$$

$$b) f(0.8 \text{ kg}) = \text{Fr. 10.50}, f(28.4 \text{ kg}) = \text{Fr. 22.50}, f(2 \text{ kg}) = \text{Fr. 10.50}$$

$$c) \mathbb{X} =]0, 30], \mathbb{Y} = \{10.50, 13.50, 22.50\}$$



$$111. a) m_c(x) = 2, s_c(x) = -\frac{1}{4}x + 2$$

$$b) m_c(x) = -x + 6, s_c(x) = -\frac{1}{3}x + 4$$

$$c) m_c(x) = -\frac{1}{2}x + \frac{15}{2}, s_c(x) = 6$$

$$d) m_c(x) = 3x - 13, s_c(x) = 2x - 8$$

$$112. a) m_c(x) = 5, s_c(x) = -\frac{5}{6}x + 5$$

$$b) m_c(x) = x - 5, s_c(x) = \frac{1}{3}x + 1$$

$$c) m_c(x) = \frac{1}{3}x + 4, s_c(x) = -\frac{2}{11}x + 4$$

$$d) m_c(x) = 7x - 45, s_c(x) = 17x - 110$$

$$113. a) S\left(\frac{8}{3}, 2\right)$$

$$c) S(-3, 1)$$

$$114. a) S\left(2, \frac{14}{3}\right)$$

$$c) S(3, 5)$$

$$b) S\left(6, \frac{10}{3}\right)$$

$$d) S(-1, -2)$$

$$b) S\left(2, \frac{8}{3}\right)$$

$$d) S(-1, 1)$$

$$115. a) U(3, 5)$$

$$c) U(-15, -8)$$

$$116. a) U(8, 6)$$

$$c) U(-7, 1)$$

$$b) U(-4, 3)$$

$$d) U(8, -2)$$

$$b) U(5, 3)$$

$$d) U(10, -20)$$

$$117. a) H(0, 0)$$

$$c) H(-12, 5)$$

$$118. a) H(164, -68)$$

$$c) H(35, 6)$$

$$b) H(-20, 4)$$

$$d) H(-18, 33)$$

$$b) H(-36, -14)$$

$$d) H(14, -5)$$

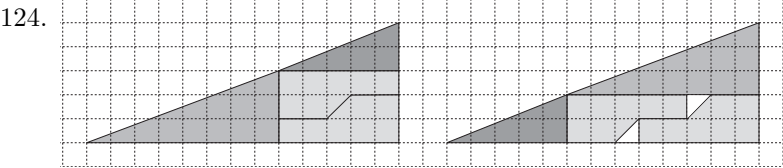
$$119. 40$$

$$120. 32$$

$$121. 25\pi$$

$$122. 169\pi$$

123. The gradient of the dark grey triangle is $\frac{2}{5}$, while that of the medium grey triangle is $\frac{3}{8}$, i. e. it is not a triangle at all, but rather a quadrilateral that is either concave or convex, depending on the circumstances.



125. Coordinates of the points f.l.t.r.:
 $(1, 0), (\frac{9}{5}, \frac{4}{5}), (\frac{12}{5}, \frac{7}{5}), (\frac{14}{5}, \frac{9}{5}), (3, 2)$
They are all lying on $y = x - 1$ ☐

126. $P(\frac{40}{11}, \frac{36}{11}), Q(\frac{71}{11}, \frac{95}{22})$
Line through PQ : $y = \frac{23}{62}x + \frac{656}{341}$
So the secret code is 9237
 P and Q must exist and have different x -coordinates.

- | | | | | | | | | | | |
|------|----|-------------------------------------|-------------------------------------|----|--------------------------|-------------------------------------|------|----|-------------------------------------|-------------------------------------|
| | | true | false | | true | false | | | true | false |
| 127. | a) | ☒ | ☐ | c) | ☐ | ☒ | 128. | a) | ☒ | ☐ |
| | b) | ☐ | ☒ | d) | ☐ | ☒ | | b) | ☐ | ☒ |

5 Systems of Linear Equations

5.1 Introduction

1.  3 p. & 78 w., 6 p. & 68 w., 9 p. & 58 w., 12 p. & 48 w., 15 p. & 38 w., 18 p. & 28 w., 21 p. & 18 w., 24 p. & 8 p.
2. $y = 6x$ as well as $2.5x + 0.75y = 49 \Rightarrow$ 1 p. & 62 w., 4 p. & 52 w., 7 p. & 42 w., 10 p. & 32 w., 13 p. & 22 w., 16 p. & 12 w., 19 p. & 2 w.
3. a) ① d) ④ 4. a) ② d) ①, ③, ④
b) ②, ③ e) none b) none e) ④
c) ② f) ①, ③, ④ c) ②, ③ f) ①
5. a) (0, 5), (5, 0), (2, 3) 6. a) (0, -9), (9, 0), (10, 1)
b) (0, -4), (2, 0), (3, 2) b) $(0, -\frac{8}{3})$, (8, 0), (14, 2)
c) (0, 0), (1, 0), (2, 0) c) $(0, -\frac{7}{6})$, $(\frac{7}{5}, 0)$, (5, 3)
d) $(0, \frac{10}{3})$, (5, 0), (2, 2) d) (0, 0), (0, 1), (0, 2)
7. ①-C, ②-D, ③-E, ④-A, ⑤-B 8. ①-C, ②-A, ③-D, ④-E, ⑤-B
9. a) (1, 5), (2, 4), (3, 3), (4, 2), (5, 1) 10. a) (1, 5), (3, 4), (5, 3), (7, 2), (9, 1)
b) (6, 1), (8, 2), (10, 3), (12, 4), ... b) none
c) (1, 2) c) (1, 11), (2, 8), (3, 5), (4, 2)
d) (1, 4), (5, 3), (9, 2), (13, 1) d) (11, 1), (14, 2), (17, 3), (20, 4), ...
11. a) e.g. $x + y = 6$ c) e.g. $x - 4y = 2$ 12. a) e.g. $x + y = 14$ c) e.g. $x + 9y = 2$
b) e.g. $5x + y = 2$ d) e.g. $3x + 5y = 1$ b) e.g. $2x + y = 6$ d) e.g. $7x - 8y = 4$
13. a) no c) no 14. a) yes c) no
b) yes d) yes b) no d) yes
15. ①-B, ②-F, ③-D, ④-A, ⑤-E, ⑥-C 16. ①-C, ②-F, ③-A, ④-E, ⑤-B, ⑥-D
17. a) e.g. $x + y = 5$, $y - x = 1$ 18. a) e.g. $x + y = -3$, $x + 4y = 0$
b) e.g. $x + 2y = -3$, $3x - 2y = -13$ b) e.g. $4x + y = 8$, $8x - y = -5$
c) e.g. $2x - y = 2$, $4x - 3y = -3$ c) e.g. $3x - y = 1$, $6x - y = -3$
d) e.g. $5x - 3y = 2$, $5x - 6y = 7$ d) e.g. $9x - 4y = 9$, $9x + 4y = -5$
19. a) (8, 4) c) (18, 9) 20. a) (-16, -8) c) (24, 12)
b) (10, 5) d) (-12, -6) b) (14, 7) d) (-10, -5)
21. a) (17, 34) b) (23, 13) 22. a) (21, 7) b) (4, 4)

5.2 Graphical Method

23. a) $\begin{cases} y = 4 \\ y = x \end{cases} \begin{matrix} (1) \\ (2) \end{matrix}$ b) $\begin{cases} 6 = x + y \\ y = 2x - 6 \end{cases} \begin{matrix} (1) \\ (2) \end{matrix}$ 24. a) $\begin{cases} y = x - 1 \\ y = 2 \end{cases} \begin{matrix} (1) \\ (2) \end{matrix}$ b) $\begin{cases} 4 = x + y \\ 2y = x + 2 \end{cases} \begin{matrix} (1) \\ (2) \end{matrix}$
Solution: (4, 4) Solution: (4, 2) Solution: (3, 2) Solution: (2, 2)

The following applies to all of the following tasks:
The upper equation in the task is denoted by (1), the lower equation by (2).

25. a)

26. a)

27.

28.

29. a)

30. a)

31. a)

32. a)

b)

b)

b)

b)

b)

b)

b)

b)

c)

c)

c)

c)

c)

c)

c)

d)

d)

d)

d)

d)

d)

d)

d)

no common
solution

common sol.
(3.5, 1.5)

common sol.
(4.75, 1.75)

no common
solution

33. a) none
b) infinitely many

c) one
d) infinitely many

34. a) infinitely many
b) none

c) one
d) none

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35. a) $y = x$

b) $y > x$

c) $x < y < x + 4$

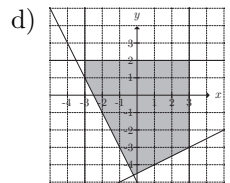
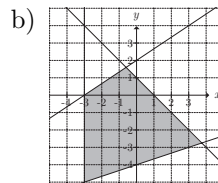
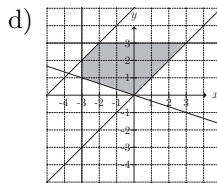
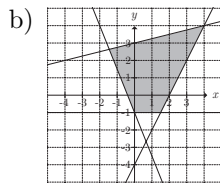
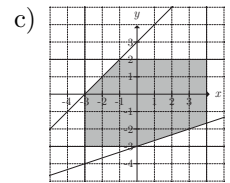
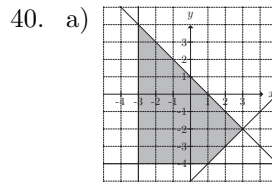
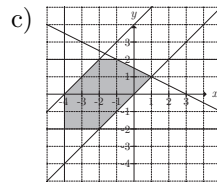
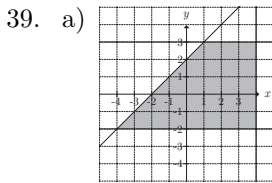
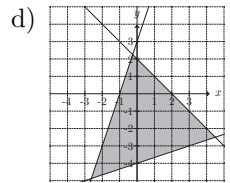
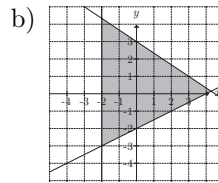
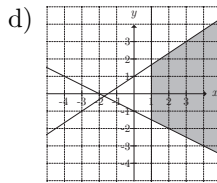
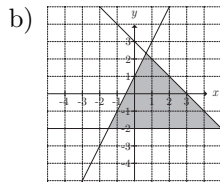
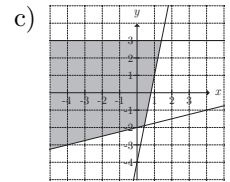
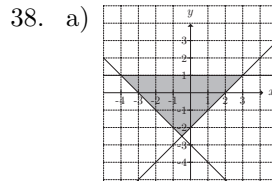
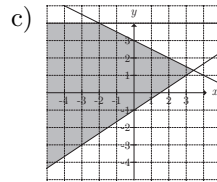
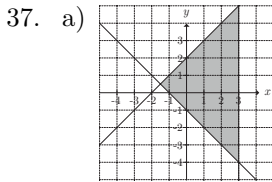
d) $\begin{cases} x - 2 < y < x \\ 1 < x < 3 \end{cases}$

36. a) $y = -x + 1$

b) $y < -x + 1$

c) $-x + 1 < y < -x + 4$

d) $\begin{cases} -x + 4 < y < -x + 6 \\ 1 < y < 4 \end{cases}$



41. a) $\begin{cases} 7x + 6y < 42 \\ 8y - 7x < 56 \\ 9x + 8y > -72 \\ 3x - 2y < 18 \end{cases}$

b) $\begin{cases} 2x + 5y < 30 \\ y - 2x < 6 \\ y > -12 \\ x < 5 \end{cases}$

42. a) $\begin{cases} 2y + x < 10 \\ 2y + 5x > 10 \\ y - 4x < -8 \\ 5y - 4x > -40 \end{cases}$

b) $\begin{cases} |y| < 6 \\ y + 2x < 6 \\ y - 2x > -6 \\ x > -8 \end{cases}$

5.3 Substitution Method

43. a) (7, 9)

c) (-2, 2)

44. a) (4, 11)

c) (3, -5)

b) (13, 25)

d) (-2, 3)

b) (-3, 1)

d) (2, 2)

45. a) (4, -2)

c) (9, 3)

46. a) (-7, 4)

c) (2, 0)

b) (12, -8)

d) (-12, 10)

b) (-5, 4)

d) (1, 1)

47. ①- C, ②- E, ③- F, ④- B, ⑤- D, ⑥- A

48. ①-E, ②-D, ③-A, ④-F, ⑤-B, ⑥-C

49. a) (-2, 8)

c) (9, 20)

50. a) (-6, -2)

c) (-4, 6)

b) (7, 10)

d) (-3, -4)

b) (6, 3)

d) (1, -2)

51. a) $(\frac{1}{2}, \frac{3}{4})$
 b) $\{(x, y) \mid 4x + y = 3\}$
 c) $(\frac{3}{5}, \frac{1}{3})$
 d) $\{\}$
52. a) (1.6, 2.8)
 b) $\{\}$
 c) $(\frac{2}{3}, \frac{5}{6})$
 d) $\{(x, y) \mid 4x - 3y = 6\}$
53. a) (8, -4)
 b) (5, 7)
 c) $(\frac{1}{2}, \frac{3}{4})$
 d) (-3, 12)
54. a) (4, 6)
 b) $(-7, \frac{1}{3})$
 c) $\{\}$
 d) (6, 13)
55. a) (13, 2)
 b) (3, -18)
 c) (4, 2)
 d) (-2, -0)
56. a) (4, -7)
 b) (-6, -8)
 c) (4, 3)
 d) (-43, -50)
57. $\square = -7$, $\square = 13$
58. $\square = 6$, $\square = -7$

5.4 Elimination Method

59. *A-I-D-F-B-L-G-C-H-J-E-K*
60. *A-D-H-B-K-F-J-E-I-L-C-G*
61. a) (8, 1)
 b) (2, 3)
62. a) (2, -3)
 b) (-4, -6)
63. a) (5, -2)
 b) (-3, 7)
64. a) (7, 12)
 b) (-8, -3)
65. a) (7, -4)
 b) (-4, -6)
66. a) (6, 3)
 b) (-1, -8)
67. a) (-3, 4)
 b) $\{\}$
 c) (-5, 0)
 d) $\{(x, y) \mid 2x - 3y = 4\}$
68. a) $\{(x, y) \mid 5x - 6y = 3\}$
 b) (-4, 13)
 c) (6, -6)
 d) $\{\}$
69. a) (7, 12)
 b) $(\frac{1}{2}, \frac{1}{3})$
70. a) (-5, 14)
 b) $(\frac{1}{4}, -\frac{2}{7})$
71. a) (1.2, 1.5)
 b) (-2, 9)
 c) $(-\frac{11}{15}, \frac{4}{3})$
72. a) (4.2, -1.7)
 b) (7, 11)
 c) $(\frac{6}{5}, -\frac{1}{4})$
73. a) $(2\sqrt{5}, -\sqrt{5})$
 b) (-5, 3)
 c) (2, 0)
 d) $(3\sqrt{2} - \sqrt{6}, 2 - \sqrt{3})$
 e) $(3\sqrt{5} + 2\sqrt{3}, -2\sqrt{15} - 6)$
74. a) $(3\sqrt{6}, -8\sqrt{6})$
 b) (0, 2)
 c) (10, 3)
 d) $(2\sqrt{6} - \sqrt{3}, 6 - 2\sqrt{2})$
 e) $(-2\sqrt{3} - 3\sqrt{2}, 10\sqrt{6} + 18)$
75. a) $(-\frac{4}{5}, -9)$
 b) $(-\frac{8}{9}, \frac{2}{3})$
76. a) (-6, $\frac{7}{11}$)
 b) $(\frac{5}{4}, -\frac{2}{3})$
77. a) (2.3, -7.5)
 b) $(\frac{3}{4}, \frac{7}{12})$
 c) (170, -48)
78. a) (-5.2, 3.9)
 b) $(\frac{1}{42}, \frac{1}{6})$
 c) $(\frac{1}{2}, -\frac{1}{13})$

5.5 Miscellaneous Exercises

75. a) $(4, -3)$ c) $(-4, \frac{1}{5})$ 76. a) $(-\frac{1}{2}, \frac{3}{2})$ c) $(\frac{2}{9}, -\frac{1}{11})$
 b) $(\frac{3}{7}, \frac{4}{5})$ d) $(-\frac{2}{3}, \frac{4}{9})$ b) $(6, 7)$ d) $(0, 3)$
77. a) $(3, -5)$ c) $(\frac{2}{3}, \frac{1}{6})$ 78. a) $(9, 4)$ c) $(-\frac{2}{5}, \frac{1}{5})$
 b) $(7, \frac{1}{2})$ d) $\{ \}$ b) $(\frac{1}{4}, 6)$ d) $\{ \}$
79. a) $(3, 10)$ 80. a) $(7, 4)$
 b) $(10, 2)$
 c) $(0, 17)$
 d) $(1, \frac{7}{2})$
 e) $\{(x, y) \mid 28x + 9y = 5\}$
 f) $(5, 9)$
 g) $\{ \}$
81. a) $(6, 5)$ 82. a) $(-4, 6)$
 b) $\{(x, y) \mid x + y = -3\}$ b) $\{ \}$
83. a) overdet. c) overdet. 84. a) 1 solution c) overdet.
 b) underdet. d) 1 solution b) underdet. d) underdet.
85. a) $\{ \}$ b) $(-6, -2)$ 86. a) $(-2, 6)$ b) $\{ \}$
87. a) $(3, 9)$ b) $(8, 3)$ 88. a) $(-5, 7)$ b) $(7, -2)$
89. a) $\dots \quad 10x + 15y - 10x - 8y = \mathbf{5 \cdot 6 - 2 \cdot 1} \quad \rightarrow \quad 7y = 28 \quad \rightarrow \quad y = 4 \quad \rightarrow \quad (-3, 4)$
 b) $\dots \xrightarrow{-2} \quad 8x - \mathbf{3} (3x - 4) = 6 \quad \rightarrow \quad 8x - 9x + 12 = 6 \quad \rightarrow \quad -x = -6 \quad \rightarrow \quad (6, 7)$
90. a) $\dots \xrightarrow{-9} \quad 45x - 8(7 + 4x) = \mathbf{9} \quad \rightarrow \quad 45x - 56 - 32x = 9 \quad \rightarrow \quad 13x = 65 \quad \rightarrow \quad (5, 3)$
 b) **Simplify first:** $\left| \begin{array}{l} 9x - 2y = 9 \\ 3y - 8x = 3 \end{array} \right| \xrightarrow{3 \cdot (1) + 2 \cdot (2)} 27x - 6y + 6y - 16x = 27 + 6 \rightarrow (3, 9)$
91. a) e. g. $\left| \begin{array}{l} x + y = -4 \\ 5x + y = 0 \end{array} \right|$ d) e. g. $\left| \begin{array}{l} x + y = 10 \\ 2x + 2y = 20 \end{array} \right|$ 92. a) e. g. $\left| \begin{array}{l} x + y = 4 \\ 2x + y = 2 \end{array} \right|$ d) e. g. $\left| \begin{array}{l} x + y = 5 \\ 2x + 2y = 10 \end{array} \right|$
- b) e. g. $\left| \begin{array}{l} x + y = 0 \\ x + y = 1 \end{array} \right|$ e) e. g. $\left| \begin{array}{l} 4x - 3y = -15 \\ 8x - 6y = -30 \end{array} \right|$ b) e. g. $\left| \begin{array}{l} x - y = 0 \\ x - y = 1 \end{array} \right|$ e) e. g. $\left| \begin{array}{l} 3x + 4y = 28 \\ 6x + 8y = 56 \end{array} \right|$
- c) e. g. $\left| \begin{array}{l} x + y = 1 \\ 2x + 2y = 2 \end{array} \right|$ f) e. g. $\left| \begin{array}{l} 3x + 2y = 23 \\ 6x + 4y = 46 \end{array} \right|$ c) e. g. $\left| \begin{array}{l} x - y = 0 \\ x - y = 0 \end{array} \right|$ f) e. g. $\left| \begin{array}{l} x + 3y = 14 \\ 2x + 6y = 28 \end{array} \right|$
93. a) $(\frac{1}{6}, \frac{1}{3})$ b) $(\frac{1}{4}, -\frac{1}{2})$ 94. a) $(\frac{1}{3}, \frac{1}{9})$ b) $(-\frac{1}{7}, \frac{1}{6})$
95. a) $(-\frac{1}{4}, -\frac{1}{5})$ c) $(\pm 2\sqrt{3}, \pm\sqrt{5})$ 96. a) $(\frac{2}{3}, \frac{1}{6})$ c) $(\pm 2\sqrt{2}, \pm\sqrt{3})$
 b) $(\frac{1}{2}, \frac{3}{8})$ d) $(24, 146)$ b) $(1, 2)$ d) $(100, 50)$
97. ①-A, ②-D, ③-F, ④-E, ⑤-C, ⑥-B 98. ①-B, ②-D, ③-F, ④-E, ⑤-A, ⑥-C

5.6 Systems With Parameters

99. a) $a = -1$: $(1.5, 0.5)$, $a = 0.5$: $(3, 3.5)$
 b) $\left(\frac{a-2}{a-1}, \frac{a^2-2}{a-1}\right)$
 c) Division by zero. One of the lines always has a gradient of 1, the other a . If $a = 1$, the two lines are parallel, but the y -intercept is then different.
100. a) $a = -1$: $(-2, 2)$, $a = 0.5$: $(0.4, 0.2)$
 b) $\left(\frac{2a}{a+2}, \frac{2a^2}{a+2}\right)$
 c) Division by zero. One of the lines always has a gradient of -2 , the other a . If $a = -2$, the two lines are parallel, but the y -intercept is then different.
101. a) $(1, -a + 1)$ c) $\left(\frac{c+1}{c}, -\frac{1}{c}\right)$
 b) $(-2b, 2 - b)$ d) $\left(\frac{b}{a}, \frac{b^2-ab}{a}\right)$
102. a) $(-1, a + b)$ c) $(1, 1 - c)$
 b) $\left(\frac{b-1}{b+1}, \frac{2}{b+1}\right)$ d) $\left(\frac{1}{a}, 0\right)$
103. a) $\left(\frac{a-b}{a+b}, \frac{2}{a+b}\right)$ c) $\left(-\frac{a^2}{b}, -\frac{a^2}{b}\right)$
 b) $(q, 2q - p)$ d) $\left(\frac{cd(d-c)}{c+d}, \frac{cd(c-d)}{c+d}\right)$
104. a) $\left(\frac{s-3}{s-2}, \frac{s}{s-2}\right)$ c) $\left(\frac{a}{b}, \frac{a^2+a-b^2}{b}\right)$
 b) $\left(\frac{1}{2}, \frac{e-c}{2(d+f)}\right)$ d) $\left(\frac{p^2+q^2}{p-q}, \frac{p^2+q^2}{p-q}\right)$
105. a) $a = -1$: $\{\}$
 $a = 1$: $\{(x, y) \mid x + y = 1\}$
 else: $\left(\frac{1}{1+a}, \frac{1}{1+a}\right)$
 b) $b = -1$: $\{(x, y) \mid x - y = 1\}$
 $b = 0$: $\{\}$
 else: $\left(\frac{1-b}{b}, \frac{b-b^2-1}{b^2}\right)$
 c) $a \in \{0, -2\}$: $\{\}$
 $a = 2$: $\{(x, y) \mid x - 2y = -2\}$
 else: $\left(-\frac{2a}{a+2}, \frac{a}{a+2}\right)$
 d) $a = -1$: $\{(x, y) \mid bx - x - y = 0\}$
 $a \neq -1 \wedge b = 0$: $\{\}$
 else: $\left(\frac{a+1}{b}, -\frac{a+1}{b}\right)$
106. a) $a = 0$: $\{\}$
 $a = 1$: $\{(x, y) \mid x + y = 1\}$
 else: $\left(a + 1, -\frac{1}{a}\right)$
 b) $c = 3$: $\{(x, y) \mid x - y = 3\}$
 else: $(c + 4, 4)$
 c) $a \in \{0, -1\}$: $\{\}$
 $a = 1$: $\{(x, y) \mid x - y = -3\}$
 else: $\left(-\frac{3}{a+1}, \frac{3}{a+1}\right)$
 d) $a = 0 \wedge b \notin \{-4, 2\}$: $\{\}$
 $a = 0 \wedge b = -4$: $\{(x, y) \mid y = 4\}$
 $b = 2$: $\{(x, y) \mid 2ax + y = -2\}$
 else: $\left(\frac{b+4}{a(b-2)}, \frac{2(b-8)}{b-2}\right)$

5.7 Systems of Equations With More Than Two Variables

107. a) e.g. $\begin{vmatrix} 24x - 11z = 7 \\ 10x + z = 13 \end{vmatrix}$ b) e.g. $\begin{vmatrix} -13y + 5z = 97 \\ -y - 16z = 4 \end{vmatrix}$ 108. a) e.g. $\begin{vmatrix} -7x + 8y = 0 \\ 10x + 5y = 26 \end{vmatrix}$ b) e.g. $\begin{vmatrix} 6x + 4z = 7 \\ x + 6z = 9 \end{vmatrix}$
109. a) $(-3, 9, 2)$ c) $(8, 6, -3)$ 110. a) $(-2, 1, 5)$ c) $(-7, 8, 4)$
 b) $(5, -4, -2)$ d) $(-3, 2, 6)$ b) $(4, -6, 7)$ d) $(-1, 0, 7)$
111. a) E.g. substitute the value of z from (3) into the top two equations.
 b) E.g. solve the system consisting of (2) and (3), then insert the values of y and z into (1).
 c) E.g. (1)–(2) yields a new equation (2') without z . (2') and (3) now form a 2×2 system.
 d) E.g. (1)–(2) as well as (2)–(3) yields a new 2×2 system without y .
112. a) E.g. insert the value of x from (1) into the lower two equations.
 b) E.g. solve the system consisting of (2) and (3), then insert the values of x and y into (1).
 c) E.g. (1)–(2) yields a new equation (2') without y . (2') and (3) now form a 2×2 system.
 d) E.g. (1)–(2) as well as (2)–(3) yields a new 2×2 system without z .

113. a) $(3, 2, 4)$ c) $(10, 7, -5)$ 114. a) $(15, -4, 3)$ c) $(-5, 3, 4)$
 b) $(-1, 5, 2)$ d) $(4, -2, 6)$ b) $(7, 2, -2)$ d) $(-5, -9, 7)$
115. a) multiply equation (2) by 2 and equation (3) by (-3)
 b) add equations $(1')$ and $(2')$ as well as equations $(1')$ and $(3')$
 c) multiply equation $(2'')$ by 5 and equation $(3'')$ by 2
 d) add equations $(2''')$ and $(3''')$ $\Rightarrow \mathbb{L} = \{(1, 3, -2)\}$
116. a) multiply equation (2) by (-4) and equation (3) by 2
 b) add equations $(1')$ and $(2')$ as well as equations $(1')$ and $(3')$
 c) multiply equation $(3'')$ by 2
 d) add equations $(2''')$ and $(3''')$ $\Rightarrow \mathbb{L} = \{(5, -2, 4)\}$
117. a) $(1, 2, 3)$ d) $(-6, 9, 1)$ 118. a) $(-3, 4, 6)$ d) $(5, -8, -2)$
 b) $(-4, 6, 7)$ e) $(12, -3, 5)$ b) $(7, 10, -1)$ e) $(-9, -2, 8)$
 c) $(2, -11, -8)$ f) $(-5, 6, 12)$ c) $(-6, 5, 10)$ f) $(-2, -3, -4)$
119. a) $(5, 4, -2)$ c) $(-9, 6, 15)$ 120. a) $(-\frac{3}{2}, 7, \frac{1}{4})$ c) $(-3, \frac{5}{6}, \frac{7}{3})$
 b) $(\frac{1}{2}, -\frac{3}{4}, 5)$ d) $(\frac{4}{5}, \frac{2}{9}, -\frac{1}{3})$ b) $(\frac{1}{5}, 3, -\frac{6}{5})$ d) $(2, 12, -16)$
121. a) $(4, 3, 5, 2)$ b) $(3, -1, 2, 4)$ 122. a) $(5, 4, -3, 2)$ b) $(2, -5, 0, -6)$
123. $(1, 2, 0, -3, -1, -2)$ 124. $(3, -1, 2, 0, 4, -2)$
 125. $(\frac{13}{2}, \frac{7}{2}, -\frac{5}{2}, -\frac{13}{2}, \frac{15}{2}, \frac{5}{2})$ 126. $(\frac{15}{7}, -\frac{4}{7}, \frac{9}{7}, \frac{6}{7}, \frac{11}{7}, \frac{12}{7})$
127. a) $\square \neq 10$ and $\square \neq -18$ 128. a) $\square = 24$ and $\square \neq 15$
 b) $\square = 10$ or $\square = -18$ b) $\square = 24$ and $\square = 15$
 c) impossible c) $\square \neq 24$

5.8 Determinants

129. a) 9 c) 24 130. a) 1 c) -144
 b) -46 d) 66 b) 0 d) 78
131. a) $\begin{vmatrix} 3 & -4 \\ 6 & 5 \end{vmatrix}, \det = 39, \text{ yes}$ 132. a) $\begin{vmatrix} 7 & 9 \\ 6 & -4 \end{vmatrix}, \det = -82, \text{ yes}$
 b) $\begin{vmatrix} -5 & 8 \\ -4 & 7 \end{vmatrix}, \det = -3, \text{ yes}$ b) $\begin{vmatrix} -8 & 4 \\ -6 & 3 \end{vmatrix}, \det = 0, \text{ no}$
 c) $\begin{vmatrix} 8 & -6 \\ 4 & -3 \end{vmatrix}, \det = 0, \text{ no}$ c) $\begin{vmatrix} 6 & -2 \\ -3 & -7 \end{vmatrix}, \det = -48, \text{ yes}$
 d) $\begin{vmatrix} 2 & 3 \\ -4 & -5 \end{vmatrix}, \det = 2, \text{ yes}$ d) $\begin{vmatrix} 1 & -4 \\ -2 & 8 \end{vmatrix}, \det = 0, \text{ no}$
133. a) $\begin{vmatrix} 6 & 1 \\ 9 & 2 \end{vmatrix}, \begin{vmatrix} -5 & 1 \\ -4 & 2 \end{vmatrix}, \begin{vmatrix} 6 & -5 \\ 9 & -4 \end{vmatrix}, (-2, 7)$ 134. a) $\begin{vmatrix} 5 & 8 \\ 3 & 4 \end{vmatrix}, \begin{vmatrix} 2 & 8 \\ 6 & 4 \end{vmatrix}, \begin{vmatrix} 5 & 2 \\ 3 & 6 \end{vmatrix}, (10, -6)$
 b) $\begin{vmatrix} 11 & 5 \\ 9 & 4 \end{vmatrix}, \begin{vmatrix} 1 & 5 \\ 0 & 4 \end{vmatrix}, \begin{vmatrix} 11 & 1 \\ 9 & 0 \end{vmatrix}, (-4, 9)$ b) $\begin{vmatrix} 15 & -4 \\ 5 & 8 \end{vmatrix}, \begin{vmatrix} 5 & -4 \\ 4 & 8 \end{vmatrix}, \begin{vmatrix} 15 & 5 \\ 5 & 4 \end{vmatrix}, (\frac{2}{5}, \frac{1}{4})$
 c) $\begin{vmatrix} 2 & 5 \\ 3 & 7 \end{vmatrix}, \begin{vmatrix} 1 & 5 \\ 2 & 7 \end{vmatrix}, \begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix}, (3, -1)$ c) $\begin{vmatrix} 7 & -5 \\ 4 & -3 \end{vmatrix}, \begin{vmatrix} 5 & -5 \\ 2 & -3 \end{vmatrix}, \begin{vmatrix} 7 & 5 \\ 4 & 2 \end{vmatrix}, (5, 6)$
 d) $\begin{vmatrix} 4 & 3 \\ 8 & -9 \end{vmatrix}, \begin{vmatrix} 6 & 3 \\ -8 & -9 \end{vmatrix}, \begin{vmatrix} 4 & 6 \\ 8 & -8 \end{vmatrix}, (\frac{1}{2}, \frac{4}{3})$ d) $\begin{vmatrix} -2 & 9 \\ -1 & 5 \end{vmatrix}, \begin{vmatrix} 3 & 9 \\ 3 & 5 \end{vmatrix}, \begin{vmatrix} -2 & 3 \\ -1 & 3 \end{vmatrix}, (12, 3)$

135. a) 26 c) 0 136. a) 5 c) -48
b) 21 d) 87 b) 0 d) 45
137. a) $\det = 1$, yes c) $\det = 0$, no 138. a) $\det = -3$, yes c) $\det = -54$, yes
b) $\det = 2$, yes d) $\det = 0$, no b) $\det = 0$, no d) $\det = 130$, yes

5.9 Word Problems

- | | | | |
|-------------|-------------|--------------|-------------|
| 139. 12, 18 | 140. 17, 41 | 141. 11, 23 | 142. 42, 67 |
| 143. 14, 62 | 144. 11, 71 | 145. $-8, 9$ | 146. 18, 19 |
| 147. 12, 56 | 148. 24, 52 | 149. 36 | 150. 47 |

- | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---------|--|----|---|---|---|---|---|---|---|---|----|--|---|---|----|----|---|---|---|---|---|---------|--|---|---|---|---|---|---|---|---|---|----|---|---|----|---|---|---|----|----|---|---|
| 151. a) | <table><tr><td>8</td><td>3</td><td>4</td></tr><tr><td>1</td><td>5</td><td>9</td></tr><tr><td>6</td><td>7</td><td>2</td></tr></table> | 8 | 3 | 4 | 1 | 5 | 9 | 6 | 7 | 2 | b) | <table><tr><td>6</td><td>5</td><td>10</td></tr><tr><td>11</td><td>7</td><td>3</td></tr><tr><td>4</td><td>9</td><td>8</td></tr></table> | 6 | 5 | 10 | 11 | 7 | 3 | 4 | 9 | 8 | 152. a) | <table><tr><td>6</td><td>1</td><td>8</td></tr><tr><td>7</td><td>5</td><td>3</td></tr><tr><td>2</td><td>9</td><td>4</td></tr></table> | 6 | 1 | 8 | 7 | 5 | 3 | 2 | 9 | 4 | b) | <table><tr><td>9</td><td>10</td><td>5</td></tr><tr><td>4</td><td>8</td><td>12</td></tr><tr><td>11</td><td>6</td><td>7</td></tr></table> | 9 | 10 | 5 | 4 | 8 | 12 | 11 | 6 | 7 |
| 8 | 3 | 4 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 5 | 9 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 6 | 7 | 2 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 6 | 5 | 10 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 11 | 7 | 3 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 4 | 9 | 8 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 6 | 1 | 8 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 7 | 5 | 3 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 2 | 9 | 4 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 9 | 10 | 5 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 4 | 8 | 12 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 11 | 6 | 7 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |

- | | | | |
|---|---------------------------|---|--------------------------------|
| 153. 26 y., 12 y. | 154. 31 y., 55 y. | 155. 18 y., 27 y. | 156. 33 y., 18 y. |
| 157. 15 y., 5 y. | 158. 48 y., 16 y. | 159. 12 cm, 6 cm | 160. 18 cm, 77 cm ² |
| 161. 16 cm, 9 cm | 162. 26 cm, 12 cm | 163. 20 cm, 6 cm | 164. 18 cm, 7 cm |
| 165. 19°, 104°, 76°, 161° | 166. 81°, 100°, 54°, 125° | 167. 40°, 30° | 168. 18°, 54° |
| 169. 8 dm, 12 dm, 24 dm × 48 dm | | 170. 7.2 dm, 14.4 dm, 28.8 dm × 43.2 dm | |
| 171. 200 000.—, 150 000.— | | 172. 350 000.—, 280 000.— | |
| 173. M.: 900 000.—, D.: 200 000.— | | 174. L.: Fr. 240 000.—, S.: Fr. 140 000.— | |
| 175. <i>p</i> : 2.5 %, K.: 123 000.—, M.: 117 000.— | | 176. N.: 2 %, E.: 1.5 % | |
| 177. W.: 40 %, R.: 70 % | | 178. S.: 36 %, R.: 63 % | |
| 179. A: +5 %, B: −7 % | | 180. A: 20 litres, B: 50 litres | |
| 181. 1. section: 24 km, 2. section: 18 km | | 182. 1. section: 16 km, total: 93 km | |
| 183. 10 km | | 184. J.: 840 m, W.: 210 m | |
| 185. P.: 1.7 m/s, G.: 0.3 m/s | | 186. A.: 3 m/s, W.: 7 m/s | |

187. a)  b) 
188. a)  b) 

- | | |
|--|---|
| 189. 468 | 190. 459 |
| 191. 14 cm, 12 cm, 8 cm | 192. 15 cm, 11 cm, 6 cm |
| 193. of A counterclockwise in cm:
5, 7, 7, 1, 1, 5 | 194. of A counterclockwise in cm:
11, 7, 7, 6, 6, 11 |
| 195. A.: $-.65$, P.: $-.80$, K.: $-.20$ | 196. G.: 6 g, S.: 9 g, B.: 5 g |
| 197. 47° , 123° , 82° , 108° | 198. 68° , 95° , 110° , 87° |

5.10 Repetition

- | | | true | false | | true | false |
|------|----|-------------------------------------|-------------------------------------|----|-------------------------------------|-------------------------------------|
| 199. | a) | ☐ | ☒ | c) | ☐ | ☒ |
| | b) | ☒ | ☐ | d) | ☒ | ☐ |

- | | | true | false | | true | false |
|------|----|-------------------------------------|-------------------------------------|----|-------------------------------------|-------------------------------------|
| 200. | a) | ☐ | ☒ | c) | ☐ | ☒ |
| | b) | ☒ | ☐ | d) | ☒ | ☐ |

- | | | true | false | | true | false |
|------|----|-------------------------------------|-------------------------------------|----|-------------------------------------|-------------------------------------|
| 201. | a) | ☒ | ☐ | c) | ☐ | ☒ |
| | b) | ☐ | ☒ | d) | ☒ | ☐ |

- | | | true | false | | true | false |
|------|----|-------------------------------------|-------------------------------------|----|-------------------------------------|-------------------------------------|
| 202. | a) | ☐ | ☒ | c) | ☒ | ☐ |
| | b) | ☒ | ☐ | d) | ☐ | ☒ |

6 Quadratic Equations

6.1 Pure Quadratic Equations and Product Equations

- | | | | |
|-------------------------------|-----------------------------------|--------------------------------|-------------------------------|
| 1. a) ± 0.4 | c) $\{\}$ | 2. a) ± 1.7 | c) $\pm 4\sqrt{2}$ |
| b) $\pm \frac{5}{4}$ | d) $\pm 5\sqrt{5}$ | b) $\pm \frac{7}{4}$ | d) $\{\}$ |
| 3. a) $\pm \frac{5}{2}$ | c) $\pm \frac{10\sqrt{3}}{3}$ | 4. a) $\pm \frac{15}{14}$ | c) $\pm \frac{4\sqrt{2}}{7}$ |
| b) $\pm \frac{11}{3}$ | d) $\pm \sqrt{5}$ | b) $\pm \frac{9}{5}$ | d) $\pm \sqrt{7}$ |
| 5. a) ± 1 | c) ± 8 | 6. a) ± 5 | c) $\pm 3\sqrt{3}$ |
| b) $\pm \frac{\sqrt{6}}{3}$ | d) ± 7 | b) $\pm \frac{3\sqrt{2}}{2}$ | d) ± 11 |
| 7. a) ± 4 | c) ± 4 | 8. a) $\{\}$ | c) ± 5 |
| b) $\{\}$ | d) $\pm 4\sqrt{2}$ | b) $\pm \sqrt{53}$ | d) $\pm \frac{3\sqrt{35}}{5}$ |
| 9. a) $\pm \sqrt{6}$ | b) ± 4 | 10. a) $\pm \sqrt{7}$ | b) $\pm 4\sqrt{2}$ |
| 11. a) $-5, 3$ | c) $-4, 0$ | 12. a) $-9, 6$ | c) $-3.5, \frac{1}{3}$ |
| b) $-7, 2.5$ | d) $-\frac{13}{3}, -\frac{16}{5}$ | b) $0, \frac{11}{3}$ | d) $-1.5, \frac{5}{3}$ |
| 13. a) $0, 7$ | c) $0, 2$ | 14. a) $-1, 0$ | c) $0, \frac{1}{2}$ |
| b) $-8, 0$ | d) $-\frac{3}{2}, 0$ | b) $-10, 0$ | d) $-8, 0$ |
| 15. a) $3, 9$ | c) $-3, 10$ | 16. a) $-1, 7$ | c) $-2, 11$ |
| b) $-5, 6$ | d) $-2, 6$ | b) $-3, 8$ | d) $4, 10$ |
| 17. a) e. g. $(x-2)(x+4) = 0$ | | 18. a) e. g. $(x+8)(x+10) = 0$ | |
| b) e. g. $(x+3.5)(x+5) = 0$ | | b) e. g. $(x-0.5)(x-2) = 0$ | |
| c) e. g. $(x-6)(x+6) = 0$ | | c) e. g. $(x+4)x = 0$ | |
| d) e. g. $x(x-3) = 0$ | | d) e. g. $(x-3)(x+3) = 0$ | |
| 19. a) $x^2 = 16$ | c) $x^2 = 13$ | 20. a) $x^2 = 225$ | c) $x^2 = 169$ |
| b) $x^2 = \frac{25}{4}$ | d) $x^2 = 324$ | b) $x^2 = 5$ | d) $x^2 = 11$ |
| 21. a) 2 | c) 2 | 22. a) 0 | c) 2 |
| b) 1 | d) 1 | b) 2 | d) 1 |
| 23. a) $-$ | | 24. a) $+$ | |
| b) impossible | | b) $+$ | |
| c) $-$ | | c) $+$ | |
| d) $+$ | | d) $-$ | |

$$25. \quad \begin{array}{lcl} \text{a)} & 4x^2 & = 64 & | \sqrt{} \\ & 2x & = \pm 8 & | : 2 \\ & x & = \pm 4 \end{array}$$

$$\text{b)} \quad \begin{array}{lcl} x(x+5) & = & 0 \\ x_1 & = & 0 \\ x_2 & = & -5 \end{array}$$

$$\text{c)} \quad \begin{array}{lcl} \frac{x^2+1}{x+3} - \frac{10}{x+3} & = & 0 & | \cdot (x+3) \\ x^2 - 9 & = & 0 & | + 9 \\ x^2 & = & 9 & | \sqrt{} \\ x & = & \pm 3 \end{array}$$

test:

$$x = 3 \quad \checkmark$$

$$x = -3 \quad \notin \mathbb{D}$$

$$26. \quad \begin{array}{lcl} \text{a)} & x^2 + 25 & = 0 & | - 25 \\ & x^2 & = -25 & | \sqrt{} \\ & \{ \} \end{array}$$

$$\text{b)} \quad \begin{array}{lcl} \frac{1}{3}x^2 - 15 & = & 0 & | \cdot 3 \\ x^2 - 45 & = & 0 & | + 45 \\ x^2 & = & 45 & | \sqrt{} \\ x & = & \pm 3\sqrt{5} \end{array}$$

$$\text{c)} \quad \begin{array}{lcl} (x+4)^2 + 9 & = & 0 & | - 9 \\ (x+4)^2 & = & -9 & \\ \{ \} \end{array}$$

$$27. \quad \text{a)} \quad 4 \text{ solutions: } (\pm 3, \pm 2)$$

$$\text{b)} \quad 4 \text{ solutions: } (\pm\sqrt{5}, \pm\sqrt{3})$$

$$\text{c)} \quad 1 \text{ solution: } (0, 0)$$

$$\text{d)} \quad \text{no solution: } \{ \}$$

$$29. \quad \text{a)} \quad \text{no solution: } \{ \}$$

$$\text{b)} \quad 8 \text{ solutions: } (\pm 2, \pm\sqrt{2}, \pm\sqrt{5})$$

$$28. \quad \text{a)} \quad 2 \text{ solutions: } (\pm 2, 0)$$

$$\text{b)} \quad 2 \text{ solutions: } (0, \pm\sqrt{6})$$

$$\text{c)} \quad \text{no solution: } \{ \}$$

$$\text{d)} \quad 4 \text{ solutions: } (\pm\sqrt{7}, \pm\sqrt{7})$$

$$30. \quad \text{a)} \quad 8 \text{ solutions: } (\pm\sqrt{6}, \pm\sqrt{10}, \pm 2)$$

$$\text{b)} \quad \text{no solution: } \{ \}$$

6.2 Completing the Square

$$31. \quad \text{a)} \quad -1$$

$$\text{b)} \quad \pm 1.5$$

$$\text{c)} \quad -1, 3$$

$$\text{d)} \quad 2 \pm \sqrt{6}$$

$$33. \quad \text{a)} \quad 1, 11$$

$$\text{b)} \quad -14, 8$$

$$\text{c)} \quad -2 \pm \sqrt{15}$$

$$\text{d)} \quad 1 \pm 3\sqrt{2}$$

$$35. \quad \text{a)} \quad 4$$

$$\text{b)} \quad 6.25$$

$$\text{c)} \quad c^2$$

$$32. \quad \text{a)} \quad -5, -1$$

$$\text{b)} \quad -11, 1$$

$$\text{c)} \quad \frac{1}{3}$$

$$\text{d)} \quad \{ \}$$

$$34. \quad \text{a)} \quad -11, 3$$

$$\text{b)} \quad 1, 17$$

$$\text{c)} \quad 5 \pm \sqrt{21}$$

$$\text{d)} \quad -10 \pm 4\sqrt{2}$$

$$36. \quad \text{a)} \quad 81$$

$$\text{b)} \quad 2.25$$

$$\text{c)} \quad \frac{a^2}{4}$$

$$37. \quad \text{a)} \quad x^2 - 6x + 9 = (x - 3)^2$$

$$\text{b)} \quad x^2 + 8x + 16 = (x + 4)^2$$

$$\text{c)} \quad x^2 + x + \frac{1}{4} = \left(x + \frac{1}{2}\right)^2$$

$$\text{d)} \quad x^2 + 4dx + 4d^2 = (x + 2d)^2$$

$$38. \quad \text{a)} \quad x^2 - 20x + 100 = (x - 10)^2$$

$$\text{b)} \quad x^2 + \frac{1}{2}x + \frac{1}{16} = \left(x + \frac{1}{4}\right)^2$$

$$\text{c)} \quad x^2 - x + \frac{1}{4} = \left(x - \frac{1}{2}\right)^2$$

$$\text{d)} \quad x^2 - 6mx + 9m^2 = (x + 3m)^2$$

$$39. \quad \text{a)} \quad -7, 11$$

$$\text{b)} \quad -19, 9$$

$$\text{c)} \quad 1 \pm 4\sqrt{3}$$

$$\text{d)} \quad -6 \pm 9\sqrt{3}$$

$$40. \quad \text{a)} \quad -17, 9$$

$$\text{b)} \quad -14, 36$$

$$\text{c)} \quad -3 \pm 6\sqrt{2}$$

$$\text{d)} \quad 7 \pm 5\sqrt{5}$$

$$41. \quad \text{a)} \quad -13, 1$$

$$\text{b)} \quad -6, 14$$

$$\text{c)} \quad -11, 5$$

$$\text{d)} \quad -21, -3$$

$$42. \quad \text{a)} \quad -13, 9$$

$$\text{b)} \quad 4, 6$$

$$\text{c)} \quad -22, 4$$

$$\text{d)} \quad -16, 0$$

43. a) $2x^2 + 20x + 50 = 2(x+5)^2$
 b) $3x^2 + 24x + 48 = 3(x+4)^2$
 c) $-\frac{2}{x} - 6x - 18 = -\frac{1}{2}(x+6)^2$
 d) $0.1x^2 - 0.2x + 0.1 = 0.1(x-1)^2$
44. a) $4x^2 - 24x + 36 = 4(x-3)^2$
 b) $\frac{1}{4}x^2 + 2x + 4 = \frac{1}{4}(x+4)^2$
 c) $-3x^2 + 12x - 12 = -3(x-2)^2$
 d) $1.5x^2 + 15x + 37.5 = 1.5(x+5)^2$
45. a) $-3 \pm 3\sqrt{2}$ c) $-2 \pm 5\sqrt{2}$
 b) $-6, 8$ d) $\frac{1}{2} \pm \frac{4}{5}\sqrt{5}$
46. a) $0, 2$ c) $-8.5, 4.5$
 b) $-6 \pm 15\sqrt{3}$ d) $1, 13$
47. a) $-12, 32$ c) $-2.5, 1.5$
 b) $-5 \pm 10\sqrt{2}$ d) $1, -\frac{1}{3}$
48. a) $-1 \pm \sqrt{17}$ c) $-0.4, 1.2$
 b) $\frac{17}{3}, -\frac{13}{3}$ d) $-\frac{31}{10}, \frac{29}{10}$
49. $\mathcal{E}, \mathcal{B}, \mathcal{D}, \mathcal{A}, \mathcal{C}$
50. $\mathcal{B}, \mathcal{D}, \mathcal{A}, \mathcal{E}, \mathcal{C}$
51. $\mathcal{C}, \mathcal{B}, \mathcal{A}, \mathcal{E}, \mathcal{F}, \mathcal{D}$
52. $\mathcal{A}, \mathcal{C}, \mathcal{D}, \mathcal{E}, \mathcal{B}, \mathcal{F}$
53. a) $x^2 + 14x = 51$
 $x^2 + 14x + 49 = 100$
 $(x+7)^2 = 100$
 $x_1 = -17, x_2 = 3$
- b) $4x^2 + 24x = -20$
 $x^2 + 6x = -5$
 $x^2 + 6x + 9 = -5 + 9$
 $(x+3)^2 = 4$
 $x+3 = \pm 2$
 $x_1 = -5, x_2 = -1$
- c) $-x^2 + 8x = -16$
 $x^2 - 8x = 16$
 $x^2 - 8x + 16 = 16 + 16$
 $(x-4)^2 = 32$
 $x-4 = \pm 4\sqrt{2}$
 $x = 4 \pm 4\sqrt{2}$
54. a) $x^2 - 10x = 39$
 $x^2 - 10x + 25 = 64$
 $(x-5)^2 = \pm 8$
 $x_1 = -3, x_2 = 13$
- b) $2x^2 + 44x = -192$
 $x^2 + 22x = -96$
 $x^2 + 22x + 121 = 25$
 $(x+11)^2 = 25$
 $x+11 = \pm 5$
 $x_1 = -16, x_2 = -6$
- c) $x^2 - 5x = 9.75$
 $x^2 - 5x + 6.25 = 9.75 + 6.25$
 $(x-2.5)^2 = 16$
 $x-2.5 = \pm 4$
 $x_1 = -1.5, x_2 = 6.5$
55. $x = \frac{-p \pm \sqrt{p^2 - 4q}}{2}$
56. $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

6.3 Quadratic Formula and Solvability

57. a) $a = 4,$
 $b = -1,$
 $c = -3$
 $-\frac{3}{4}, 1$
- b) $a = 4,$
 $b = -7,$
 $c = -15$
 $-\frac{5}{4}, 3$
- c) $a = 2,$
 $b = -9,$
 $c = -35$
 $-2.5, 7$
- d) $a = 2,$
 $b = -7,$
 $c = 6.125$
 1.75
58. a) $a = 9,$
 $b = 17,$
 $c = -2$
 $-2, \frac{1}{9}$
- b) $a = -3,$
 $b = -4,$
 $c = 7$
 $-\frac{7}{3}, 1$
- c) $a = -20,$
 $b = -17,$
 $c = -3$
 $-\frac{3}{5}, -\frac{1}{4}$
- d) $a = 9,$
 $b = -10,$
 $c = -16$
 $-\frac{8}{9}, 2$
59. a) $-\frac{1}{4}, 2$ c) $-\frac{9}{2}, \frac{1}{6}$
- b) $-3, 2.5$ d) $-3, 7$
60. a) $-3, 6$ c) $-2, -\frac{1}{5}$
- b) $-\frac{11}{4}, 2$ d) $-1, \frac{1}{2}$

61. a) $-1.5, 1$ c) $4, 11$ 62. a) $-2.5, 5$ c) $-\frac{3}{2}, 5$
 b) $-2, \frac{1}{3}$ d) $-\frac{1}{2}, \frac{4}{3}$ b) $-4, \frac{1}{2}$ d) $-7, -\frac{9}{7}$
63. a) $-\frac{1}{10}, 3$ c) $-\frac{5}{2}, 2$ 64. a) $1.5, 12.5$ c) $-\frac{2}{3}, \frac{11}{3}$
 b) $-\frac{6}{5}, -1$ d) $\frac{3}{4}, \frac{4}{3}$ b) $-5, 12$ d) $-2.5, 1.5$
65. a) $\frac{2 \pm \sqrt{2}}{2}$ c) $\frac{-27 \pm 3\sqrt{89}}{4}$ 66. a) $\frac{3 \pm \sqrt{21}}{4}$ c) $3 \pm \sqrt{14}$
 b) $-1, -\frac{2}{3}$ d) $\frac{-11 \pm \sqrt{73}}{12}$ b) $\frac{-1 \pm \sqrt{41}}{10}$ d) $5 \pm \sqrt{31}$
67. a) $\frac{1 \pm \sqrt{129}}{8}$ c) $\frac{1}{2}, \frac{1}{4}$ 68. a) $-1, -\frac{1}{7}$ c) $\frac{-1 \pm \sqrt{2}}{4}$
 b) $\frac{-7 \pm \sqrt{37}}{6}$ d) $-1 \pm \sqrt{6}$ b) $\frac{-6 \pm \sqrt{21}}{5}$ d) $\frac{-5 \pm \sqrt{15}}{10}$
69. a) $-\sqrt{2}, 2\sqrt{2}$ 70. a) $-3\sqrt{7}, \sqrt{7}$
 b) $-3\sqrt{2}, 2\sqrt{2}$ b) $\sqrt{3}, -\frac{5\sqrt{3}}{3}$
71. a) $-2 - \sqrt{2}, \frac{2 + \sqrt{2}}{2}$ 72. a) $\frac{3 - \sqrt{5}}{2}, \frac{-23 + 3\sqrt{5}}{22}$
 b) $\sqrt{3}, 4$ b) $-\sqrt{5}, 2$
 c) $-1, -\sqrt{3}$ c) $-\sqrt{5}, -1$
73. a) 1 c) 2 74. a) 2 c) 0
 b) 0 d) 2 b) 0 d) 1
75. ①- $\mathcal{D}$, ③- $\mathcal{A}$, ④- $\mathcal{E}$, ⑤- $\mathcal{B}$, ⑥- $\mathcal{C}$, remains: ②, $\mathcal{F}$
 suitable for ② e.g. $6x^2 + 17x + 4 = 0$
 suitable for $\mathcal{F}$ e.g. $4x^2 - 6x + 2 = 0$
76. ①- $\mathcal{B}$, ②- $\mathcal{E}$, ③- $\mathcal{F}$, ④- $\mathcal{A}$, ⑥- $\mathcal{C}$, remains: ⑤, $\mathcal{D}$
 suitable for ⑤ e.g. $\frac{7}{2}x^2 - 10x + \frac{1}{2} = 0$
 suitable for $\mathcal{D}$ e.g. $\frac{1}{2}x^2 - 2x = 0$
77. a) $-; -\frac{1}{2}, \frac{1}{3}$ 78. a) impossible
 b) $+$; $-7, 2.5$ b) $+$; $-3\sqrt{2}, \frac{1}{2}\sqrt{2}$
 c) impossible $-; -\frac{3}{2}\sqrt{2}, -\sqrt{2}$
 d) $+$; $-\sqrt{3}, \frac{1}{3}$ c) $+$; $-\frac{7}{3}, 2$
 d) $-; \pm 4.5$
79. a) $] - \infty, -2\sqrt{2}[\cup] 2\sqrt{2}, \infty[$ 80. a) $] - 4\sqrt{3}, 4\sqrt{3}[$
 b) $\frac{4}{3}, 0$ b) $] \frac{16}{3}, \infty[$
 c) $] - \infty, \frac{9}{4}]$ c) $] - \infty, -4]$
 d) $] - \infty, 0[\cup] 0, \frac{27}{4}[$ d) $] - \infty, \frac{1}{3}[$
81. a) $] - \infty, -2] \cup [0, \infty[$ 82. a) $] - \infty, 0[\cup] 2, \infty[$
 b) $] - \infty, -\frac{5}{2}] \cup [\frac{5}{2}, \infty[$ b) $] - \infty, 0[\cup [8, \infty[$
83. a) $\frac{1}{3}$ c) $\{\}$ 84. a) $-1.5, 4$ c) $\frac{3}{4}$
 b) $-3, \frac{1}{4}$ d) $-3.5, 1$ b) $\{\}$ d) $\{\}$
85. a) $-\sqrt{2} - 1$ 86. a) $-\sqrt{3} - 1, 2\sqrt{3} + 2$
 b) $-\sqrt{6}, \frac{2\sqrt{6}}{3}$ b) $-\sqrt{10} + 2, \frac{2\sqrt{10}-4}{3}$
 c) $3\sqrt{3}$ c) $\{\}$
 d) $\{\}$ d) $\sqrt{6}$

87. a) $-6, 0$ c) $\pm \frac{5}{2}$ 88. a) $0, 10$ c) $\{\}$
 b) $-\frac{1}{3}, 0$ d) $\{\}$ b) $\pm \frac{\sqrt{30}}{5}$ d) $0, \frac{1}{4}$
89. a) $-\frac{1}{5}, 0$ c) $0, \frac{\sqrt{5}}{2}$ 90. a) $\pm\sqrt{2}$ c) $0, \frac{1}{5}$
 b) $0, 12$ d) $\pm\sqrt{7}$ b) $-3, 0$ d) $-\frac{\sqrt{3}}{3}, 0$
91. a) $(x+6)(x-4)$ c) $(x+9)(x+5)$ 92. a) $(x-2)(x-18)$ c) $(x+8)(x+6)$
 b) $(x-3)(x-7)$ d) $(x+10)(x-1)$ b) $(x+3)(x-4)$ d) $(x+9)(x-3)$
93. a) $-1, 4$ c) $-6, -3$ 94. a) $-5, 10$ c) $-1, 2$
 b) $2, 6$ d) $-12, 7$ b) $5, 6$ d) $-2, 3$
95. a) $b = -1, c = -12$ 96. a) $b = 3, c = -40$
 b) $b = -3 - \sqrt{7}, c = 3\sqrt{7}$ b) $b = 1 + \sqrt{7}, c = \sqrt{7}$
97. a) $\pm 5\sqrt{2}$ c) $-4, 3.5$ 98. a) -2 c) $-6, 9$
 b) $-4.5, 1.5$ d) -3 b) $-5, -4$ d) ± 8
99. a) ± 5 c) $-12, 8$ 100. a) $\pm 10\sqrt{2}$ c) $-2, 8$
 b) $-2\sqrt{2}, 3\sqrt{2}$ d) $\pm 4\sqrt{6}$ b) $6, 18$ d) $-4\sqrt{3}, 1$
101. ①- $\mathcal{C}$, ②- $\mathcal{B}$, ③- $\mathcal{A}$, ④- $\mathcal{F}$, ⑤- $\mathcal{E}$, remains: ⑥, $\mathcal{D}$ 102. ①- $\mathcal{B}$, ②- $\mathcal{F}$, ③- $\mathcal{A}$, ④- $\mathcal{E}$, ⑤- $\mathcal{D}$, remains: ⑥, $\mathcal{C}$
 suitable for ⑥: $2(x-6)(x+4) = 0$ suitable for ⑥: $\frac{1}{2}(x+10)(x-3.6) = 0$
 suitable for $\mathcal{D}$: $2x^2 + 15.2x - 48 = 0$ suitable for $\mathcal{C}$: $\frac{1}{2}x^2 + 8x - 18 = 0$
103. a) $-2, 27$ c) $-\frac{1}{2}, 8$ 104. a) $\frac{3}{4}$ c) $-4, \frac{1}{3}$
 b) ± 5 d) $-\frac{9}{2}, \frac{5}{2}$ b) $-8, -1$ d) $-1, -\frac{1}{2}$
105. a) $\{\}$ c) $-\frac{18}{5}, -\frac{1}{6}$ 106. a) $-5, 3$ c) 0
 b) $-4, -1$ d) $\pm\sqrt{3}$ b) $-\frac{8}{3}, \frac{3}{2}$ d) $\{\}$
107. a) 2 mistakes c) 2 mistakes 108. a) 2 mistakes c) 2 mistakes
 b) 2 mistakes d) 3 mistakes b) 1 mistake d) 4 mistakes
109. $\frac{\pm\sqrt{-ac}}{a}$, if $ac \leq 0$; $\{\}$ otherwise 110. $x_1 = 0, x_2 = -\frac{b}{a}$
111. true false true false 112. true false true false
 a) ☐ ☒ c) ☒ ☐
 b) ☒ ☐ d) ☒ ☐
113. true false true false 114. true false true false
 a) ☒ ☐ c) ☐ ☒
 b) ☐ ☒ d) ☒ ☐

6.4 Substitution and Biquadratic Equations

115. a) $-\frac{21}{2}, -6$ c) 7.2, 17.6 116. a) $-82, 18$ c) $\{\}$
 b) $-\frac{10}{3}, -\frac{1}{3}$ d) $\{\}$ b) $\frac{19}{4}, \frac{23}{4}$ d) $-\frac{5}{6}, \frac{7}{3}$
117. a) 61 118. a) 2
 b) $-\frac{3}{2}, \frac{9}{2}$ b) $\frac{1}{3}, \frac{10}{3}$
 c) $-12, 24$ c) $-2, \frac{1}{4}$
 d) $-2, 4$ d) $-18, 0$
 e) they are equal e) they are equal
 f) identical before the reverse substitution f) identical before the reverse substitution
119. a) $(x+3)^2 + 9(x+3) + 20 = 0$ 120. a) $(x - \frac{1}{2})^2 + 9(x - \frac{1}{2}) + 20 = 0$
 b) $(\frac{1}{2}x)^2 + 9(\frac{1}{2}x) + 20 = 0$ b) $(3x)^2 + 9(3x) + 20 = 0$
121. a) $2(\frac{x}{4})^2 - 9(\frac{1}{4}x) - 5 = 0$ 122. a) $2(4x)^2 - 9(4x) - 5 = 0$
 b) $2(x-1)^2 - 9(x-1) - 5 = 0$ b) $2(x+6)^2 - 9(x+6) - 5 = 0$
123. a) $\pm 2, \pm 3$ c) ± 4 124. a) ± 11 c) ± 1
 b) $\pm 1, \pm 5$ d) $\{\}$ b) ± 6 d) $\pm 2, \pm 10$
125. a) $\pm\sqrt{3}$ c) 0 126. a) $\{\}$ c) ± 1
 b) $\pm\sqrt{15}, \pm 2\sqrt{2}$ d) $\pm\sqrt{10}, \pm 2$ b) $0, \pm 7$ d) $0, \pm 1$
127. a) $\pm\frac{\sqrt{10}}{2}$ c) $\pm\frac{\sqrt{6}}{6}, \pm\frac{1}{4}$ 128. a) $\pm\frac{\sqrt{14}}{2}, \pm\frac{\sqrt{10}}{2}$ c) $\{\}$
 b) $\pm\frac{\sqrt{3}}{2}$ d) $\pm\frac{2}{3}$ b) $\pm\frac{3}{2}$ d) $\pm\frac{\sqrt{33}}{3}, \pm 1$

6.5 Rational Equations

129. a) $\mathbb{R} \setminus \{-2, 7\}$ c) $\mathbb{R} \setminus \{3, 8\}$ 130. a) $\mathbb{R} \setminus \{0, \frac{1}{5}\}$ c) $\mathbb{R} \setminus \{-2, \frac{1}{3}\}$
 b) $\mathbb{R} \setminus \{-3, -4\}$ d) $\mathbb{R} \setminus \{-0.5, 7\}$ b) $\mathbb{R} \setminus \{4, 6\}$ d) $\mathbb{R} \setminus \{\pm 3\}$
131. a) -6 c) -7 132. a) $-6, \frac{1}{2}$ c) -8
 b) $\pm 2\sqrt{6}$ d) $-\frac{1}{3}, 4$ b) $\pm 5\sqrt{2}$ d) $-4, 8$
133. a) $\frac{5}{2}, -1$ c) $-7, 1$ 134. a) $0, 3$ c) $0, 2$
 b) $\{\}$ d) 3 b) -3 d) $4, -2$
135. a) 2 c) $\{\}$ 136. a) $2, -4$ c) $\{\}$
 b) 3 d) $\{\}$ b) -6 d) -7
137. a) -4 c) -6 138. a) -0.5 c) -6
 b) -5 d) -9 b) 7 d) -6

6.6 Root Equations

139. a) $] - \infty, -\sqrt{5}] \cup [\sqrt{5}, \infty[$
 b) $] - \infty, -4] \cup [1, \infty[$
 c) $\mathbb{R}$
 d) $] - \infty, -3] \cup [0, \infty[$
140. a) $[-4, 4]$
 b) $] - \infty, 1] \cup [4, \infty[$
 c) $[-6, 0]$
 d) $\{\}$
141. ①- $\mathcal{B}$ solutions 2 and 6,
 ②- $\mathcal{C}$ solutions 1 and 2,
 ③- $\mathcal{A}$ solutions 0 and 3,
 ④- $\mathcal{D}$ solution 3
142. ①- $\mathcal{B}$ solution 5,
 ②- $\mathcal{C}$ solution 7,
 ③- $\mathcal{D}$ solutions 5 and 8,
 ④- $\mathcal{A}$ solutions 0 and 6
143. a) $-9, 6$ c) $5, 7$
 b) $4, 5$ d) 4
144. a) $-5, 10$ c) 7
 b) $9, 11$ d) 1
145. a) 6 c) 4
 b) 9 d) $\{\}$
146. a) 6 c) 6
 b) 8 d) $-\frac{1}{2}, 2$
147. a) $1, 37$ d) 9
 b) $\frac{2}{3}$ e) 0
 c) $\{\}$ f) $\frac{1}{4}$
148. a) 4 d) 15
 b) $\frac{3}{4}$ e) 0
 c) $\{\}$ f) $\frac{121}{64}$

6.7 Parameter Equations

149. a) for m : $-4n, 2n$
 for n : $-\frac{1}{4}m, \frac{1}{2}m$
 b) for a : $-4b, 2b$
 for b : $-\frac{1}{4}a, \frac{1}{2}a$
150. a) for k : $-\frac{1}{4}p, \frac{3}{2}p$
 for p : $-4k, \frac{2}{3}k$
 b) for c : $\frac{2}{3}d, \frac{3}{2}d$
 for d : $\frac{3}{2}c, \frac{2}{3}c$
151. a) $-3a, 2a$ c) $-\frac{3}{c}, 4$
 b) $-\frac{b}{2}, 4b$ d) $\frac{3}{d}, -\frac{5}{d}$
152. a) $-7a, \frac{a}{2}$ c) $-11, \frac{1}{4c}$
 b) $-6b, \frac{b}{3}$ d) $\pm \frac{1}{2d}$
153. a) $-1, -\frac{1}{b}$ c) $\frac{1}{2}, \frac{-4d+1}{2}$
 b) $\frac{2}{c}, \frac{10}{c}$ d) $-\frac{2}{a}, a+1$
154. a) $-\frac{3}{a}, \frac{2}{a}$ c) $\{\}$
 b) $-\frac{3}{4b}, \frac{5}{2}b$ d) $\frac{2}{9}d, \frac{1}{6d}$
155. $a = 7$: solution: -3.5
 $a = -7$: solution: 3.5
156. $a = 13$: solution: -6.5
 $a = -13$: solution: 6.5
157. $a = 25$: solutions: $-5, -20$
 $a = -25$: solutions: $20, 5$
158. $a = 27$: solutions: $-18, -9$
 $a = -27$: solutions: $9, 18$
159. For all a with $|a| \geq 10$
 solutions: $x = \frac{-a \pm \sqrt{a^2 - 100}}{10}$
160. $a = -20$: solutions: $4, 16$
161. a) $a = 26; x_2 = 6.5$
 b) $a = 4; x_2 = 11$
 c) $a = 4; x_2 = -\sqrt{2}$
 d) $a = \frac{23}{3}; x_2 = -\frac{1}{6}$
162. a) $a = 4.5; x_2 = -4.5$
 b) $a = 19; x_2 = \frac{1}{3}$
 c) $a = 12; x_2 = 4\sqrt{3}$
 d) $a = 23; x_2 = \frac{2}{3}$

163. for $a = 4$: $x_2 = -8$
for $a = 0$: $x_2 = 0$
165. a) no sol.: $b \in]-4\sqrt{2}, 4\sqrt{2}[$
1 sol.: $b = \pm 4\sqrt{2}$
2 sol.: $b \in]-\infty, -4\sqrt{2}[\cup]4\sqrt{2}, \infty[$
b) no sol.: $b > 3$
1 sol.: $b = 3$
2 sol.: $b < 3$
c) 1 sol.: $b = 0$
no sol.: otherwise
d) 1 sol.: $b = 0$
2 sol.: otherwise
167. a) $-\frac{8+3a}{12}, \frac{3+a}{4}$ c) $\pm 2a$
b) $-\frac{8p}{3}, 4p$ d) $\pm \frac{\sqrt{b}}{2b}, \pm \frac{\sqrt{2b}}{2}$
169. a) $-m, \frac{m}{2}$ c) $\frac{a^2}{2}$
b) $-\frac{4}{b}, 6b$ d) $\pm \frac{\sqrt{4b^2+1}}{2}$
171. a) for $c > -4$: $2 \pm \sqrt{4+c}$,
for $c = -4$: 2 ,
for $c < -4$: $\{\}$
b) for $b \neq 0$: $\frac{-1 \pm \sqrt{2}}{b}$,
for $b = 0$: $\{\}$
c) for $a \in]-\infty, -1[\cup]-1, 1[$:
 $\frac{-a+1 \pm \sqrt{2-2a}}{a+1}$,
for $a = 1$: 0 ,
for $a = -1$: $\frac{1}{2}$,
for $a > 1$: $\{\}$
d) for $d \in]-\infty, -6[\cup]6, \infty[$: $\frac{d \pm \sqrt{d^2-36}}{18}$,
for $d \in]-6, 6[$: $\{\}$,
for $d = 6$: $\frac{1}{3}$,
for $d = -6$: $-\frac{1}{3}$
173. a) for all a : $\frac{1}{2}a, a$
b) for all b : $\{\}$
c) for $c \neq 0$: $\frac{1}{4c}, \frac{1}{4}$,
for $c = 0$: $\frac{1}{4}$
d) for all d : $\frac{-d-4 \pm \sqrt{d^2+16}}{4}$
164. for $a = 6$: $x_2 = 4$
for $a = 10$: $x_2 = 12$
166. a) 2 sol.: $b < 0$
1 sol.: $b = 0$
no sol.: otherwise
b) no sol.: $b < -4.5$
1 sol.: $b = -4.5$
2 sol.: $b > -4.5$
c) 1 sol.: $b = 1$
2 sol.: otherwise
d) 1 sol.: $b = 0$
2 sol.: otherwise
168. a) $0, \frac{1}{a}$ c) $\pm \sqrt{6}a, \pm a$
b) $-\frac{1}{p}, \frac{4-3p}{2p^2-2p}$ d) $\pm \frac{3}{b}, \pm b$
170. a) $-\frac{a}{2}, \frac{a}{4}$ c) $-b, 2b$
b) $-b \pm b\sqrt{2}$ d) $p^2, 17p^2$
172. a) for $a \in]-\infty, -2[\cup]2, \infty[$: $\frac{-a \pm \sqrt{a^2-4}}{2}$
for $a = -2$: 1
for $a = 2$: -1
for $a \in]-2, 2[$: $\{\}$
b) for $b = -9$: 3
for $b > -9$: $3 \pm \sqrt{9+b}$
for $b < -9$: $\{\}$
c) for $c > -2 \vee c \neq 2$: $\frac{-c-2 \pm 2\sqrt{c+2}}{2c-4}$,
for $c = -2$: 0 ,
for $c = 2$: $-\frac{1}{4}$
for $c < -2$: $\{\}$
d) for all d : $-\frac{d}{2}, -\frac{d}{3}$
174. a) for $a \neq 0$: $-\frac{2}{5a}, -\frac{1}{a}$,
for $a = 0$: $\{\}$
b) for all b : $\{\}$
c) for $c \neq -1$: $\frac{-1}{c+1}, -1$,
for $c = -1$: -1
d) for $d < 2.5 \vee d \neq 0$: $\frac{-5+d \pm \sqrt{25-10d}}{-2d^2}$,
for $d = 2.5$: $\frac{1}{5}$,
for $d = 0$: $\frac{1}{20}$
for $d > 2.5$: $\{\}$

175. a) for $a = -1 : -2$,
 for $a = -9 : -6$,
 for $a \in]-\infty, -9[\cup]-1, \infty[: \frac{-a+3 \pm \sqrt{a^2+10a+9}}{-2}$,
 for $a \in]-9, -1[: \{\}$
- b) for $b = \frac{1}{9} : -6$,
 for $b = 1 : 2$,
 for $b = 0 : -2$,
 for $b \in]-\infty, 0[\cup]0, \frac{1}{9}[\cup]\frac{1}{9}, \infty[: \frac{-3b+1 \pm \sqrt{9b^2-10b+1}}{-b}$,
 for $b \in]\frac{1}{9}, 1[: \{\}$
- c) for $c = -\frac{1}{4} : 2$,
 for $c = -1 : -1$,
 for $c = 0 : \frac{1}{2}$,
 for $c \in]-\infty, -1[\cup]-\frac{1}{4}, 0[\cup]0, \infty[: \frac{-2c-1 \pm \sqrt{4c^2+5c+1}}{c}$,
 for $c \in]-1, -\frac{1}{4}[: \{\}$
- d) for $d = \sqrt{6} : \frac{\sqrt{6}}{12}$,
 for $d = -\sqrt{6} : -\frac{\sqrt{6}}{12}$,
 for $d = 0 : \text{equation not defined}$,
 for $d = 2 : -\frac{1}{4}$,
 for $d = -2 : \frac{1}{4}$,
 for $d \in]-\infty, -\sqrt{6}[\cup]-2, 0[\cup]0, 2[\cup]\sqrt{6}, \infty[: \frac{-5+d^2 \pm \sqrt{d^4-10d^2+24}}{2d}$,
 for $d \in]-\sqrt{6}, -2[\cup]2, \sqrt{6}[: \{\}$
176. a) for $a = -\frac{2}{3} : -\frac{1}{9}$,
 for $a = -\frac{4}{3} : -\frac{1}{3}$,
 for $a = -\frac{2}{15} : -\frac{5}{12}$,
 for $a \in]-\infty, -\frac{4}{3}[\cup]-\frac{2}{3}, -\frac{2}{15}[\cup]-\frac{2}{15}, \infty[: \frac{-3a+2 \pm \sqrt{9a^2+18a+8}}{15a+2}$,
 for $a \in]-\frac{4}{3}, -\frac{2}{3}[: \{\}$
- b) for $b = -\frac{1}{8} : -\frac{1}{4}$,
 for $b = -\frac{1}{2} : \frac{1}{2}$,
 for $b < -\frac{1}{2} \vee b > -\frac{1}{8} : \frac{-4b-1 \pm \sqrt{16b^2+10b+1}}{2}$,
 for $b \in]-\frac{1}{2}, -\frac{1}{8}[: \{\}$
- c) for $c = -1 : -1$,
 for $c = -4 : \frac{1}{2}$,
 for $c = 0 : -\frac{1}{4}$,
 for $c \in]-\infty, -4[\cup]-1, 0[\cup]0, \infty[: \frac{2+c \pm \sqrt{c^2+5c+4}}{c}$,
 for $c \in]-4, -1[: \{\}$
- d) for $d = -1 : 6$,
 for $d = -4 : 12$,
 for $d < -4 \vee d > -1 : -2d+4 \pm 2\sqrt{d^2+5d+4}$,
 for $d \in]-4, -1[: \{\}$

6.8 Word Problems

- | | |
|----------------------------------|-----------------------------------|
| 177. -20, 16 | 178. -18, 21 |
| 179. (9, -6), (-25, -40) | 180. (14, -11), (-75, -100) |
| 181. -3, 20 | 182. -3, 40 |
| 185. $-\frac{8}{9}, \frac{9}{8}$ | 186. $\frac{3}{7}, \frac{7}{3}$ |
| 189. 7.29 | 190. 0, 1.3 |
| 193. 18 | 194. 9 |
| 197. 178 cm | 198. 22 dm |
| 201. 240 mm ² | 202. 84 dm |
| 205. 38 cm | 206. 65 cm |
| 209. 1536 mm ² | 210. 28.8 cm |
| | 183. 38 |
| | 184. 59 |
| | 187. $\frac{21}{22}$ |
| | 188. $-\frac{1}{3}, \frac{15}{8}$ |
| | 191. 27 |
| | 192. 35 |
| | 195. 45 |
| | 196. 83 |
| | 199. 62 dm |
| | 200. 136 cm |
| | 203. 60 cm ² |
| | 204. 588 mm ² |
| | 207. 69 cm |
| | 208. 52 cm |
| | 211. 111 476 dm ³ |
| | 212. 450 mm |

6.9 Vieta's formulas

213. $x_1 + x_2 = -\frac{b}{a}$
 $x_1 \cdot x_2 = \frac{c}{a}$
 $a(x - x_1)(x - x_2) = ax^2 + bx + c$
214. -
215. a) sum: -1
product: -15.75
b) sum: -20
product: 75
c) no solutions
216. a) no solutions
b) sum: $-\frac{7}{3}$
product: -2
c) sum: 11
product: -12
217. a) $-0.8(x - 1)(x + 4) = 0$
b) $\frac{2}{3}(x - 3)^2 = 0$
c) impossible
d) $6(x - \frac{1}{6})(x + \frac{3}{4}) = 0$
218. a) impossible
b) $\frac{1}{5}(x + 6)(x - 7) = 0$
c) $8(x + \frac{3}{4})(x - \frac{1}{8}) = 0$
d) $\frac{1}{3}(x - \frac{12}{5})(x + 2) = 0$
219. a) $4x^2 + 31x - 8 = 0$
b) $3x^2 + 36x - 39 = 0$
c) $\frac{1}{5}x^2 - \frac{4}{5}x - 12 = 0$
d) $\frac{3}{2}x^2 + 15x - 300 = 0$
220. a) $-4x^2 + 60x + 216 = 0$
b) $\frac{1}{3}x^2 - \frac{11}{3}x - 4 = 0$
c) $\frac{4}{5}x^2 + \frac{32}{5}x + 12 = 0$
d) $-x^2 - 2x + 55.25 = 0$

221. a) $\square : -\frac{3}{4}$
 2. solution: 12
 equation: $-\frac{3}{4}(x+4)(x-12) = 0$

b) $\square : \frac{2}{5}$
 2. solution: -1
 equation: $\frac{2}{5}(x-2)(x+1) = 0$

c) $\square : 2.7$
 2. solution: -4
 equation: $-0.3(x+4)(x+5) = 0$

d) $\square : 2$
 2. solution: -1.2
 equation: $2(x+1.2)(x-2) = 0$

222. a) $\square : 18$
 2. solution: -1.5
 equation: $3(x+1.5)(x-4) = 0$

b) $\square : 0.6$
 2. solution: -2
 equation: $0.6(x+2)(x-7) = 0$

c) $\square : 220$
 2. solution: -10
 equation: $-2(x-11)(x+10) = 0$

d) $\square : 9$
 2. solution: -2
 equation: $5\left(x - \frac{1}{5}\right)(x+2) = 0$

223. Sum of the solutions: $x_1 + x_2 + x_3 = -\frac{b}{a}$
 Sum of the pairwise products of the solutions: $x_1x_2 + x_1x_3 + x_2x_3 = \frac{c}{a}$
 Product of the solutions: $x_1x_2x_3 = -\frac{d}{a}$

224. Sum of the solutions: $x_1 + x_2 + x_3 + x_4 = -\frac{b}{a}$
 Sum of the pairwise products of the solutions: $x_1x_2 + x_1x_3 + x_1x_4 + x_2x_3 + x_2x_4 + x_3x_4 = \frac{c}{a}$
 Sum of the products of three solutions each: $x_1x_2x_3 + x_1x_2x_4 + x_1x_3x_4 + x_2x_3x_4 = -\frac{d}{a}$
 Product of the solutions: $x_1x_2x_3x_4 = \frac{e}{a}$

7 Quadratic Functions

7.1 Quadratic Functions

1.

a)

b)

yes

no

c)

d)

yes

no

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2.

a)

b)

yes

no

c)

d)

yes

no

☐

☒

☒

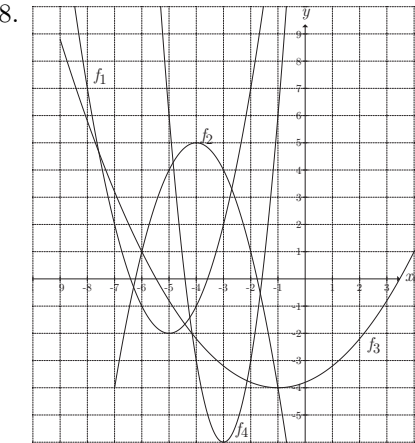
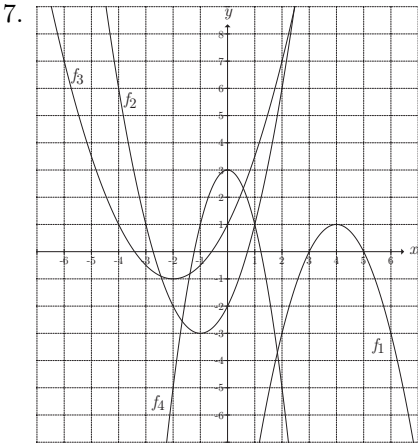
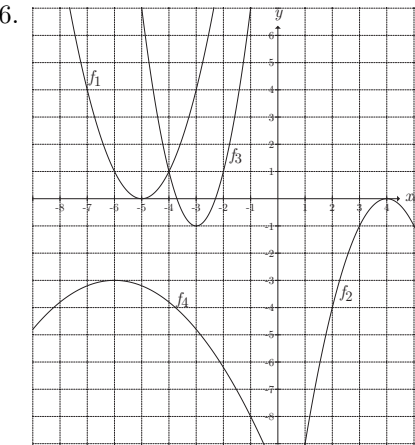
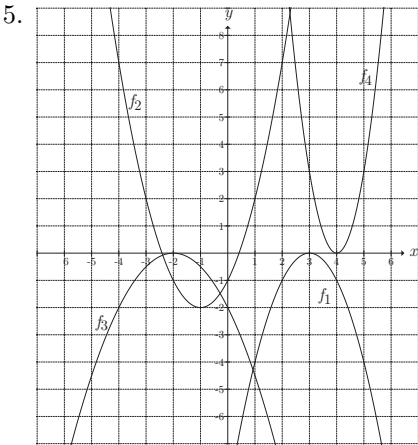
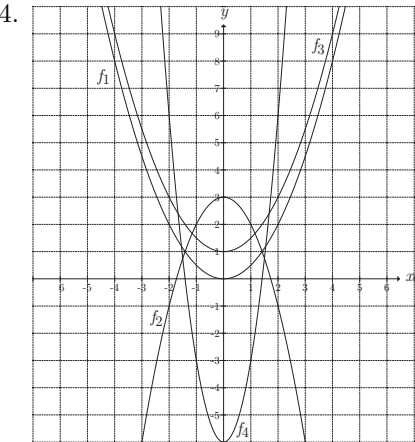
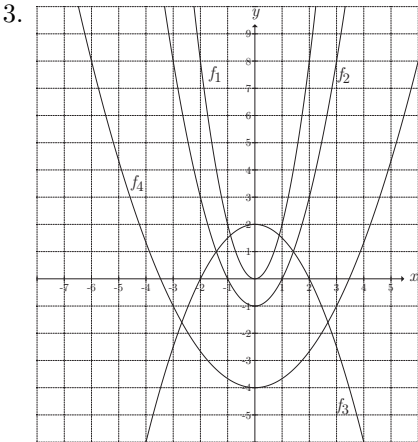
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9. ①-B, ②-D, ③-A, ④-C

10. ①-D, ②-A, ③-C, ④-B

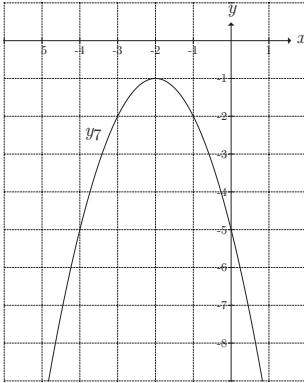
11. $f_1(x) = (x+1)^2,$

$f_2(x) = -3(x-4)^2 + 6,$

$f_3(x) = -\frac{1}{2}(x+2)^2 + 4,$

$f_4(x) = 2(x-3)^2 - 4$

13. $a: x \mapsto y_5, b: x \mapsto y_3, c: x \mapsto y_1,$
 $d: x \mapsto y_4, e: x \mapsto y_8, f: x \mapsto y_2,$
 $g: x \mapsto y_6$



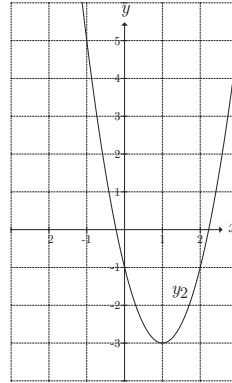
12. $f_1(x) = -(x-2)^2,$

$f_2(x) = \frac{1}{2}(x+3)^2 - 5,$

$f_3(x) = 3(x+1)^2 - 7,$

$f_4(x) = -2x^2 - 1$

14. $a: x \mapsto y_4, b: x \mapsto y_1, c: x \mapsto y_3,$
 $d: x \mapsto y_5, e: x \mapsto y_7, f: x \mapsto y_8,$
 $g: x \mapsto y_6$



15. a)

true	false
☒	☐

 c)

true	false
☐	☒

b)

☒	☐
-------------------------------------	--------------------------

 d)

☒	☐
-------------------------------------	--------------------------

17. a)

true	false
☒	☐

 c)

true	false
☐	☒

b)

☒	☐
-------------------------------------	--------------------------

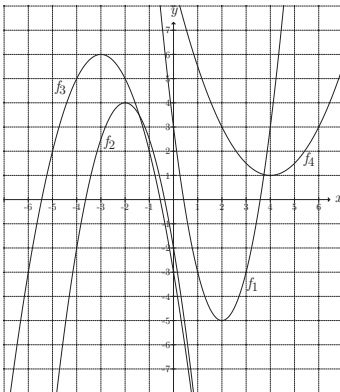
 d)

☒	☐
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19. a) $y = 4x^2 - 16x + 19$
b) $y = -2x^2 + 20x - 51$
c) $y = \frac{1}{4}x^2 + 4x$
d) $y = -\frac{3}{2}x^2 - 6x - 10$

21. $\mathcal{B}, \mathcal{D}, \mathcal{C}, \mathcal{A}$ 22. $\mathcal{D}, \mathcal{C}, \mathcal{A}, \mathcal{B}$

25. a) $(5, 3)$ c) $(-6, -7)$
b) $(-2, 4)$ d) $(8, 2)$

27. $S_1(2, -5), S_2(-2, 4), S_3(-3, 6), S_4(4, 1)$ 

16. a)

true	false
☒	☐

 c)

true	false
☐	☒

b)

☐	☒
--------------------------	-------------------------------------

 d)

☐	☒
--------------------------	-------------------------------------

18. a)

true	false
☐	☒

 c)

true	false
☐	☒

b)

☐	☒
--------------------------	-------------------------------------

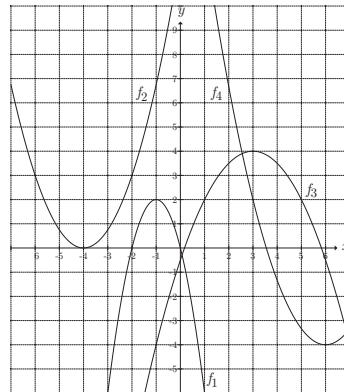
 d)

☒	☐
-------------------------------------	--------------------------

20. a) $y = 5x^2 + 40x + 68$
b) $y = -\frac{2}{5}x^2 + 8x - 25$
c) $y = \frac{2}{3}x^2 - 8x + 26$
d) $y = -3x^2 + 6x + 2$

23. $\mathcal{D}, \mathcal{B}, \mathcal{E}, \mathcal{A}, \mathcal{C}$ 24. $\mathcal{E}, \mathcal{A}, \mathcal{D}, \mathcal{B}, \mathcal{C}$

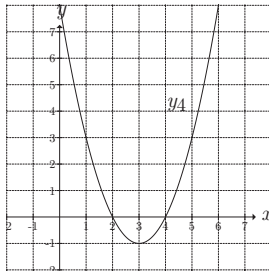
26. a) $(-11, -9)$ c) $(-1, -2)$
b) $(\frac{1}{3}, 4)$ d) $(-10, -5)$

28. $S_1(-1, 2), S_2(-4, 0), S_3(3, 4), S_4(6, -4)$ 

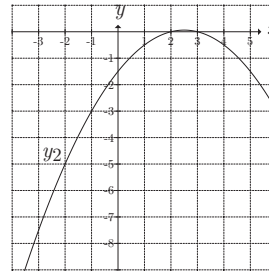
29. a) $f(x) = 3x^2 - 6x + 5$
 $= 3(x^2 - 2x) + 5$
 $= 3(x^2 - 2x + 1) + 5 - 3$
 $= 3(x - 1)^2 + 2$
 $S(1, 2)$
 b) $f(x) = -\frac{2}{3}x^2 - 8x - 23$
 $= -\frac{2}{3}(x^2 + 12x) - 23$
 $= -\frac{2}{3}(x^2 + 12x + 36) + 24 - 23$
 $= -\frac{2}{3}(x + 6)^2 + 1$
 $S(-6, 1)$
30. a) $f(x) = -\frac{3}{4}x^2 - 6x - 15$
 $= -\frac{3}{4}(x^2 + 8x) - 15$
 $= -\frac{3}{4}(x^2 + 8x + 16) - 15 + 12$
 $= -\frac{3}{4}(x + 4)^2 - 3$
 $S(-4, -3)$
 b) $f(x) = 4x^2 - 24x + 31$
 $= 4(x^2 - 6x) + 31$
 $= 4(x^2 - 6x + 9) + 31 - 36$
 $= 4(x - 3)^2 - 5$
 $S(3, -5)$
31. $S\left(-\frac{p}{2}, \frac{4q-p^2}{4}\right)$
32. $S\left(-\frac{b}{2a}, \frac{4ac-b^2}{4a}\right)$
33. a) (3, 1)
 b) (3, -10)
 c) (3, -18)
 d) (3, $c - 18$), x -values all equal
34. a) (-2, -3)
 b) (-2, 6)
 c) (-2, 2)
 d) (-2, $c + 2$), x -values all equal
35. a) (5, -24)
 b) (5, -11.5)
 c) (5, 51)
 d) (5, $1 - 25a$), x -values all equal
36. a) (-1, -5)
 b) (-1, -7)
 c) (-1, -3.5)
 d) (-1, $-4 - a$), x -values all equal
37. a) $c > 48$ b) $c > -54$
38. a) $c > -12.5$ b) $c > 40$
39. a) 8 b) 1.5
40. a) 4 b) -2.5
41. a) (2, -4) c) (0, 4)
 b) (-1, 2) d) (1, -4)
42. a) (-5, -6) c) (-2, -3)
 b) (4, 2) d) (5, 1)
43. a) $P \in f, Q \notin f, R \notin f$
 b) $P \notin f, Q \notin f, R \in f$
44. a) $P \notin f, Q \in f, R \in f$
 b) $P \in f, Q \notin f, R \in f$
45. a) $P(-5, -3.5), Q_1(2, 28), Q_2(-14, 28), R_1(-2, 4), R_2(-10, 4)$
 b) $P(-5, 0.75), Q_1(-2, 3), Q_2(-6, 3), R(-4, 0)$
46. a) $P(0, -7), Q_1(3, -34), Q_2(-9, -34), R_1(-1, -2), R_2(-5, -2)$
 b) $P(-6, -9), Q(-3, -3), R_1(1, -\frac{41}{3}), R_2(-7, -\frac{41}{3})$
47. a) -3 c) 14
 b) 10 d) 1.2
48. a) 16.5 c) 0.7
 b) 24 d) 5
49. a) $f(x) = 2(x - 3)^2 - 5$
 b) $f(x) = 5(x - 1)^2$
 c) $f(x) = \frac{1}{6}(x + 11)^2 - 4$
 d) $f(x) = \frac{2}{3}(x - \frac{11}{4})^2 + \frac{13}{4}$
50. a) $f(x) = -\frac{1}{2}(x + 8)^2 - 4$
 b) $f(x) = -4(x + 1.5)^2 + 2$
 c) $f(x) = \frac{6}{7}(x + 7)^2 - 11$
 d) $f(x) = -\frac{1}{4}x^2 + 3$

51. a) $f(x) = -3x^2 - 6x + 3, S(-1, 6)$
 b) $f(x) = 0.3x^2 + 4.2x + 11.7, S(-7, -3)$
 c) $f(x) = 1.5x^2 - 6x + 7, S(2, 1)$
 d) $f(x) = -0.5x^2 + 3x - 8.5, S(3, -4)$
52. a) $f(x) = 0.6x^2 + 3.6x + 5.4, S(-3, 0)$
 b) $f(x) = -2x^2 + 28x - 93, S(7, 5)$
 c) $f(x) = 2.5x^2 - 10x + 10, S(2, 0)$
 d) $f(x) = 0.1x^2 + x - 5.5, S(-5, -8)$
53. o s n o s n
 a) ☒ ☐ ☐ c) ☐ ☐ ☒
 b) ☐ ☒ ☐ d) ☐ ☒ ☐
54. o s n o s n
 a) ☒ ☐ ☐ c) ☐ ☒ ☐
 b) ☐ ☐ ☒ d) ☒ ☐ ☐
55. a) $3, -\frac{1}{2}$ c) $-\frac{13}{2}, -4$
 b) $\{\}$ d) $0, 2$
56. a) $2, 4$ c) $\{\}$
 b) $-4, 2.5$ d) $-\frac{1}{3}, 6$
57. a) $X_1(-5, 0), X_2(1, 0), Y(0, -20)$
 b) $X(-1, 0), Y(0, -\frac{2}{3})$
 c) $X_1(1, 0), X_2(2.5, 0), Y(0, \frac{1}{2})$
 d) $X_1(-4.5, 0), X_2(5, 0), Y(0, 67.5)$
58. a) $X_1(-3, 0), X_2(6, 0), Y(0, -7.2)$
 b) $X_1(-5, 0), X_2(7.5, 0), Y(0, 75)$
 c) $X_1(-10, 0), X_2(-1, 0), Y(0, -\frac{10}{3})$
 d) $X_1(-3, 0), X_2(\frac{2}{3}, 0), Y(0, 8)$
59. a) $y = \frac{1}{4}x^2 - 4$
 b) $y = -2x^2 - 11x + 6$
 c) $y = -3x^2 + 2x + 1$
 d) $y = 0.8x^2 + 1.6x - 38.4$
60. a) $y = 5x^2 + 15x - 20$
 b) $y = \frac{2}{3}x^2 + \frac{28}{3}x + 30$
 c) $y = -\frac{3}{4}x^2 - \frac{45}{4}x - \frac{81}{2}$
 d) $y = -2x^2 - 12x - 16$
61. a) $x_1 = -3, x_2 = -7, S(-5, -2)$
 b) $x_1 = 0, x_2 = 1, S(\frac{1}{2}, \frac{3}{4})$
 c) $x_1 = 2, x_2 = -4, S(-1, -\frac{27}{4})$
 d) $x_1 = -5, x_2 = 5, S(0, -150)$
62. a) $x_1 = -4, x_2 = 1, S(-1.5, -12.5)$
 b) $x_1 = -6, x_2 = -8, S(-7, 4)$
 c) $x_1 = -3, x_2 = -9, S(-6, -3)$
 d) $x_1 = 0, x_2 = -6, S(-3, \frac{9}{2})$
63. a) SF: $f(x) = -0.5x^2 + x + 4$
 VF: $f(x) = -0.5(x - 1)^2 + 4.5$
 b) SF: $f(x) = -3x^2 + 30x - 72$
 VF: $f(x) = -3(x - 5)^2 + 3$
 c) SF: $f(x) = \frac{1}{4}x^2 + \frac{5}{4}x$
 VF: $f(x) = \frac{1}{4}(x + \frac{5}{2})^2 - \frac{25}{16}$
 d) SF: $f(x) = 2x^2 - 162$
 VF: $f(x) = 2x^2 - 162$
64. a) SF: $f(x) = 2x^2 + 8x - 24$
 VF: $f(x) = 2(x + 2)^2 - 32$
 b) SF: $f(x) = 0.8x^2 - 0.8$
 VF: $f(x) = 0.8x^2 - 0.8$
 c) SF: $f(x) = \frac{2}{3}x^2 + 6x + 12$
 VF: $f(x) = \frac{2}{3}(x + 4.5)^2 - 1.5$
 d) SF: $f(x) = \frac{3}{2}x^2 + 18x + 48$
 VF: $f(x) = \frac{3}{2}(x + 6)^2 - 6$
65. a) $f(x) = \frac{1}{2}(x + 3)(x + 5)$
 b) not possible
 c) $f(x) = \frac{2}{3}(x + 5)(x + 9)$
 d) $f(x) = 2(x - 4)(x + 4)$
66. a) $f(x) = 0.6(x + 6)(x + 1)$
 b) $f(x) = -4(x + 7)(x - 7)$
 c) $f(x) = 2(x + 2.5)(x + 5)$
 d) $f(x) = -(x - \frac{2}{3})(x + \frac{3}{2})$
67. a) $f(x) = 0.4(x - 2)(x + 6)$
 b) $f(x) = -(x + 4)(x - 5)$
 c) $f(x) = -\frac{1}{2}(x - 7)(x + 3)$
 d) $f(x) = 3(x - 1)(x + 5)$
68. a) $f(x) = \frac{3}{2}(x + 4)(x - 1)$
 b) not possible
 c) $f(x) = \frac{1}{2}(x - 6.5)(x - 2.5)$
 d) $f(x) = 2(x - 4)(x + 10)$

69. $a: x \mapsto y_3$, $b: x \mapsto y_1$, $c: x \mapsto y_6$,
 $d: x \mapsto y_5$, $e: x \mapsto y_2$



70. $a: x \mapsto y_3$, $b: x \mapsto y_1$, $c: x \mapsto y_6$,
 $d: x \mapsto y_4$, $e: x \mapsto y_5$



71. a) impossible
 b) possible
 c) possible

72. a) possible
 b) impossible
 c) impossible

73. a) $[-4, 2]$
 b) $] -\infty, -2[\cup] 3, \infty[$
 c) $[-4, 6.5]$
 d) $] -\infty, -8] \cup] 3, \infty[$

74. a) $] -\infty, -4] \cup] 2.5, \infty[$
 b) $\mathbb{R} \setminus \{1\}$
 c) $] -12, -3[$
 d) $[-1, 6]$

75. a) $] -3, 7[$
 b) $] -\infty, -2] \cup] 8, \infty[$
 c) $\{\}$
 d) $] -\infty, -9] \cup] 6, \infty[$

76. a) $] -\infty, -4[\cup] 3, \infty[$
 b) $[-6, 18]$
 c) $\{\}$
 d) $] -\infty, -4] \cup] 7, \infty[$

- | | | | | | |
|--------|-------------------------------------|-------------------------------------|----|--------------------------|-------------------------------------|
| | true | false | | true | false |
| 77. a) | ☐ | ☒ | c) | ☐ | ☒ |
| b) | ☒ | ☐ | d) | ☐ | ☒ |

- | | | | | | |
|--------|--------------------------|-------------------------------------|----|-------------------------------------|-------------------------------------|
| | true | false | | true | false |
| 78. a) | ☐ | ☒ | c) | ☒ | ☐ |
| b) | ☐ | ☒ | d) | ☐ | ☒ |

- | | | | | | |
|--------|-------------------------------------|--------------------------|----|-------------------------------------|-------------------------------------|
| | true | false | | true | false |
| 79. a) | ☒ | ☐ | c) | ☒ | ☐ |
| b) | ☒ | ☐ | d) | ☐ | ☒ |

- | | | | | | |
|--------|-------------------------------------|--------------------------|----|-------------------------------------|-------------------------------------|
| | true | false | | true | false |
| 80. a) | ☒ | ☐ | c) | ☐ | ☒ |
| b) | ☒ | ☐ | d) | ☒ | ☐ |

81. $y = \frac{1}{2}(x-1)^2 + 9$

82. $y = -3(x+3)^2 - 1$

83. $y = 0.4x^2 + 1.6x + 3.6$

84. $y = -1.2x^2 - 9.6x - 11.2$

85. a) $y = 3(x+5)^2 + 2$

86. a) $y = 2(x+3)^2 - 4$

b) $y = -3(x-5)^2 - 2$

b) $y = -2(x-3)^2 + 4$

c) $y = -3(x+5)^2 - 2$

c) $y = 2(x-3)^2 - 4$

87. a) $y = 0.75x^2 - 9x + 23$

88. a) $y = 1.5x^2 - 12x + 16$

b) $y = -0.75x^2 - 9x - 23$

b) $y = -1.5x^2 - 12x - 16$

c) $y = -0.75x^2 + 9x - 23$

c) $y = 1.5x^2 + 12x + 16$

89. a) $y = -4(x+1)^2 + 2$

90. a) $y = \frac{1}{2}(x-3)^2 + 4$

b) $y = 4(x-11)^2 + 2$

b) $y = -\frac{1}{2}(x+15)^2 + 4$

c) $y = -4(x+1)^2 + 4$

c) $y = \frac{1}{2}(x-3)^2 - 8$

d) $y = -4(x-9)^2 + 6$

d) $y = \frac{1}{2}(x+1)^2 - 2$

91. $d = -5$
93. $d_1 = 5, d_2 = 9$
95. $y = (x + 1)^2 + 7$
97. $y = 3x^2 - 54x + 244$
99. $y = 6(x + 4)^2$
100. $y = -\frac{1}{6}(x - 1)^2$
101. $y = -\frac{2}{5}(x - 2)^2$
102. $y = 18x^2$
103. $d_1 = 3, d_2 = 8$
104. $d_1 = 3, d_2 = 6$
105. a) yes, $(-1, -1.5)$ c) not possible
b) yes, $(-2.5, 1.5)$ d) yes, $(-3, -3)$
106. a) yes, $(3, 1.5)$ c) yes, $(2.5, -4)$
b) yes, $(-1, 2)$ d) not possible
107. a) yes, $\begin{pmatrix} -6 \\ 1 \end{pmatrix}$ c) not possible
b) yes, $\begin{pmatrix} 8 \\ 9 \end{pmatrix}$ d) yes, $\begin{pmatrix} -3 \\ -1.5 \end{pmatrix}$
108. a) not possible c) yes, $\begin{pmatrix} 3 \\ 10 \end{pmatrix}$
b) yes, $\begin{pmatrix} -10 \\ -4 \end{pmatrix}$ d) yes, $\begin{pmatrix} 5 \\ -9 \end{pmatrix}$
109. a) $(-2, 40), (6, 8)$
b) $(-6, -11), (6, -35)$
c) $(-2, 0), (6, 3.2)$
d) $(6, 4)$
110. a) $(-7, -4), (-2, 1)$
b) $(-7, -3.25), (-2, 8)$
c) $(-5, 4), (3, 0)$
d) $\{\}$
111. a) $(-4, -2), (-1, 7)$
b) $(-7, -4), (-3, -4)$
c) $\{\}$
d) $(3, 4)$
112. a) $(-2, 5), (2, 5)$
b) $(3, 10)$
c) $(-2, 4), (-7, -1)$
d) $\{\}$
113. a) $y = 3$ c) $y = -24$
b) $y = 0$ d) $y = 12.5$
114. a) $y = -2$ c) $y = 2$
b) $y = 4$ d) $y = -3$
115. a) $m = 0.5, B(4, 6.5)$
b) $m = 11, B(-2, 3)$
c) $m_1 = 4, B_1(1, 6),$
 $m_2 = 2, B_2(-1, 0)$
d) $m_1 = 10, B_1(-1, -21),$
 $m_2 = 6, B_2(1, -5)$
116. a) $m = -0.5, B(1, -4)$
b) $m = 30, B(-9, -6)$
c) $m_1 = 18, B_1(2, 27),$
 $m_2 = -6, B_2(-2, 3)$
d) $m_1 = 26, B_1(5, 163)$
 $m_2 = 6, B_2(-5, 3)$
117. a) $y = 8x + 15$ b) $y = 6x - 19$
118. a) $y = 4x - 21$ b) $y = -3x - 1$
119. a) $c \in]-\infty, 7[$ c) $m \in]-2, 2[$
b) $c \in]12, \infty[$ d) $m \in]0, 3[$
120. a) $c \in]-\infty, -3[$ c) $m \in]0, 1.5[$
b) $c \in]-\infty, -4[$ d) $m \in]\frac{2}{3}, 2[$
121. true false true false
a) ☒ ☐ c) ☐ ☒
b) ☐ ☒ d) ☐ ☒
122. true false true false
a) ☒ ☐ c) ☒ ☐
b) ☒ ☐ d) ☒ ☐
123. true false true false
a) ☒ ☐ c) ☐ ☒
b) ☒ ☐ d) ☐ ☒
124. true false true false
a) ☐ ☒ c) ☒ ☐
b) ☒ ☐ d) ☒ ☐

7.2 The Parabola as a Geometric Locus

125. -

126. -

127. $y = \frac{1}{4p}x^2$

128. $F(u, \frac{1}{4a} + v), \ell : y = -\frac{1}{4a} + v$ or $F(-\frac{b}{2a}, c + \frac{1-b^2}{4a}), \ell : y = c - \frac{1+b^2}{4a}$

129. a) $F(0, \frac{1}{6}), \ell : y = -\frac{1}{6}$

130. a) $F(0, \frac{1}{2}), \ell : y = -\frac{1}{2}$

b) $F(0, 5.25), \ell : y = 4.75$

b) $F(0, 1.75), \ell : y = 2.25$

c) $F(-3, 0), \ell : y = -2$

c) $F(5, \frac{7}{2}), \ell : y = \frac{9}{2}$

d) $F(1, \frac{17}{8}), \ell : y = \frac{15}{8}$

d) $F(4, \frac{11}{4}), \ell : y = \frac{13}{4}$

131. a) $y = -\frac{1}{12}(x-4)^2 + 3$

132. a) $y = -\frac{1}{8}(x-3)^2 + 8$

b) $y = \frac{1}{8}(x+2)^2 + 2$

b) $y = \frac{1}{12}(x+2)^2$

133. a) $y = \frac{1}{8}x^2 - \frac{5}{2}x + \frac{41}{2}$

134. a) $y = -\frac{1}{8}x^2 - x + 4$

b) $y = \frac{1}{8}x^2 - \frac{3}{4}x + \frac{57}{8}$

b) $y = 0.25x^2 - 2.5x + 12.25$

135. $y = \frac{1}{4}x^2 - 2x + 5$

136. $y = \frac{1}{4}x^2 - 5x + 32$

137. a) $F(0, 2.5), \ell : y = -2.5$

b) The reflected rays all pass through F .

c) They are all parallel to the axis of symmetry of the parabola and thus also parallel to the original incoming rays.

138. a) $y = x - 2.5$

b) $y = -x - 2.5, (-5, 2.5)$

c) $(0, -2.5)$

d) Tangent: $y = -1.2x - 3.6$,

Perpendicular tangent with point of contact: $y = \frac{5}{6}x - \frac{125}{72}, (\frac{25}{6}, \frac{125}{72})$,

Intersection of the tangents: $(-\frac{11}{12}, -2.5)$

e) They are all on the directrix.

139. $(0, 2)$

140. $(0, 1)$

141. $(0, 2.25)$

142. $(0, 1.5)$

143. y_1 : intersection point $(0, 0), (2, 1)$, midpoint $(1, 0.5)$

y_2 : intersection point $(-2, 1), (4, 4)$, midpoint $(1, 2.5)$

y_3 : intersection point $(-4, 4), (6, 9)$, midpoint $(1, 6.5)$

y_4 : intersection point $(-6, 9), (8, 16)$, midpoint $(1, 12.5)$

y : intersection points $(1 - \sqrt{4c+1}, \frac{1}{2}(1 - \sqrt{4c+1}) + c), (1 + \sqrt{4c+1}, \frac{1}{2}(1 + \sqrt{4c+1}) + c)$, midpoint $(1, c + \frac{1}{2})$

144. $P(p, p^2), Q(q, q^2)$, line equation $y = mx + c$ through P and Q yields a 2×2 -system of equations:

$$\begin{aligned} p^2 &= mp + c \\ q^2 &= mq + c \end{aligned} \Rightarrow p^2 - q^2 = m(p - q) \Rightarrow m = p + q, p^2 = p^2 + pq + c \Rightarrow c = -pq$$

7.3 Applications

145. a) $x_{\min} = 3, y_{\min} = 4$
 $x_{\max} = 1 \wedge 5, y_{\max} = 6$
 b) $x_{\min} = 2, y_{\min} = 4.5$
 $x_{\max} = -1, y_{\max} = 12$
 c) $x_{\min} = 3, y_{\min} = 4$
 $x_{\max}$ does not exist
 d) $x_{\min} = 3, y_{\min} = 4$
 $x_{\max}$ does not exist
147. a) Minimum position: $x = 2$,
 Minimum: $y = -100$
 b) Maximum position: $x = -1.5$,
 Maximum: $y = \frac{75}{16}$
 c) Minimum position: $x = -5$,
 Minimum: $y = -\frac{1}{2}$
 d) Maximum position: $x = -4$,
 Maximum: $y = 2$
149. a) Minimum position: $x = 1$,
 Minimum: $y = \frac{235}{64}$
 b) Maximum position: $x = 1$,
 Maximum: $y = 10$
 c) Minimum position: $x = 3$,
 Minimum: $y = -\frac{8}{3}$
 d) Maximum position: $x = 4$,
 Maximum: $y = 3.8$
151. ①-C, ②-E, ③-D, ④-A, ⑤-B
153. a) 1.65 m c) 8.75 m
 b) 19.6 m d) 4.7 m
155. $f(x) = -0.06x^2 + 0.8x + 1.6$
157. a) 1.8 m
 b) 42.09 m
 c) nein
159. a) 4 m
 b) $0.\bar{1}$ m
 c) 16 m
 d) 8 m
161. $f(x) = -\frac{1}{24}x^2 + x$
146. a) $x_{\min} = 0, y_{\min} = -7$
 $x_{\max} = 2, y_{\max} = 1$
 b) $x_{\min} = -3, y_{\min} = -49$
 $x_{\max} = 2, y_{\max} = 1$
 c) $x_{\min}$ does not exist
 $x_{\max} = 2, y_{\max} = 1$
 d) $x_{\min}$ does not exist
 $x_{\max} = 2, y_{\max} = 1$
148. a) Maximum position: $x = 1$,
 Maximum: $y = 50$
 b) Minimum position: $x = -2.5$,
 Minimum: $y = -1.8$
 c) Minimum position: $x = -6$,
 Minimum: $y = -4$
 d) Maximum position: $x = -1.5$,
 Maximum: $y = 5$
150. a) Minimum position: $x = -2$,
 Minimum: $y = -\frac{125}{64}$
 b) Maximum position: $x = -1$,
 Maximum: $y = 4.23$
 c) Minimum position: $x = -2$,
 Minimum: $y = -3.92$
 d) Maximum position: $x = 1$,
 Maximum: $y = \frac{21}{8}$
152. ①-D, ②-C, ③-E, ④-B, ⑤-A
154. a) 1.75 m c) 7.1 m
 b) 15.9 m d) 5.3 m
156. $f(x) = -0.05x^2 + 0.9x + 1.8$
158. a) $f(x) = -0.0472(x - 25)^2 + 31$
 b) 49.7 m
160. a) 20 m
 b) 5 m
 c) 2.45 m
 d) 8 m or 12 m
162. $f(x) = -\frac{2}{225}x^2$

163. a) $y = 0.0006x^2$
 b) 47.04 m
 c) 120 m
 d) 90 m
165. a) 2.2 m
 b) yes
167. a) Fr. 19.20 to Fr. 64.80,
 maximum at Fr. 42.—
 b) As prices rise, demand falls.
169. a) $f(x) = -\frac{1}{6}(x-3)^2 + 1.5$
 b) yes
164. $f(x) = \frac{1}{375}(x-75)^2$
166. a) $y = -0.8x^2$
 b) < 2.952 m
 c) yes, very tight
168. a) $f(x) = -6(x-220)^2 + 117\,600$,
 360 units
 b) 170 units
170. a) $f(x) = -\frac{1}{12}(x-6)^2 + 3$
 b) $2.\bar{6}$ m

7.4 Optimization Problems

171. numbers 2 and -2 , product -4
173. numbers -3 and 3 , half product -4.5
175. $(\frac{1}{8}, -\frac{1}{2})$, product $-\frac{1}{16}$
177. $(-1, 1)$, minimum 2
179. $(\frac{3}{8}, \frac{1}{8})$, maximum $\frac{1}{8}$
181. Side lengths: 5.25 cm (square)
 area: 27.5625 cm^2
183. Length 30 cm, width 25 cm, height 15 cm,
 surface 3150 cm^2
185. $\frac{100}{\pi} \text{ m} \approx 31.83 \text{ m}$
187. perpendicular to the house wall 6.25 m, parallel to the house wall 12.5 m
189. adjacent to the bicycle shelter 2.5 m, parallel to the garage wall 5 m
191. $a = 3.75 \text{ cm}$, $b = 11.25 \text{ cm}$
193. 8.75 cm
195. 24 cm high and 32 cm long
197. 83.4 cm long and 69.5 cm wide
199. (1.25, 2.5)
201. $-\frac{4}{3}$
203. $C(3.5, 2.25)$, area 10.125
205. $C(1, 8)$, $u = 20$
207. $u = 2$, $\overline{PQ} = 4$
172. numbers -3.5 and 3.5 , product -12.25
174. -4.5 and 4.5 , product -6.75
176. (1, 2), product 2
178. (2, -2), minimum 8
180. $(\frac{4}{3}, \frac{8}{3})$, minimum $-\frac{16}{3}$
182. Side lengths 16.5 cm (square)
 area: 272.25 cm^2
184. base edge 15 cm, height 30 cm,
 surface 2700 cm^2
186. $\frac{200}{\pi} \text{ m} \approx 63.66 \text{ m}$
188. perpendicular to the garage wall 10.5 m,
 parallel to the garage wall 21 m
190. 6 m each
192. $x = 7 \text{ cm}$, $A = 196 \text{ cm}^2$
194. 18.25 cm
196. 6 cm long and 7.5 cm high
198. 75 cm each
200. (3, 1.5)
202. 1
204. $C(24, 5.2)$, area 135.2
206. $C(2, 2)$, $u = 12$
208. $u = \frac{1}{2}$, $\overline{PQ} = \frac{47}{12}$

8 Powers

8.1 Powers With Natural Exponents

1. $-6^4 < -3^5 < 0^{11} < 11^0 < (-6)^2 < 10^2 < 2^{10} < 4^6$
2. $-3^5 < (-5)^3 < -7^2 < 0^5 < 3^0 < 2^7 < (-5)^4 < (-3)^6$
3. a) 81 d) $-\frac{1}{64}$
- b) -64 e) 0
- c) 343 f) $-\frac{8}{125}$
4. a) $\frac{9}{49}$ d) 1
- b) -32 e) -256
- c) $\frac{1}{81}$ f) -512
5. a) -1 000 000 d) 4096
- b) 256 e) 0.0081
- c) 1024 f) -216
6. a) 27 d) 625
- b) 1 000 000 e) -27
- c) 81 f) -0.343
7. a) 2^{81} d) $10^{10\,000}$
- b) 5^{1024} e) 2^{192}
- c) 2^4 f) 2^{16}
8. a) 3^{16} d) 5^{512}
- b) 3^{25} e) 6^{64}
- c) 3^{27} f) 2^{512}
9. a) $<$ c) $=$
- b) $>$ d) $=$
10. a) $<$ c) $<$
- b) $>$ d) $<$
11. a) 2^6 d) 2^8
- b) 3^4 e) 6^3
- c) 7^3 f) 13^2
12. a) 3^6 d) 5^3
- b) 5^5 e) 2^{11}
- c) 3^5 f) 5^4
13. a) 2^{10} c) 2^{26}
- b) 3^{14} d) 6^{12}
14. a) 21^2 c) 5^{16}
- b) 2^7 d) 6^{13}
15. a) 382 d) 4096
- b) -49 e) 243
- c) 0 f) 729
16. a) -16 d) 64
- b) 729 e) -616
- c) 256 f) -32
17. a) $-\frac{32}{243}$ d) $\frac{625}{81}$
- b) 97 e) 0.0256
- c) 25 f) 0
18. a) $\frac{81}{16}$ d) $\frac{3125}{32}$
- b) -431 e) 45
- c) 27 f) -1.44
19. ①- $\mathcal{B}$, ②- $\mathcal{F}$, ③- $\mathcal{G}$, ④- $\mathcal{E}$, ⑤- $\mathcal{D}$, ⑥- $\mathcal{A}$, ⑦- $\mathcal{C}$
20. ①- $\mathcal{E}$, ②- $\mathcal{C}$, ③- $\mathcal{D}$, ④- $\mathcal{F}$, ⑤- $\mathcal{B}$, ⑥- $\mathcal{G}$, ⑦- $\mathcal{A}$
21. a) a^{15} c) not possible
- b) x^{11} d) 6^m
22. a) not possible c) b^{3n}
- b) 5^n d) 5^n
23. a) 0 c) a^4b^4
- b) 1 d) $2a^2b^2$
24. a) 0 c) $-p^6q^6$
- b) -1 d) $-2p^3q^3$
25. a) 7^{4n-16} e) 16
- b) $18a^7$ f) $169a^4b^2c^8$
- c) 37^{2+n} g) $-27a^6b^3c^3$
- d) $16a^4$ h) $9a^3$
26. a) $8a^5$ e) $3x^3$
- b) 5^{3n-9} f) $18a^6$
- c) 13^{8n+6} g) $-8a^6b^9c^{12}$
- d) $125a^6b^3c^3$ h) $49a^2b^2c^4$

27. a) $a^2 - 2a + 1$ c) $a^2 + 10a + 25$ 28. a) -1 c) $a^2 - 12a + 36$
 b) 1 d) $4a^2 + 3$ b) $a + 6$ d) $-4a^2$
29. $27x^9 - 1$ 30. $\frac{12x^5 - 4x^2 - 1}{64}$

8.2 Powers With Integer Exponents

31. a) $\frac{1}{2025}$ d) $\frac{1}{256}$ 32. a) $\frac{1}{625}$ d) $-\frac{1}{216}$
 b) $\frac{1}{49}$ e) $-\frac{1}{1024}$ b) $\frac{1}{256}$ e) not defined
 c) $-\frac{1}{128}$ f) $-\frac{1}{64}$ c) $\frac{1}{27}$ f) $\frac{1}{10000}$
33. a) $\frac{1}{64}$ d) $\frac{1}{10000}$ 34. a) $-\frac{1}{25}$ d) $\frac{1}{81}$
 b) $\frac{1}{729}$ e) 81 b) 512 e) $-\frac{1}{361}$
 c) -256 f) $-\frac{1}{324}$ c) $\frac{1}{36}$ f) 2025
35. a) $\frac{10000}{81}$ d) $\frac{16}{25}$ 36. a) $-\frac{8}{27}$ d) $\frac{125}{729}$
 b) $\frac{64}{27}$ e) $\frac{9}{25}$ b) $\frac{81}{16}$ e) $\frac{125}{27}$
 c) $-\frac{8000}{729}$ f) $-\frac{343}{8}$ c) 25 f) $-\frac{32}{243}$
37. a) 3^{-2} e) 6^{-3} 38. a) 10^{-3} e) 3^{-6}
 b) 2^{-3} f) 2^{-6} b) 2^{-11} f) 2^{-7}
 c) 5^{-3} g) 2^{-1} c) 3^{-5} g) 7^{-2}
 d) 3^{-3} h) 2025^{-1} d) 50^{-1} h) 5^{-4}

39.

3^7	3^2	3^6	3^{-5}
3^{-4}	3^5	3^1	3^8
3^{-3}	3^4	3^0	3^9
3^{10}	3^{-1}	3^3	3^{-2}

40.

5^2	5^{-3}	5^1	5^{-10}
5^{-9}	5^0	5^{-4}	5^3
5^{-8}	5^{-1}	5^{-5}	5^4
5^5	5^{-6}	5^{-2}	5^{-7}

41. a) $\frac{1}{32}$ d) $\frac{1}{9}$ 42. a) $\frac{1}{125}$ d) $\frac{1}{625}$
 b) 27 e) $\frac{1}{144}$ b) $-\frac{1}{343}$ e) $\frac{1}{64}$
 c) $\frac{1}{121}$ f) $\frac{-1}{256}$ c) 729 f) 4
43. a) $\frac{1}{4096}$ d) 1 44. a) $\frac{1}{4}$ d) $\frac{16}{81}$
 b) 0 e) $\frac{1}{36}$ b) $\frac{1}{576}$ e) $\frac{1}{7}$
 c) -9 f) 81 c) $-\frac{1}{4096}$ f) $\frac{1}{169}$
45. a) 64 d) $\frac{1}{10000}$ 46. a) 625 d) -1
 b) $\frac{1}{343}$ e) 8 b) -128 e) 1
 c) $\frac{1}{100}$ f) $-\frac{1}{9}$ c) $\frac{1}{9}$ f) 3
47. a) 1 d) $\frac{1}{243}$ 48. a) $\frac{5}{2}$ d) $\frac{343}{27}$
 b) 27 e) $\frac{9}{16}$ b) $\frac{9}{16}$ e) $-\frac{9}{4}$
 c) $\frac{3}{4}$ f) $-\frac{81}{256}$ c) $\frac{125}{512}$ f) $\frac{16}{81}$
49. a) $\frac{1}{4}$ c) $\frac{1}{32}$ 50. a) $\frac{27}{64}$ c) 256
 b) $\frac{27}{64}$ d) 2 b) $\frac{49}{4}$ d) 1

51. ①- $\mathcal{E}$, ②- $\mathcal{B}$, ③- $\mathcal{A}$, ④- $\mathcal{C}$, ⑤- $\mathcal{D}$

53. a) false c) false
b) true d) true

55. a) -8 56. a) 1
b) 7 b) -9
c) -1 c) -1

59. a) -3 60. a) $\frac{5}{2}$
b) -1 b) $-\frac{1}{8}$
c) $\mathbb{Z}$ c) $\{\}$

61. a) a^4 d) $\frac{1}{3^{4x}}$
b) $\frac{1}{b^2}$ e) 16
c) $\frac{1}{b}$ f) a^3

63. a) d^{3d-3} c) $\frac{9}{4}$
b) 2^{-88x} d) 0

65. a) a^{-24} c) $\frac{8}{9c^{11}}$
b) $-b^{-3}$ d) d^{k^2-2k}

67. a) 1 c) $\frac{1}{a^6} - \frac{1}{a^8}$
b) $\frac{21}{x^2} - 2$ d) $\frac{1}{27}$

69. a) -8 c) -6
b) -2 d) 5

71. a) $\frac{a^2}{9b^2}$ d) $\frac{2x^2y^3}{15}$
b) $\frac{c^6}{14a^2b^8}$ e) $\frac{b^2}{9a^4}$
c) $\frac{d^8}{a^4b^2c^2}$ f) $\frac{125y^{15}}{x^9z^3}$

73. a) $4 + 4x^{-2} + x^{-4}$
b) $4x^{-4} - 4x^{-6} + x^{-8}$
c) $x^2 + 2 + x^{-2}$
d) $25 - 10x^{-5} + x^{-10}$
e) $4x^{-6} - 12x^{-3}y^2 + 9y^4$
f) $1 - 2x^{-2} + x^{-4}$

52. ①- $\mathcal{E}$, ②- $\mathcal{C}$, ③- $\mathcal{A}$, ④- $\mathcal{D}$, ⑤- $\mathcal{B}$

54. a) true c) false
b) false d) false

57. a) 2^{3a-6} 58. a) 11^{-5a+5}
b) 6^{17a-5} b) 4^{-5a+5}
c) 4^{-a-6} c) 7^{-10a}

62. a) $\frac{1}{a^8}$ d) $\frac{1}{b^6}$
b) $\frac{36}{x^4}$ e) b^4
c) $\frac{1}{12^{2x}}$ f) a^{20}

64. a) m^{8m-5} c) $1 - \frac{1}{a^2}$
b) 5^{-4x} d) 0

66. a) r^{60} c) $\frac{1}{9t^{14}}$
b) $17s^{-4}$ d) u^{4x^2-2x}

68. a) $\frac{9}{x^2} + 3x^8$ c) $\frac{1}{36}$
b) -1 d) $-\frac{1}{16}$

70. a) $3, -1$ c) $-2, -5$
b) 9 d) -6

72. a) $\frac{64a^6}{c^3}$ d) $\frac{4x^{10}}{b^4}$
b) $\frac{26a^7c}{b}$ e) $\frac{b^9}{216a^9}$
c) $\frac{25}{144x^8z^4}$ f) $\frac{1}{a^3b^3c^6d^6}$

74. a) $x^{-6} - 2x^{-3}y^{-6} + y^{-12}$
b) $9x^{-2} + 42x^{-1}y^{-5} + 49y^{-10}$
c) $9 + 6x^{-3} + x^{-6}$
d) $x^2 - 2xy^{-4} + y^{-8}$
e) $x^{-4} - 2x^{-2}y^{-2} + y^{-4}$
f) $x^{-2} + y^{-2} + 2x^{-1}y^{-1}$

8.3 Roots

		a)	b)
	radicand	4096	343
75.	degree	12	3
	term	$\sqrt[12]{1024}$	$\sqrt[3]{343}$
	result	2	7

		a)	b)
76.	radicand	625	729
	degree	4	6
	term	$\sqrt[4]{625}$	$\sqrt[6]{729}$
	result	5	3

77. a) 2
b) 1
c) 4
79. a) 5
b) -2
c) 3
81. a) $\frac{3}{5}$
b) $\frac{3}{4}$
c) $\frac{1}{20}$
83. a) 16 and 17
b) 4 and 5
c) 5 and 6
85. a) 3
b) 9
c) 20
87. a) $\sqrt[3]{91}$
b) true
c) not defined
89. a) $3\sqrt[3]{5} + 4\sqrt{2}$
b) $22\sqrt[3]{4}$
91. $576\sqrt[3]{49}$
- d) not defined
e) -5
f) 100
- d) 23
e) 100
f) 2
- d) $\frac{1}{5}$
e) $\frac{5}{6}$
f) $\frac{1}{100}$
- d) 3 and 4
e) 2 and 3
f) 4 and 5
- d) $\frac{5}{3}$
e) 2
f) 4
58. a) $\sqrt[3]{91}$
b) true
c) not defined
90. a) $4\sqrt[3]{6} + 2\sqrt[5]{25}$
b) $32\sqrt[3]{2}$
92. $16\sqrt{13}$
78. a) 4
b) 0
c) 3
80. a) not defined
b) 6
c) 1000
82. a) $\frac{1}{2}$
b) $\frac{3}{5}$
c) $\frac{1}{2}$
84. a) 13 and 14
b) 10 and 11
c) 6 and 7
86. a) 6
b) 2
c) 10
- d) 3
e) 10
f) 3
- d) 26
e) 7
f) not defined
- d) $\frac{3}{10}$
e) $\frac{4}{3}$
f) $\frac{1}{5}$
- d) 2 and 3
e) 8 and 9
f) 2 and 3
- d) $\sqrt[3]{3}$
e) $\frac{2}{5}$
f) 8
- d) $\sqrt[3]{192}$
b) $\sqrt[3]{117}$
c) true

8.4 Powers With Rational Exponents

93. a) $\sqrt[5]{4}$
b) $\sqrt[4]{5}$
c) $\sqrt[3]{36}$
95. a) $\sqrt[4]{8}$
b) $\sqrt[4]{125}$
c) $\sqrt[3]{123}$
97. a) $a^{\frac{5}{4}}$
b) $\frac{1}{3}$
c) 8
99. a) 19
b) 3
c) 4
- d) $\sqrt[3]{7}$
e) $\sqrt{1000}$
f) $\sqrt[3]{243}$
- d) $\sqrt[10]{2048}$
e) 1
f) $\sqrt[20]{512}$
- d) $\frac{1}{b}$
e) $\frac{c}{b}$
f) $-2a^2$
- d) 9
e) 2197
f) 3375
94. a) $\sqrt[5]{128}$
b) $\sqrt[3]{81}$
c) $\sqrt[4]{3}$
96. a) $\sqrt[5]{2401}$
b) $\sqrt[3]{128}$
c) $\sqrt{134}$
98. a) $\frac{3}{2}$
b) a^2
c) $\frac{3}{4}$
100. a) 17
b) 3
c) 3
- d) $\sqrt[6]{2}$
e) $\sqrt{32}$
f) $\sqrt[4]{512}$
- d) $\sqrt[3]{49}$
e) $\sqrt[4]{216}$
f) $\sqrt[3]{625}$
- d) $\frac{1}{10}$
e) $a^{-\frac{5}{2}}$
f) b
- d) 243
e) 25
f) 128

101. a) $\frac{1}{16}$ d) $\frac{1}{243}$ 102. a) $\frac{1}{14}$ d) $\frac{1}{512}$
 b) 10 e) 2187 b) 1000 e) 243
 c) $\frac{1}{3}$ f) $\frac{1}{1728}$ c) $\frac{1}{5}$ f) $\frac{1}{256}$
103. a) $\frac{1}{3}$ d) $\frac{1}{128}$ 104. a) $\frac{1}{5}$ d) $\frac{1}{81}$
 b) $\frac{3}{2}$ e) $\frac{3}{5}$ b) $\frac{10}{9}$ e) $\frac{3}{4}$
 c) 1 f) $\frac{5}{6}$ c) $\frac{1}{4}$ f) $\frac{5}{2}$
105. a) $\frac{1}{8}$ d) $\frac{3}{2}$ 106. a) 3 d) 3
 b) $\frac{1}{5}$ e) $\frac{5}{2}$ b) $\frac{5}{6}$ e) $\frac{25}{22}$
 c) $\frac{3}{4}$ f) 20 c) $\frac{2}{3}$ f) $\frac{1}{12}$
107. a) $32^{\frac{1}{6}} = 2^{\frac{5}{6}}$ d) $3^{-\frac{9}{2}}$ 108. a) $9^{\frac{1}{3}} = 3^{\frac{2}{3}}$ d) $4^{-\frac{1}{2}}$
 b) $1024^{\frac{1}{4}} = 4^{\frac{5}{4}}$ e) $5^{\frac{1}{2}}$ b) $625^{\frac{1}{3}} = 5^{\frac{4}{3}}$ e) 9
 c) $5^{\frac{1}{4}}$ f) 3^{-3} c) $512^{\frac{1}{2}} = 8^{\frac{3}{2}}$ f) 27
109. a) 12 d) 3 110. a) $4^{-13.8}$ d) 25
 b) $3^{\frac{1}{6}}$ e) $5^{\frac{1}{14}}$ b) 1 e) $2^{-\frac{13}{7}}$
 c) 16 f) 5 c) 2 f) 3
111. a) 1 c) 1 112. a) a c) a
 b) $a^{\frac{17}{12}}$ d) a^6 b) $a^{\frac{1}{12}}$ d) $a^{-\frac{1}{12}}$
113. a) $a^{-\frac{3}{4}}$ b) $a^{\frac{2}{5}}$ 114. a) $a^{-\frac{31}{15}}$ b) $a^{\frac{1}{12}}$
115. ①-②-④-⑥, ③-⑤-⑦-⑧ 116. ①-④-⑤-⑦-⑧, ②-⑥, ③
117. a) $\frac{1}{9}$ c) 11 118. a) 2 c) 31
 b) 2 d) 13 b) 4 d) -8
119. a) 12 c) 4 120. a) -20 c) 88
 b) 21 d) 60 b) 15 d) 30
121. a) -2 c) $\frac{216}{5}$ 122. a) -4 c) 864
 b) 731 d) $\frac{1}{2}$ b) $\frac{3}{4}$ d) 38
123. a) $\sqrt[25]{5}$ c) $\sqrt[8]{8}$ 124. a) $\sqrt[9]{3}$ c) $\sqrt[3]{16}$
 b) $\sqrt[3]{12}$ d) $\sqrt[8]{5}$ b) $\sqrt[3]{2}$ d) $\sqrt[9]{4}$
125. $\frac{y^{\frac{17}{10}} z^{\frac{3}{32}}}{x^{\frac{9}{4}}}$ 126. $\frac{x^{\frac{19}{20}} z^{\frac{1}{8}}}{y^{\frac{1}{2}}}$

8.5 Power Equations

127. a) 5 d) $\frac{1}{3}$ 128. a) ± 3 d) $\pm \frac{1}{2}$
 b) $\pm \frac{1}{14}$ e) ± 6 b) -2 e) $\pm \frac{1}{5}$
 c) ± 19 f) no solution c) 7 f) $\frac{1}{4}$
129. a) $\frac{243}{2}$ c) $\frac{81}{625}$ 130. a) $\frac{1}{9}$ c) 206
 b) $-\sqrt[3]{2} = -2^{\frac{1}{3}}$ d) 85 b) 2 d) $\frac{256}{2401}$

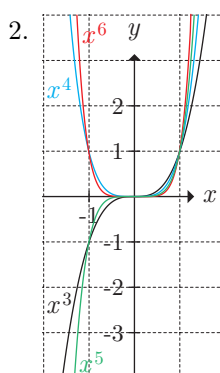
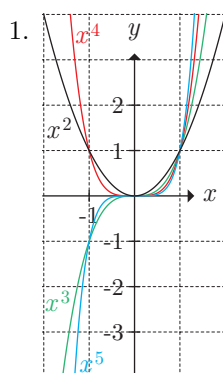
131. a) ± 3 c) no solution 132. a) 4 c) -2
 b) -3 d) $\frac{4}{3}$ b) ± 2 d) $\pm\sqrt{2}$
133. a) $-\frac{2}{3}, 4$ c) no solution 134. a) no solution c) $\frac{1}{81}$
 b) 5 d) $\frac{1}{1296}$ b) 15 d) $-\frac{29}{20}$
135. a) $\frac{2}{3}$ d) $\frac{80}{3}$ 136. a) -2 d) 68
 b) $\frac{5}{3}$ e) -1 b) 2 e) $-\frac{1}{2}$
 c) -2 f) $-\frac{1}{2}$ c) $\frac{2}{3}$ f) $\frac{3}{4}$
137. a) 3 d) -4 138. a) -2 d) $\frac{3}{4}$
 b) -1 e) $-\frac{4}{9}$ b) $\frac{2}{7}$ e) $-\frac{3}{11}$
 c) $\frac{2}{7}$ f) $-\frac{9}{5}$ c) $\frac{1}{3}$ f) $\frac{5}{8}$
139. a) 2, 3 b) 0, 1 140. a) 0, 2 b) $-4, -1$
141. $r = 6 \text{ cm}$ 142. $r = h = 9 \text{ cm}$ 143. $\frac{15}{7}, \frac{53}{21}$ 144. $\frac{\sqrt{2}-12}{8}, \frac{-\sqrt{2}-12}{8}$

8.6 Powers With Real Exponents

145. ①- $\mathcal{D}$, ②- $\mathcal{C}$, ③- $\mathcal{B}$, ④- $\mathcal{A}$ 146. ①- $\mathcal{A}$, ②- $\mathcal{D}$, ③- $\mathcal{B}$, ④- $\mathcal{C}$
147. a) 9 d) 1024 148. a) 32 d) 27
 b) 512 e) 49 b) 125 e) 81
 c) 32 f) 27 c) 9 f) 343
149. a) $625^{\sqrt{7}}$ 150. a) $\left(\frac{4}{7}\right)^{\sqrt{3}}$
 b) 1 b) $13^{\sqrt{14}}$
 c) $12^{\sqrt{11}}$ c) $\left(\frac{8}{11}\right)^{\sqrt{3}}$
 d) $\left(\frac{3}{7}\right)^{2\sqrt{2}}$ d) $\sqrt{3}^{3\sqrt{6}}$
 e) $\sqrt{4}^{3\sqrt{3}} = 2^{3\sqrt{3}} = 8^{\sqrt{3}}$ e) $2^{4\sqrt{5}}$
 f) $5^{2\sqrt{3}} = 25^{\sqrt{3}}$ f) 3^{π}
151. a) 1 152. a) $4^3 \sqrt[4]{7} = 64 \sqrt[4]{7}$
 b) $2^{\sqrt{5}}$ b) $\left(\frac{9}{5}\right)^{2\sqrt{7}} = \left(\frac{81}{25}\right)^{\sqrt{7}}$
 c) $\left(\frac{4}{7}\right)^{2\sqrt{2}} = \left(\frac{16}{49}\right)^{\sqrt{2}}$ c) 1
 d) $\left(\frac{2}{3}\right)^{2\sqrt{5}} = \left(\frac{4}{9}\right)^{\sqrt{5}}$ d) $\sqrt{2}^{2\sqrt{2}} = 2^{\sqrt{2}}$
 e) $3^5 \sqrt[5]{5} = 243 \sqrt[5]{5}$ e) $\left(\frac{7}{8}\right)^{7\sqrt{7}}$
 f) $\sqrt{3}^{3\sqrt{3}}$ f) $6^{2\sqrt{2}} = 36^{\sqrt{2}}$
153. a) 3 c) 4 154. a) 5 c) 3
 b) 4 d) 9 b) 2 d) 1
155. a) 16 c) $8^{\sqrt{2}}$ 156. a) 81 c) $\pi^{-\sqrt{241}}$
 b) 3^{12} d) 9^8 b) $25^{\sqrt{3}}$ d) $16^{\sqrt{6}}$
157. 9261 158. $\sqrt{13}^{4\sqrt{3}}$

9 Power Functions

9.1 Power Functions With Natural Exponents



3. a: 3, b: 5, c: 3, d: 4, e: 5, f: 8, g: 8, h: 4

5. a) A: 64, B: 3, C: -3, D: 0.5

b) A: 16, B: $\frac{1}{625}$, C: $\frac{1}{625}$, D: ± 3

7. a) $n = 3$

d) -

b) $n = 3$

e) n even

c) $n \geq 5$

f) n odd

9. a) $n \geq 7$

c) 1, 3, 5

b) 2, 4, 6

11. a) 11, 13, 15, ...

c) 10, 12, 14, ...

b) $n \leq 9$

4. a: 5, b: 7, c: 2, d: 4, e: 5, f: 7, g: 4, h: 2

6. a) A: 81, B: ± 7 , C: -, D: 0.01

b) A: -1, B: 2, C: -32, D: $-\frac{1}{2}$

8. a) $n = 3$

d) -

b) -

e) n odd

c) $n \leq 6$

f) n even

10. a) 1, 3, 5, 7, 9, 11

c) 2, 4, 6, 8, 10

b) $n \geq 12$

12. a) $n \leq 4$

c) 5, 7, 9, ...

b) 6, 8, 10, ...

13. a) $y = \frac{1}{8}x^4 - 6$

c) $y = \frac{1}{27}x^5 + 1$

b) $y = \frac{1}{9}x^3 + 5$

d) $y = 8x^4 - \frac{1}{4}$

14. a) $y = \frac{1}{2}x^3 + 7$

c) $y = \frac{1}{4}x^3 - 3$

b) $y = 8x^4 + \frac{3}{2}$

d) $y = \frac{1}{16}x^3 + 3$

15. a) true false

c) true false

b) ☒ ☐

d) ☐ ☒

b) ☒ ☐

d) ☒ ☐

16. a) true false

c) true false

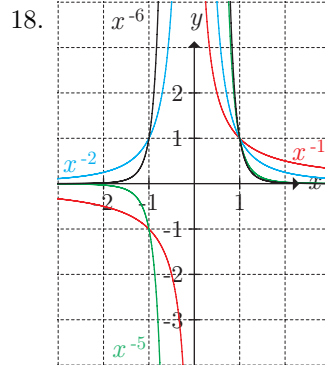
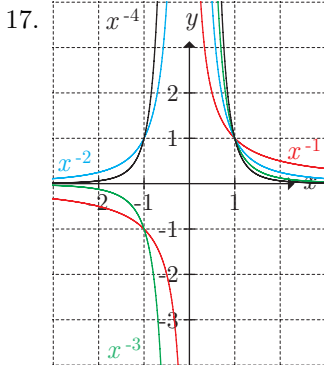
b) ☐ ☒

d) ☐ ☒

b) ☐ ☒

d) ☐ ☒

9.2 Power Functions With Integer Exponents



19. $a: -9, b: -5, c: -9, d: -5, e: -4,$
 $f: -2, g: -2, h: -4$

21. a) $A: \frac{1}{4}, B: \frac{1}{7}, C: -6, D: 8$
 b) $A: \frac{1}{16}, B: 256, C: \pm 3, D: -$

23. a) $0 < x < 1$ d) $n \geq 5$
 b) n even e) $n \in \mathbb{N} \setminus \{2, 4, 6\}$
 c) $n = 2$ f) n even

20. $a: -5, b: -1, c: -8, d: -5, e: -2,$
 $f: -1, g: -2, h: -8$

22. a) $A: \frac{1}{36}, B: 25, C: \pm \frac{1}{6}, D: \pm \frac{3}{2}$
 b) $A: \frac{1}{27}, B: -2, C: 216, D: \frac{1}{4}$

24. a) $x \geq 1$ d) $n \in \{1, 2, 3, 4\}$
 b) n odd e) $n \in \mathbb{N} \setminus \{2, 4\}$
 c) $n = 4$ f) n odd

9.3 The Inverse Function

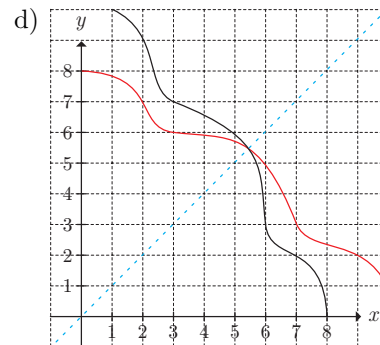
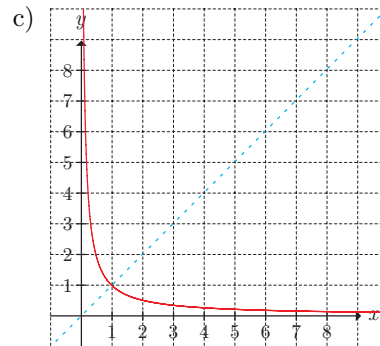
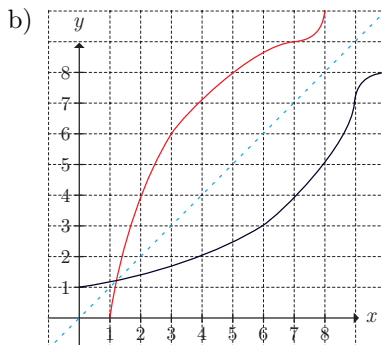
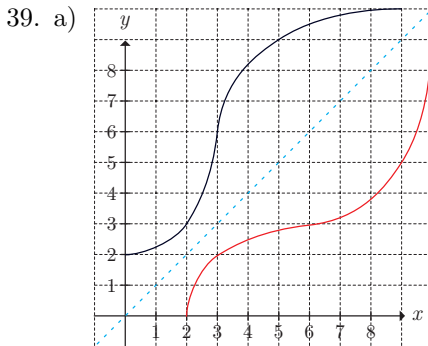
25. a) no c) yes 26. a) no c) no
 b) no d) no b) yes d) no

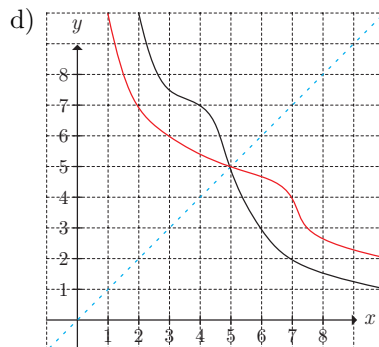
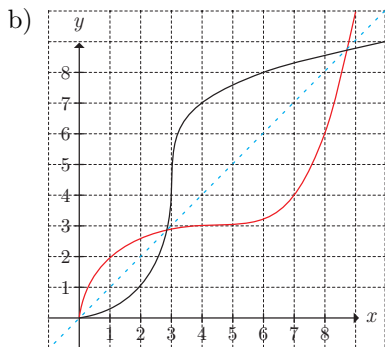
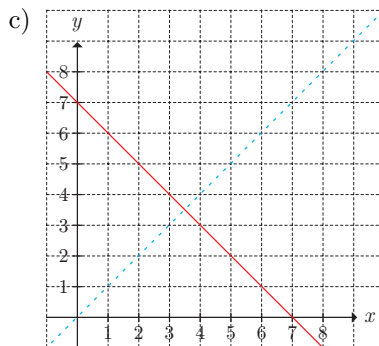
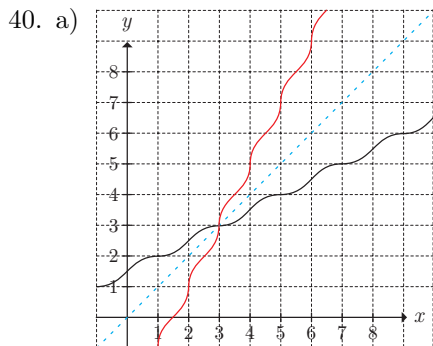
27. a) Not really bijective, as sometimes several coats hang on the same hook or numbers are not assigned.
 b) Not bijective, as different kinds may be the same price.
 c) Bijective if the car park is fully occupied, otherwise not.
 d) By definition bijective

28. a) Not bijective, as there are different books with the same number of pages.
 b) Yes, if only the AHV numbers of living Swiss women are considered.
 c) Not bijective, as there are more than four people.
 d) Not bijective, since, for example, 2 and -2 result in the same square number.

29. a) f^{-1} halves the argument: $y = \frac{x}{2}$
 b) f^{-1} triples the argument: $y = 3x$
 c) f^{-1} reduces the argument by 8: $y = x - 8$
 d) f^{-1} increases the argument by 2 and then divides it by 5: $y = (x + 2) : 5$
 e) f^{-1} subtracts 3 from the argument and then generates the opposite number: $y = -(x - 3)$

30. a) f^{-1} quadruples the argument: $y = 4x$
 b) f^{-1} divides the argument by 8: $y = \frac{x}{8}$
 c) f^{-1} increases the argument by 7: $y = x + 7$
 d) f^{-1} reduces the argument by 5 and doubles it afterwards: $y = (x - 5) \cdot 2$
 e) f^{-1} adds 1 to the argument and then generates the opposite number: $y = -(x + 1)$
31. a) $y = 4(x + 6)$ c) $y = -\frac{x-3}{8}$ 32. a) $y = \frac{x-2}{4}$ c) $y = -\frac{7}{2}(x + 4)$
 b) $y = \frac{3}{2}x$ d) $y = -\frac{4}{3}(x + 5)$ b) $y = \frac{4}{9}x$ d) $y = \frac{x-\frac{1}{2}}{4}$
33. a) $\mathbb{X} = \mathbb{R}, y = \frac{1}{3}x - \frac{5}{3}$
 b) $\mathbb{X} = \mathbb{R}, y = 4x - 19$
 c) $\mathbb{X} = \mathbb{R}_0^+, y = \sqrt{x}$
 d) $\mathbb{X} = [4, \infty[, y = \sqrt{x-6} + 4$
 e) $\mathbb{X} = \mathbb{R} \setminus \{0\}, y = \frac{2}{x-3}$
 f) $\mathbb{X} = [3, \infty[, y = \left(\frac{x-7}{6}\right)^2 + 3 = \frac{x^2-14x+157}{36}$
34. a) $\mathbb{X} = \mathbb{R}, y = 3x + 20$
 b) $\mathbb{X} = \mathbb{R}, y = \frac{1}{9}x - \frac{22}{9}$
 c) $\mathbb{X} = \mathbb{R}_0^+, y = x^2$
 d) $\mathbb{X} = [5, \infty[, y = \frac{x^2}{9} + 5$
 e) $\mathbb{X} = [4, \infty[, y = \sqrt{2(x-1)} + 4$
 f) $\mathbb{X} = \mathbb{R} \setminus \{0\}, y = \frac{1}{2(x+4)}$
35. a) $\mathbb{X} = [2, \infty[, y = \sqrt{-4(x-3)} + 2$
 b) $\mathbb{X} = [-5, \infty[, y = \sqrt[4]{\frac{1}{3}(x+11)} - 5$
 c) $\mathbb{X} = [-1, \infty[, y = \frac{1}{24}(x-7)^2 - 1$
 d) $\mathbb{X} =]9, \infty[, y = \sqrt{\frac{8}{x}} + 9$
36. a) $\mathbb{X} = [-6, \infty[, y = \left(\frac{1}{2}(x+8)\right)^{\frac{1}{4}} - 6$
 b) $\mathbb{X} =]1, \infty[, y = \frac{1}{3}(5-x)^{-\frac{1}{2}} + 1$
 c) $\mathbb{X} = [-7, \infty[, y = \sqrt[6]{8(x-9)} - 7$
 d) $\mathbb{X} = [2, \infty[, y = \frac{1}{4}(x+3)^2 + 2$
37. a) yes b) no c) no d) yes e) no
38. a) yes b) no c) yes d) no e) no





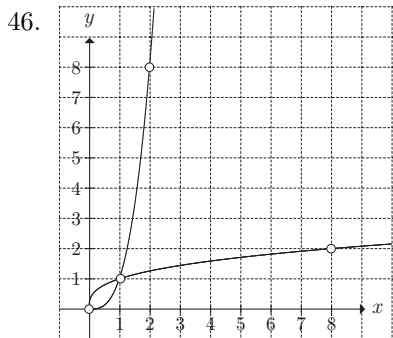
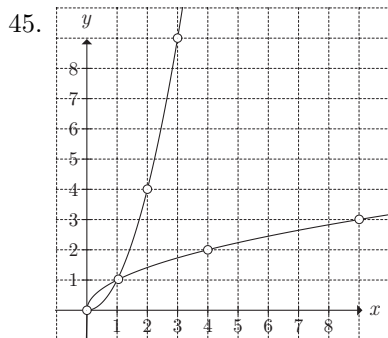
41. a) ☐ true ☒ false c) ☒ true ☐ false
b) ☒ true ☐ false d) ☒ true ☐ false

42. a) ☒ true ☐ false c) ☒ true ☐ false
b) ☐ true ☒ false d) ☒ true ☐ false

9.4 Power Functions With Rational Exponents

43. $f^{-1}: y = x^{\frac{1}{2}}, g^{-1}: y = x^{\frac{1}{5}}, h^{-1}: y = x^{\frac{1}{16}}$

44. $f^{-1}: y = x^{\frac{1}{3}}, g^{-1}: y = x^{\frac{1}{4}}, h^{-1}: y = x^{\frac{1}{11}}$

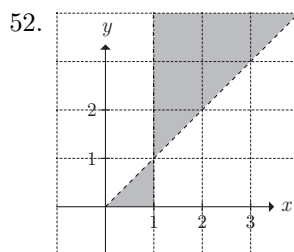
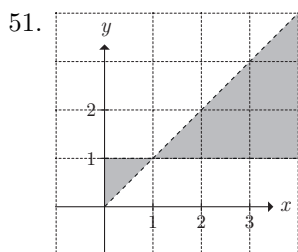


47. $a: y = \sqrt{x}, b: y = \sqrt[3]{x}, c: y = \sqrt[4]{x}, d: y = \sqrt[5]{x}$
 $A(1, 1), B(4, 2), C(8, 2), D(9, 3), E(16, 2), F(16, 4), G(25, 5), H(27, 3), I(32, 2)$

48. $(0, 0) (f, g, h), (1, 1) (f, g, h), (64, 2) (f), (128, 2) (g), (256, 2) (h), (729, 3) (f)$

49. $a: y = x^{0.1}, b: y = x^{0.4}, c: y = x^{0.9},$
 $d: y = x^{\frac{2}{3}}, e: y = x^{0.9}, f: y = x^{1.1},$
 $h: y = x^{\frac{3}{2}}, i: y = x^2$
Exponent of g lies between 1.1 and 1.5.
Exact value: 1.3

50. $a: y = x^{\frac{1}{5}}, b: y = x^{\frac{3}{5}}, c: y = x^{\frac{3}{4}},$
 $e: y = x^{1.1}, f: y = x^{\frac{5}{4}}, g: y = x^{\frac{4}{3}}$
 $h: y = x^3, i: y = x^4,$
Exponent of d lies between 0.75 and 1.1.
Exact value: 0.9



53. $a: y = x^5$, $b: y = x^{-11}$, $c: y = x^{-3}$,
 $d: y = x^{-2}$, $e: y = x^{-\frac{1}{5}}$, $f: y = x^{\frac{1}{5}}$,
 $g: y = x^{\frac{1}{3}}$, $h: y = x^{\frac{10}{9}}$. Between d and e

54. $a: y = x^{\frac{3}{2}}$, $b: y = x^8$, $c: y = x^{-4}$,
 $d: y = x^{-\frac{3}{5}}$, $e: y = x^{-\frac{2}{5}}$, $f: y = x^{\frac{1}{10}}$,
 $g: y = x^{\frac{1}{4}}$, $h: y = x^{\frac{7}{8}}$. Between c and d

55.

	true	false		true	false
a)	☒	☐	c)	☒	☐
b)	☐	☒	d)	☒	☐

56.

	true	false		true	false
a)	☒	☐	c)	☐	☒
b)	☐	☒	d)	☒	☐

9.5 Graph Modifications

57. ①- i , ②- h , ③- f , ④- g , ⑤- k , ⑥- j ,

58. ①- j , ②- f , ③- g , ④- k , ⑤- i , ⑥- h ,

59. $a: y = \sqrt{-x}$, $b: y = -\sqrt{-x}$, $c: y = -\sqrt{x}$

60. $a: y = \sqrt{-x} + 1$, $b: y = -\sqrt{-x} - 1$,
 $c: y = -\sqrt{x} - 1$

61. $a: y = (x-3)^3 - 1$, $b: y = -(x-3)^3 + 1$,
 $c: y = (x+3)^3 + 1$, $d: y = -(x+3)^3 - 1$

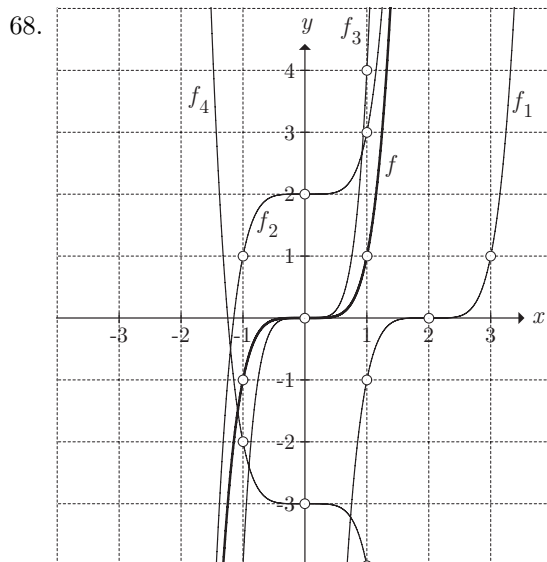
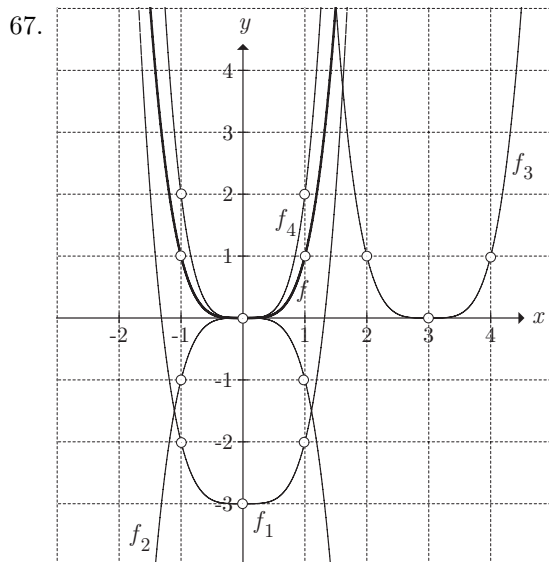
62. $a: y = -(x-4)^4 + 2$, $b: y = (x-4)^4 - 2$,
 $c: y = (x+4)^4 - 2$, $d: y = -(x+4)^4 + 2$

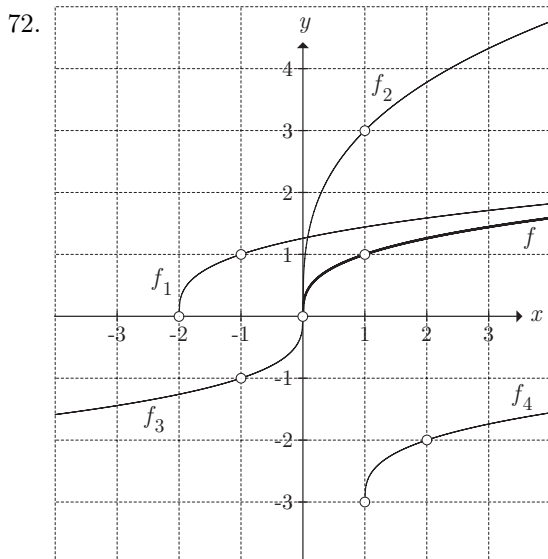
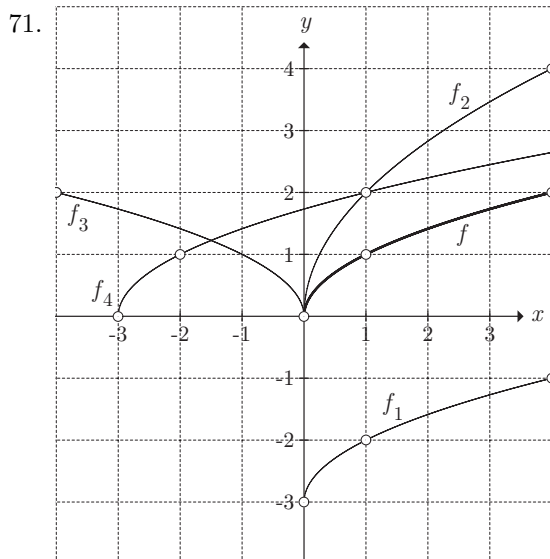
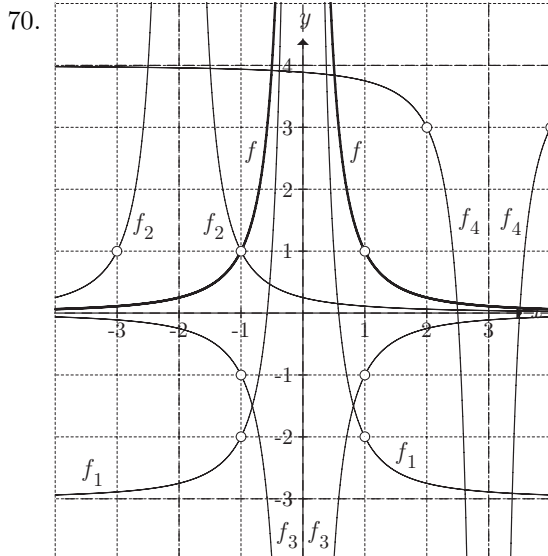
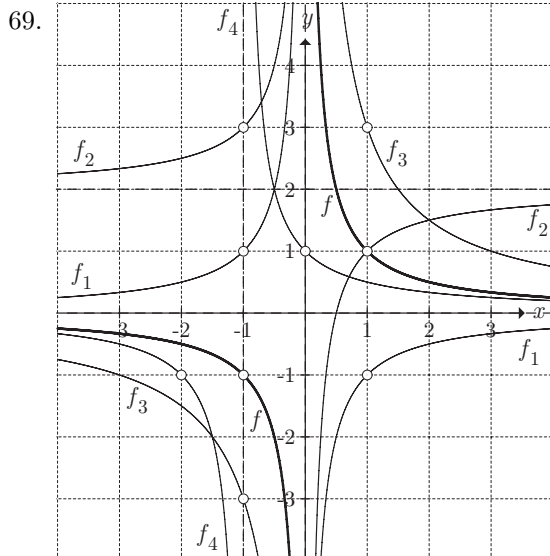
63. ①- $\mathcal{H}$, ②- $\mathcal{A}$, ③- $\mathcal{G}$, ④- $\mathcal{F}$, ⑤- $\mathcal{C}$, ⑥- $\mathcal{D}$, ⑦- $\mathcal{B}$, ⑧- $\mathcal{E}$

64. ①- $\mathcal{C}$, ②- $\mathcal{A}$, ③- $\mathcal{E}$, ④- $\mathcal{D}$, ⑤- $\mathcal{F}$, ⑥- $\mathcal{G}$, ⑦- $\mathcal{B}$, ⑧- $\mathcal{H}$

65. g_1 - $\mathcal{H}$, g_2 - $\mathcal{E}$, g_3 - $\mathcal{A}$, g_4 - $\mathcal{B}$, g_5 - $\mathcal{C}$, g_6 - $\mathcal{F}$

66. g_1 - $\mathcal{C}$, g_2 - $\mathcal{E}$, g_3 - $\mathcal{A}$, g_4 - $\mathcal{H}$, g_5 - $\mathcal{G}$, g_6 - $\mathcal{B}$





73. f_1 : reflection across the x -axis, f_2 : a shift of 14 units in the y -direction,
 f_3 : reflection at the origin, f_4 : reflection across the y -axis

74. f_1 : reflection across the y -axis, f_2 : reflection at the origin,
 f_3 : reflection across the x -axis, f_4 : a shift of 16 units in the y -direction

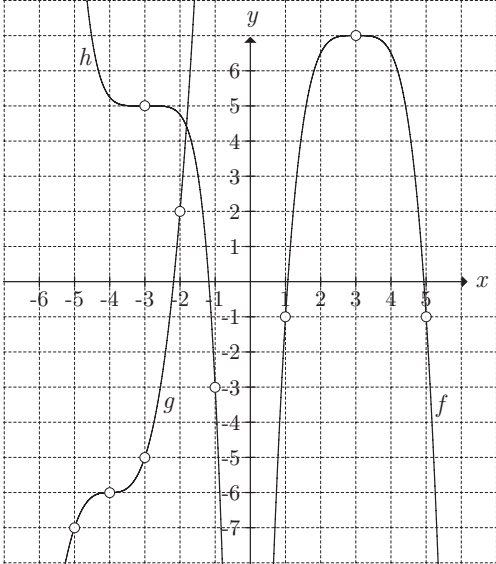
75. f_1 : reflection across the y -axis, f_2 : reflection at the origin,
 f_3 : translation by 6 units in the x -direction, f_4 : reflection across the x -axis

76. f_1 : reflection across the y -axis, f_2 : reflection at the origin,
 f_3 : shift by 10 units in the x -direction, f_4 : reflection across the x -axis

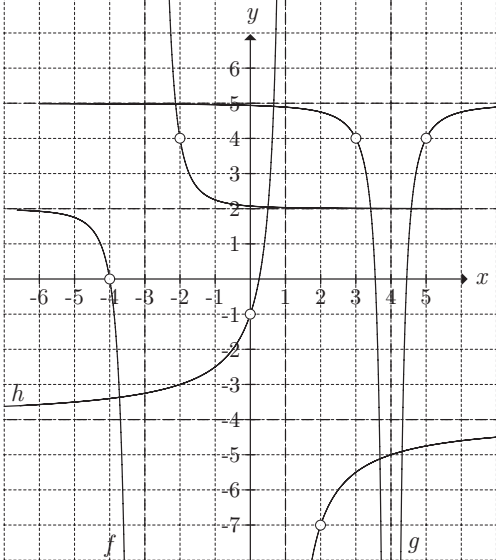
77. $f: y = -(x+2)^3 + 5$,
 $g: y = -\frac{1}{2}\sqrt{x+6} - 1$,
 $h: y = 2(x-5)^{-2} - 6$

78. $f: y = \frac{1}{2}(x+3)^4 - 7$,
 $g: y = 2\sqrt[3]{x+6} - 5$,
 $h: y = -(x-4)^{-1} + 3$

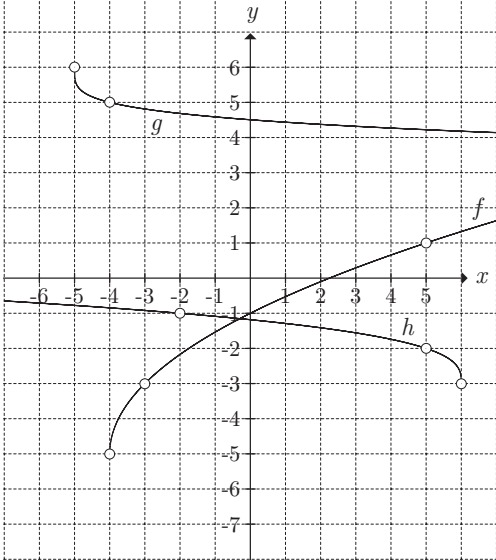
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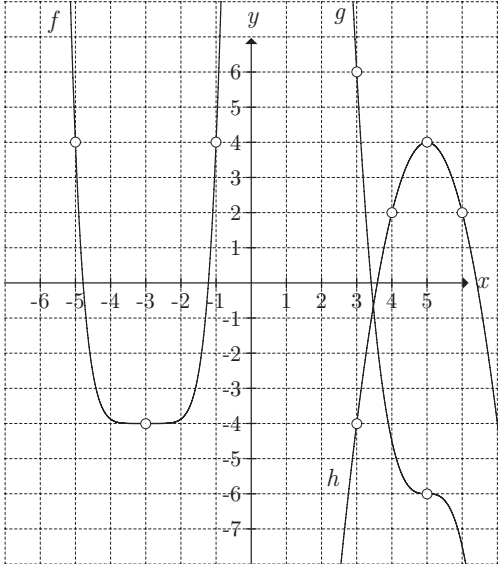
81.



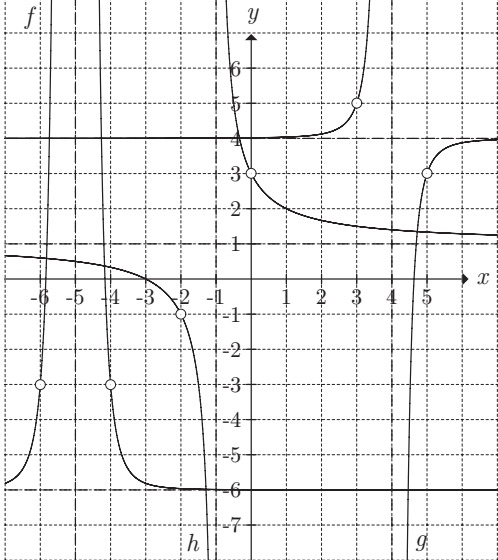
83.



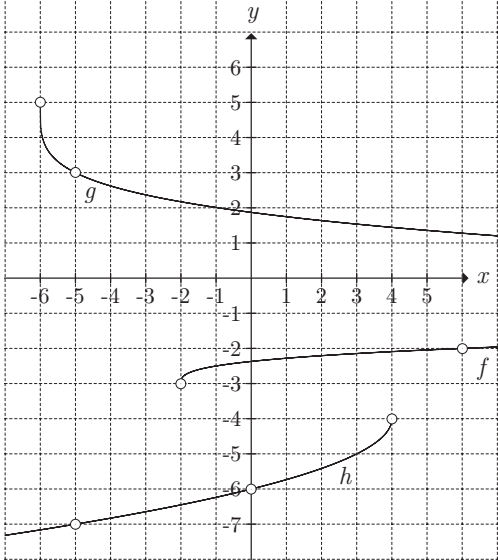
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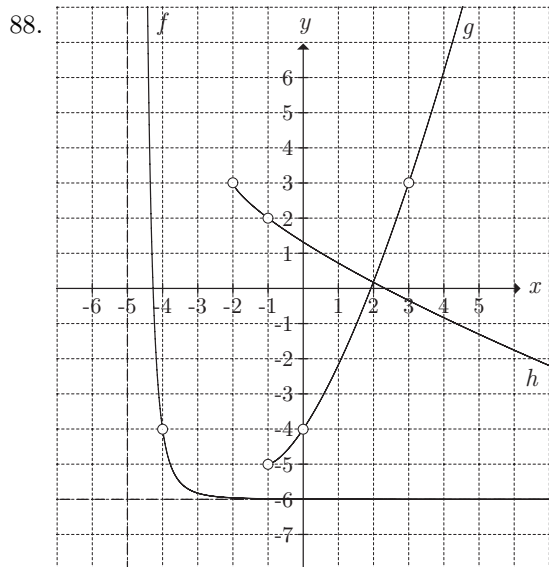
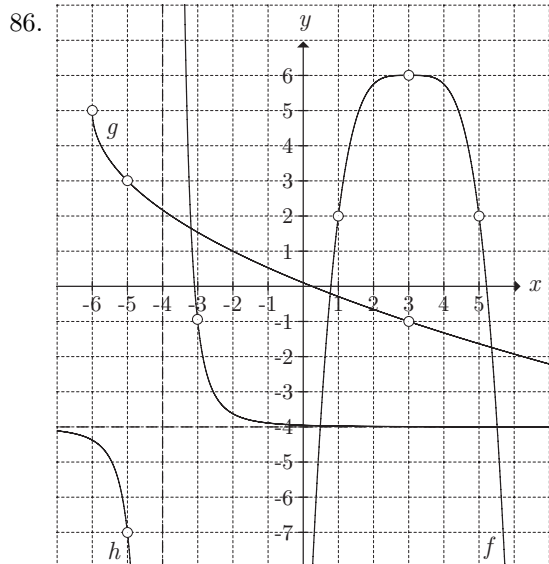
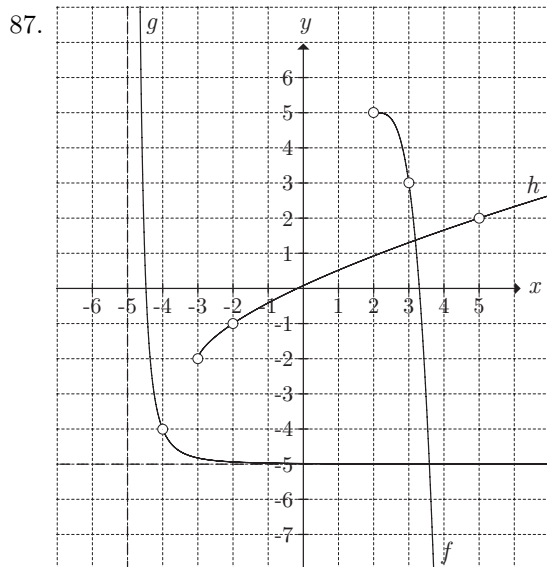
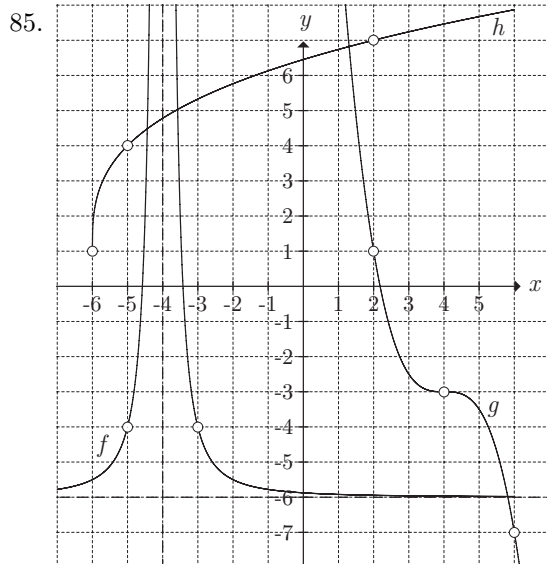


82.



84.





89. a) $y = (x + 1)^6 - 3$
 b) $y = -(x - 4)^6 + 3$
 c) $y = (x - 7)^6 - 1$
 d) $y = -(x + 4)^6 + 3$

91. a) $y = (x + 2)^{-3} + 5$
 b) $y = -(x - 7)^{-3} - 6$
 c) $y = (x + 2)^{-3} - 9$
 d) $y = (x - 18)^{-3} + 3$

90. a) $y = -(x + 1)^5 - 5$
 b) $y = (x - 1)^5 + 2$
 c) $y = -(x + 5)^5 + 8$
 d) $y = -(x - 1)^5 - 2$

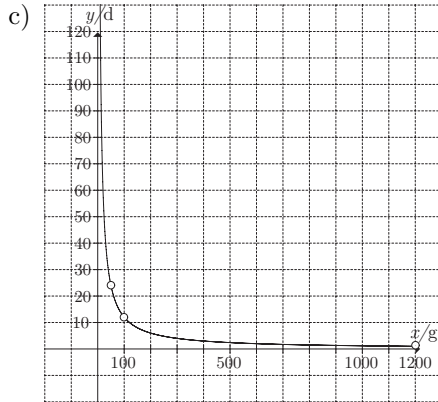
92. a) $y = -2(x - 3)^{-4} - 8$
 b) $y = -2(x - 3)^{-4} - 8$
 c) $y = 2(x + 5)^{-4} - 8$
 d) $y = -2(x + 9)^{-4} + 18$

93. a) $a = -1, b = 2, c = 2n, n \in \mathbb{N}, d = 3$
 b) $a = 2, b = 1, c = 2n + 1, n \in \mathbb{N}, d = 2$
 c) $a = \frac{3}{2^c}, b = 1, c = 2n, n \in \mathbb{N}, d = 0$
 d) $a = -\frac{1}{2^c}, b = 2, c = 2n + 1, n \in \mathbb{N}, d = 2$

95. $V = \frac{4}{3}\pi(2t)^3 = \frac{32}{3}\pi t^3, V(9\text{ s}) \approx 24\,429\text{ cm}^3$

97. a) 12 d, 24 d, 120 d

b) $y = \frac{1200}{x}, \mathbb{X} =]0, 1200], \mathbb{Y} = [1, \infty[$

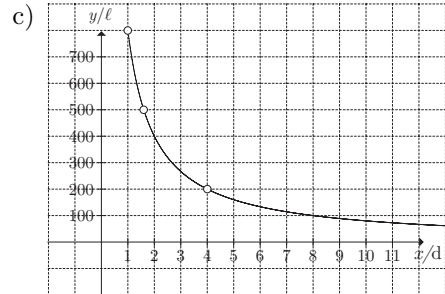


94. a) $a = 2, b = 1, c = 2n, n \in \mathbb{N}, d = 1$
 b) $a = \frac{2}{2^c}, b = 2, c = 2n + 1, n \in \mathbb{N}, d = 2$
 c) $a = -3, b = 2, c = 2n + 1, n \in \mathbb{N}, d = 2$
 d) $a = -\frac{4}{2^c}, b = 1, c = 2n, n \in \mathbb{N}, d = 4$

96. $A = \pi(3t)^2 = 9\pi t^2, A(24\text{ h}) \approx 16\,286\text{ m}^2$

98. a) 20 ℓ, 50 ℓ, 5 ℓ

b) $y = \frac{800}{x}, \mathbb{X} = [1, \infty[, \mathbb{Y} =]0, 800]$



99. $f(x) = \frac{32}{3}\pi x^3, f^{-1}(x) = \sqrt[3]{\frac{3x}{32\pi}}$
 $f(8\text{ cm}) \approx 17\,157\text{ cm}^3$
 $f^{-1}(1000\text{ cm}^3) \approx 3.1\text{ cm}$

100. $f(x) = 16\pi x^3, f^{-1}(x) = \sqrt[3]{\frac{x}{16\pi}}$
 $f(8\text{ cm}) \approx 25\,736\text{ cm}^3$
 $f^{-1}(1000\text{ cm}^3) \approx 2.7\text{ cm}$

9.6 Polynomial Functions

101. a) no polynomial b) polynomial c) polynomial d) no polynomial

102. a) no polynomial b) polynomial c) no polynomial d) no polynomial

103. a) 5, 7, 9
 b) $-2x^7, -x^5, -3x^9$

105. ①-C, ②-D, ③-A, ④-F, ⑤-E, ⑥-B

107. a) 6
 b) 6
 c) ≥ 5
 d) 9, 11, 13, ...
 e) 4, 6, 8, ...

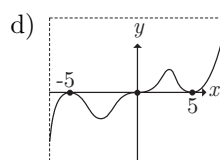
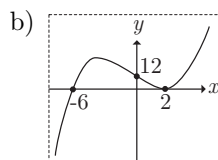
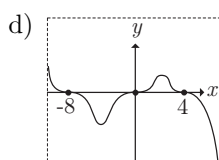
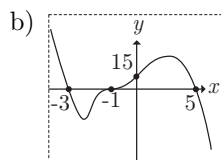
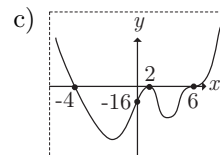
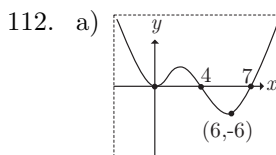
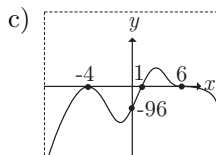
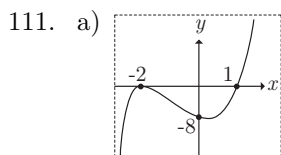
109. f_2, f_6, g_1, g_2, h_2

104. a) 6, 8
 b) $-4x^6, -x^{10}$

106. ①-F, ②-A, ③-F, ④-C, ⑤-B, ⑥-D

108. a) 4
 b) 4
 c) ≥ 3
 d) 6, 8, 10, ...
 e) 5, 7, 9, ...

110. $f_3, f_4, g_3, g_6, h_2, h_5$



113. a) $y = \frac{1}{12}x(x+4)(x-3)$

b) $y = -\frac{1}{100}(x-2)^3(x+5)^2(x-6)$

c) $y = \frac{1}{1024}x^2(x+4)^3(x-3)^3$

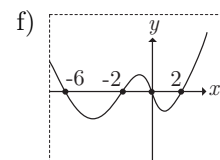
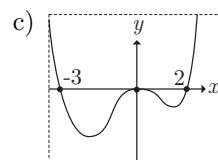
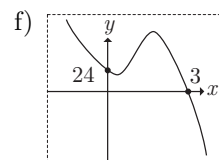
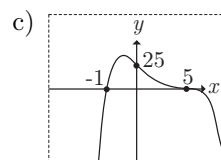
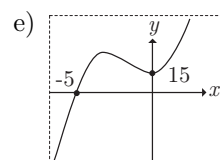
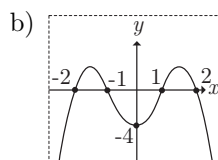
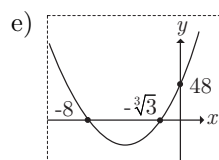
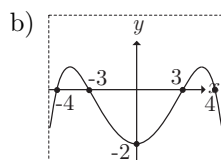
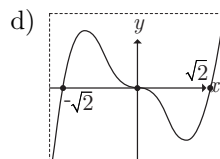
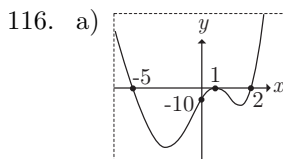
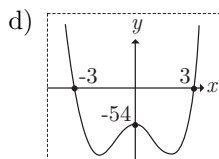
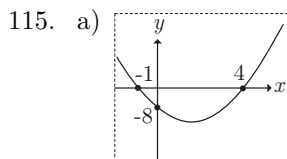
d) $y = -\frac{1}{8}(x+1)^2(x-4)^3$

114. a) $y = -\frac{1}{5}x(x-2)(x-3)(x+4)$

b) $y = \frac{1}{12}x^2(x-4)(x-7)$

c) $y = \frac{1}{5}(x+1)(x-2)^3(x-5)$

d) $y = -\frac{1}{8}(x+1)^2(x-2)^2(x-4)^2$



117. a) $y = \frac{3}{2}(x-1)^2(x+2)$

b) $y = \frac{1}{7}x^2(x-4)(x-6)$

c) $y = \frac{1}{2}(x+2)^2(x-6)$

d) $y = -(x+2)^2(x-4) - 20$

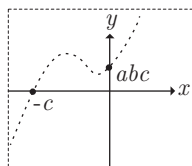
118. a) $y = -\frac{1}{2}(x-2)^3(x+3)$

b) $y = \frac{1}{6}3(x+1)(x+3)(x-4)$

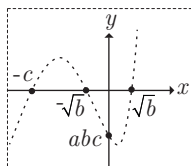
c) $y = -\frac{1}{2}x^2(x+3)(x-4) + 2$

d) $y = -\frac{1}{4}(x-1)^2(x-5)$

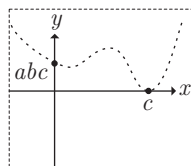
119. $a > 0, b > 0$



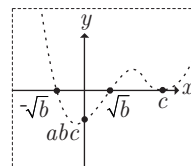
$a > 0, b < 0$



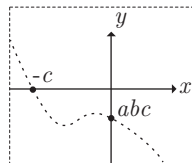
120. $a > 0, b > 0$



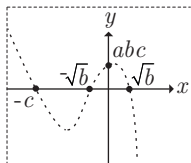
$a > 0, b < 0$



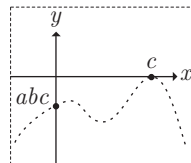
$a < 0, b > 0$



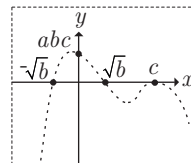
$a < 0, b < 0$



$a < 0, b > 0$



$a < 0, b < 0$



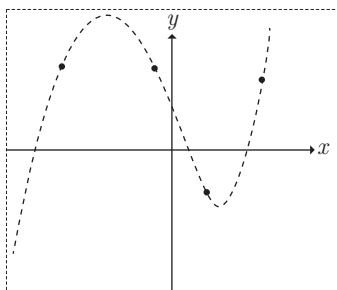
121. a) $y = -x^3 + 4x^2 + x - 6$

b) $y = x^4 - 3x^3 + x^2 - 6x + 9$

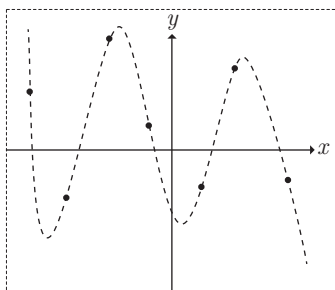
122. a) $y = x^3 - 4x^2 + 3x + 1$

b) $y = -x^4 + 4x^3 - 2x^2 - 4x + 2$

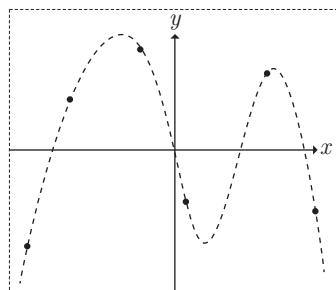
123. degree 3



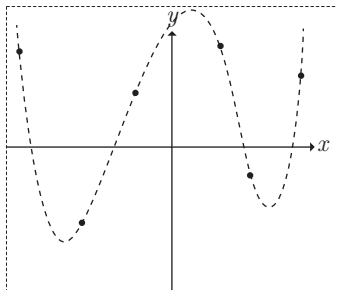
degree 5



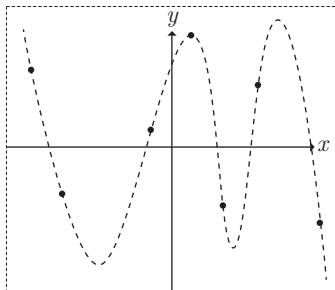
degree 4



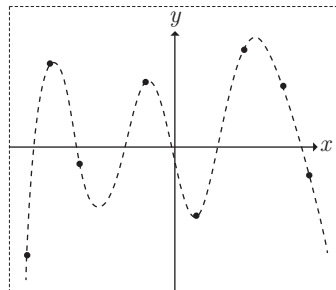
124. degree 4



degree 5



degree 6



125. a) ☐ true ☒ false c) ☐ true ☒ false
 b) ☒ true ☐ false d) ☒ true ☐ false

126. a) ☒ true ☐ false c) ☒ true ☐ false
 b) ☐ true ☒ false d) ☐ true ☒ false

10 Exponential and Logarithmic Functions

10.1 Exponential Function

1. yes, $y = 10^6 \cdot 1.09^x$, $\mathbb{X} = \mathbb{N}_0$

3. no

5. yes, $y = 5^x$, $\mathbb{X} = \mathbb{N}$, $\mathbb{Y} = \{5^n | n \in \mathbb{N}\}$

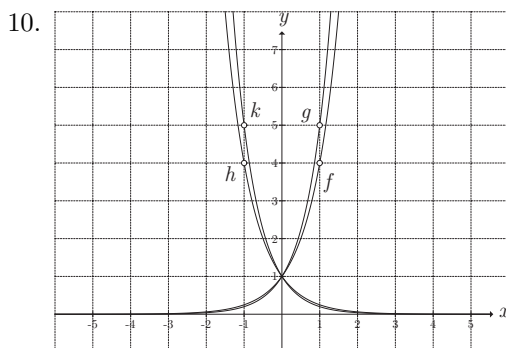
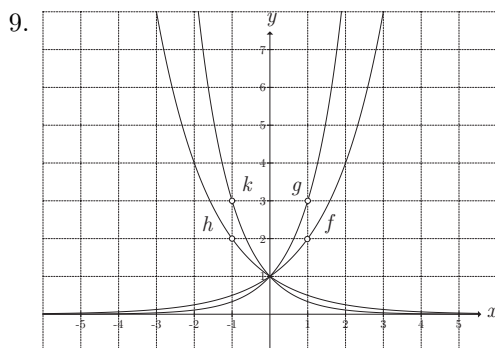
	yes	no		yes	no
7. a)	☒	☐	c)	☒	☐
b)	☐	☒	d)	☐	☒

2. no, linear

4. yes, $y = 3000 \cdot 0.9^t$, $\mathbb{X} = \mathbb{N}_0$

6. yes, $y = 2500 \cdot 1.035^{t-1}$, $\mathbb{X} = \mathbb{N}$

	yes	no		yes	no
8. a)	☐	☒	c)	☒	☐
b)	☒	☐	d)	☐	☒

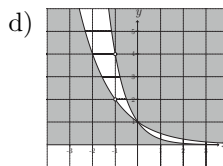
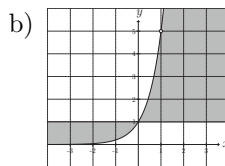
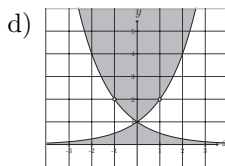
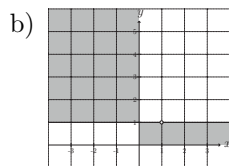
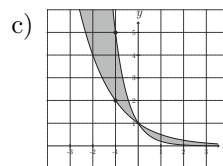
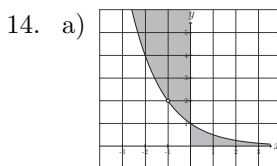
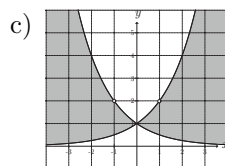
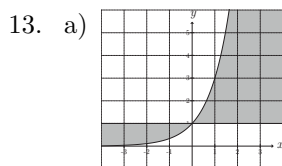


11. $f: x \rightarrow 6^x$
 $g: x \rightarrow 2^x$
 $h: x \rightarrow \left(\frac{1}{3}\right)^x$
 $k: x \rightarrow \left(\frac{1}{5}\right)^x$

12. $f: x \rightarrow 3^x$
 $g: x \rightarrow 5^x$
 $h: x \rightarrow \left(\frac{1}{2}\right)^x$
 $k: x \rightarrow \left(\frac{1}{6}\right)^x$

Note on tasks 13 and 14:

The solution is the area highlighted in grey without the border, at d) additionally also without the y -axis.



15. a) $0 < a < 2 \vee a > 3$
b) $0 < a < \frac{1}{2} \vee a > 3$

16. a) $0 < a < 4$
b) $0 < a < \frac{1}{2} \vee a > 1$

17. a) ②
b) ③

c) ④
d) ③

18. a) ③
b) ④
c) ①
d) ③

19. a) The new function value is $\frac{1}{4}$ of the original function value.
 b) The dependent variable is the square of the original dependent variable.
 c) The argument was increased by one.
 d) The independent variable was reduced by two.
20. a) The new function value is the root of the original function value
 b) The dependent variable is multiplied by eight.
 c) The argument was increased by 2.
 d) The independent variable was tripled.
21. a) ⑤
 b) $f(x) = a^x$ for all $a \in \mathbb{R}^+ \setminus \{1\}$
 c) ①
 d) ③
22. a) ②
 b) $f(x) = a^x$ for all $a \in \mathbb{R}^+ \setminus \{1\}$
 c) ⑥
 d) ④
23. a)

true	false
☒	☐

 c)

true	false
☒	☐

 b)

☐	☒
--------------------------	-------------------------------------

 d)

☒	☐
-------------------------------------	--------------------------
24. a)

true	false
☐	☒

 c)

true	false
☐	☒

 b)

☒	☐
-------------------------------------	--------------------------

 d)

☒	☐
-------------------------------------	--------------------------
25. a)

true	false
☒	☐

 c)

true	false
☒	☐

 b)

☐	☒
--------------------------	-------------------------------------

 d)

☐	☒
--------------------------	-------------------------------------
26. a)

true	false
☒	☐

 c)

true	false
☐	☒

 b)

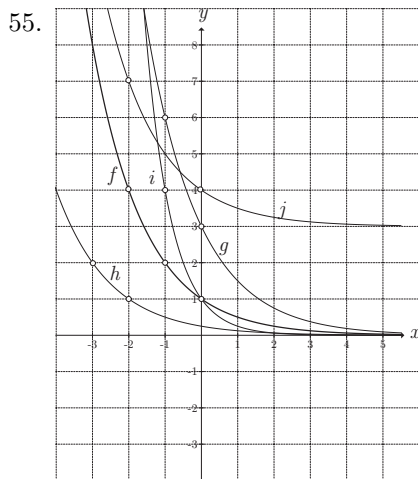
☐	☒
--------------------------	-------------------------------------

 d)

☒	☐
-------------------------------------	--------------------------
27. a) P : no, Q : yes
 b) P : no, Q : no
 c) P : yes, Q : no
 d) P : no, Q : no
28. a) P : no, Q : yes
 b) P : yes, Q : yes
 c) P : no, Q : yes
 d) P : yes, Q : yes
29. a) 4
 b) $\frac{1}{3}$
30. a) $\frac{1}{6}$
 b) 4
31. $f(x) = 4^x$
32. $f(x) = 0.5^x$
33. $f(x) = \left(\frac{2}{3}\right)^x$
34. $f(x) = 10^x$
35. a) $a = \frac{1}{2}, b = 3$
 b) $a = 3, b = -2$
36. a) $a = 4, b = 2$
 b) $a = 4, b = -3$
37. $f(x) = -3^x$
38. $f(x) = \frac{1}{2} \cdot \left(\frac{1}{5}\right)^x$
39. $f(x) = -2 \cdot 5^x$
40. $f(x) = 3 \cdot \left(\frac{1}{2}\right)^x$
41. a) linear, $f(x) = 4x + 2$
42. a) exponential function, $f(x) = 3 \cdot \left(\frac{1}{4}\right)^x$
43. a) $y = \left(\frac{1}{2}\right)^x$
 b) $y = -2 \cdot 4^x$
44. a) $y = 3^x$
 b) not possible
45. a) $y = -3^x + 4$
 b) $y = \frac{1}{2} \cdot \left(\frac{1}{3}\right)^x - 1$
46. a) $y = 5 \cdot 2^x - 20$
 b) $y = 6 \cdot \left(\frac{1}{6}\right)^x - 6$
47. $f(x) = -3^x + 2$
48. $f(x) = \left(\frac{1}{2}\right)^x - 4$

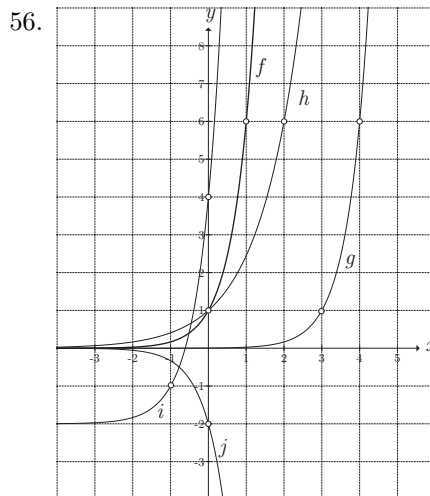
49. g : Shift by -2 in the x -direction;
 $y = 3^{x+2}$
 h : Reflection on the y -axis;
 $y = \left(\frac{1}{3}\right)^x$
 i : Shift by -4 in the y -direction;
 $y = 3^x - 4$
 j : Reflection on the x -axis;
 $y = -3^x$

51. a) Reflection on the y -axis
 b) Shift by -1 in the x -direction and by -2 in the y -direction
 c) Stretching in the y -direction by a factor of 2
 d) Compression in x -direction with factor $\frac{1}{3}$
53. a) E.g. compression in the x -direction with factor $\frac{1}{2}$, stretching in the y -direction with factor 2, followed by a shift in y -direction by -1
 b) E.g. stretching in x -direction with factor 4



50. g : Shift by 3 in the y -direction;
 $y = \left(\frac{1}{4}\right)^x + 3$
 h : Shift by 2 in the x -direction;
 $y = \left(\frac{1}{4}\right)^{x-2}$
 i : Reflection on the x -axis;
 $y = -\left(\frac{1}{4}\right)^x$
 j : Reflection at the origin;
 $y = -4^x$

52. a) Compression in x -direction with factor $\frac{1}{2}$
 b) Reflection at the origin
 c) Stretching with factor -3 in the y -direction
 d) Shift by -1 in the x -direction and by 3 in the y -direction
54. a) E.g. stretching in x -direction with factor $\frac{2}{3}$
 b) E.g. reflection at the origin and compression in the y -direction by a factor of $\frac{1}{2}$



57. $g(x) = 2^{x+2} = 2^x \cdot 2^2 = 4 \cdot 2^x$

59. a) Compression in y -direction with factor $\frac{1}{3}$
 b) Stretching in x -direction with factor 2

61. a) $y = 4^{x-3} + 5$ c) $y = 3 \cdot 4^x$
 b) $y = -4^x$ d) $y = 4^{-(x+2)}$

63. a) $y = -2^x$ c) $y = \left(\frac{1}{2}\right)^{3x} + 2$
 b) $y = 3 \cdot \left(\frac{1}{2}\right)^{x-2}$ d) $y = \frac{1}{4} \left(\left(\frac{1}{2}\right)^x + 1\right)$

58. $g(x) = 2^{2x} = 2^{x \cdot 2} = (2^x)^2$

60. a) reflection on the y -axis
 b) shift by 1 in negative x -direction

62. a) $y = 4^{-x}$ c) $y = 4^{\frac{1}{2}x} = 2^x$
 b) $y = 4^{x+2} + 6$ d) $y = -4^x + 2$

64. a) $y = 5 \left(\left(\frac{1}{2}\right)^x - 4\right)$ c) $y = \left(\frac{1}{2}\right)^{\frac{1}{2}x} + 4$
 b) $y = -2^x + 2$ d) $y = \left(\frac{1}{2}\right)^{-\frac{1}{3}x}$

65. a) asymptote: $y = 4$
 y -intercept: 5
 b) asymptote: $y = 0$
 y -intercept: -1
 c) asymptote: $y = 0$
 y -intercept: 2
 d) asymptote: $y = 2$
 y -intercept: 127
66. a) asymptote: $y = 0$
 y -intercept: 3
 b) asymptote: $y = 0$
 y -intercept: -4
 c) asymptote: $y = 0$
 y -intercept: 3
 d) asymptote: $y = 0$
 y -intercept: 1
67. a) to the left, from above
 b) to the right, from below
 c) to the right, from above
 d) to the left, from below
68. a) to the right, from above
 b) to the right, from below
 c) to the right, from above
 d) to the right, from below
69. a) $y = 2 \cdot 2^x + 4$
 b) $y = 2 \cdot \left(\frac{1}{3}\right)^x + 1$
 c) $y = 3 \cdot 4^x - 2$
 d) $y = \frac{1}{2} \cdot 5^x + 2$
70. a) $y = -3 \cdot 2^x + 4$
 b) $y = \frac{1}{5} \cdot 6^x$
 c) $y = -2 \cdot 5^x + 6$
 d) $y = 9 \cdot \left(\frac{1}{2}\right)^x - 3$
71. $y = -\frac{2}{5} \cdot 2^x + \frac{7}{5}$
72. $y = 3 \cdot \left(\frac{1}{4}\right)^x - 5$
73. a)

smi	smd
☐	☒

 c)

smi	smd
☒	☐

 b)

☒	☐
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 d)

☐	☒
--------------------------	-------------------------------------
74. a)

smi	smd
☐	☒

 c)

smi	smd
☒	☐

 b)

☐	☒
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 d)

☒	☐
-------------------------------------	--------------------------
75. a)

true	false
☐	☒

 c)

true	false
☐	☒

 b)

☒	☐
-------------------------------------	--------------------------

 d)

☐	☒
--------------------------	-------------------------------------
76. a)

true	false
☒	☐

 c)

true	false
☒	☐

 b)

☒	☐
-------------------------------------	--------------------------

 d)

☒	☐
-------------------------------------	--------------------------
77. a)

true	false
☐	☒

 c)

true	false
☒	☐

 b)

☐	☒
--------------------------	-------------------------------------

 d)

☐	☒
--------------------------	-------------------------------------
78. a)

true	false
☒	☐

 c)

true	false
☒	☐

 b)

☐	☒
--------------------------	-------------------------------------

 d)

☐	☒
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10.2 Exponential Growth and Decay

79. a) after 1 h: 4000,
 after 2 h: 8000,
 after 3 h: 16 000,
 after 4 h: 32 000,
 after 5 h: 64 000
 b) 4, 4
 c) $2000 \cdot 2^n$
 d) 2828, 5657
 e) 41.42 %
 f) 1.26
80. a) after 1 h: $5 \cdot 10^{15}$,
 after 2 h: $2.5 \cdot 10^{15}$,
 after 3 h: $1.25 \cdot 10^{15}$,
 after 4 h: $6.25 \cdot 10^{14}$,
 after 5 h: $3.125 \cdot 10^{14}$
 b) $\frac{1}{8}, \frac{1}{8}$
 c) $10^{16} \cdot \left(\frac{1}{2}\right)^n$
 d) $7.07 \cdot 10^{15}, 3.54 \cdot 10^{15}$
 e) 29.29 %
 f) 0.79

81. a) after 2 h: 37.89 mg,
after 3 h: 32.99 mg,
after 4 h: 28.72 mg,
after 5 h: 25 mg,
after 8 h: 16.49 mg
b) 24.21 %, 34.02 %
c) $50 \cdot 2^{-0.2n}$
d) 39.68 mg, 4.96 mg
e) 0.79
f) After 6.6 hours, i.e. during the 7th hour.
g) Theoretically infinitely long

83. 18 years

85.

t	-4	-3	0	1	3	5
$f(t)$	$\frac{625}{432}$	$\frac{125}{72}$	3	$\frac{18}{5}$	$\frac{648}{125}$	$\frac{23\,328}{3125}$

87. a)

t	0	1	2	3	4	6
$f(t)$	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{9}{2}$	$\frac{27}{2}$	$\frac{243}{2}$

- b) $f(t) = \frac{1}{6} \cdot 3^t$
c) 3
d) 9
e) $\sqrt{3}$

89. a)

t	-3	1	4	6	9	11
$f(t)$	$\frac{3}{256}$	$\frac{3}{16}$	1.5	6	48	192

- b) $f(t) = \frac{3}{32} \cdot 2^t$
c) 2
d) 512
e) $\sqrt{2}$

91. 16 s

93.

t	-4	-3	0	1	3	5
$f(t)$	24.88	20.74	12	10	6.94	4.82

95. a)

t	0	1	2	2.5	3	8
$f(t)$	40	20	10	$5\sqrt{2}$	5	$\frac{5}{32}$

- b) $f(t) = 40 \cdot 0.5^t$
c) $\frac{1}{2}$
d) $\frac{1}{32}$
e) $\frac{\sqrt{2}}{2}$

82. a) after 4 weeks: 192 cm^2 ,
after 8 weeks: 768 cm^2 ,
after 20 weeks: $49\,152 \text{ cm}^2$

- b) 7
c) 300 %
d) $48 \cdot 2^{\frac{t}{2}}$
e) 68 cm^2 ,
 136 cm^2 ,
 1086 cm^2
f) 64
g) 12 cm^2

84. 40 days, 45 days, 50 days

86.

t	-4	-3	0	1	3	5
$f(t)$	$\frac{128}{625}$	$\frac{32}{125}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{125}{128}$	$\frac{3125}{2048}$

88. a)

t	0	1	2	3	4	6
$f(t)$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{9}{8}$	$\frac{27}{16}$	$\frac{81}{32}$	$\frac{729}{128}$

- b) $f(t) = \frac{1}{2} \cdot 1.5^t$
c) $\frac{3}{2}$
d) $\frac{9}{4}$
e) $\frac{\sqrt{6}}{2}$

90. a)

t	-1	0	2	3	5	10
$f(t)$	$\frac{32}{243}$	$\frac{16}{81}$	$\frac{4}{9}$	$\frac{2}{3}$	$\frac{3}{2}$	$\frac{729}{64}$

- b) $f(t) = \frac{16}{81} \cdot 1.5^t$
c) $\frac{3}{2}$
d) $\frac{81}{16}$
e) $\frac{\sqrt{6}}{2}$

92. after 17 months

94.

t	-4	-3	0	1	3	5
$f(t)$	40.5	27	8	5.33	2.37	1.05

96. a)

t	0	2	2.5	3	6	8
$f(t)$	$\frac{243}{8}$	$\frac{27}{2}$	$\frac{9}{2}\sqrt{6}$	9	$\frac{8}{3}$	$\frac{32}{27}$

- b) $f(t) = \frac{243}{8} \cdot \left(\frac{2}{3}\right)^t$
c) $\frac{2}{3}$
d) $\frac{8}{27}$
e) $\frac{\sqrt{6}}{3}$

97. a)

t	-3	1	4	6	9	11
$f(t)$	$\frac{3}{256}$	$\frac{3}{16}$	1.5	6	48	192
- b) $f(t) = \frac{3}{32} \cdot 2^t$
- c) 2
- d) 512
- e) $\sqrt{2}$
99. 64, 4
101. 0.25, 0.5
103. a)

true	false
☒	☐

 c)

true	false
☐	☒
- b)

☐	☒
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 d)

☒	☐
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105. a) $a = 1.2$
- b) $a = 0.12$
107. a) +40 %
- b) -75 %
109. a) 512
- b) $2\sqrt{2}$
- c) $8^{\frac{1}{12}}$
- d) $16\sqrt{2}$
111. a) $\frac{5}{2}$
- b) $\frac{2}{3}$
113. a) $\frac{3}{4}$
- b) 57.81 %
- c) 2027
115. a) 4530
- b) 10 %
117. a) 42 256 m³
- b) 76 318 m³
- c) 3 %
119. a) Fr. 4 251 998.—
- b) Fr. 354 437.—
- c) 22 %
121. a) $f(t) = 4820 \cdot \sqrt[16]{2}^t$
- b) 7433
- c) 27 266
- d) 2027
123. a) $f(t) = 4.8 \cdot 0.75^t$
- b) 1.52 cm, 0.85 cm, 0.27 cm
- c) 17.80 %
98. a)

t	-3	1	4	6	9	11
$f(t)$	$\frac{3}{256}$	$\frac{3}{16}$	1.5	6	48	192
- b) $f(t) = \frac{3}{32} \cdot 2^t$
- c) 2
- d) 512
- e) $\sqrt{2}$
100. 27, 3
102. $\frac{4}{9}, \frac{2}{3}$
104. a)

true	false
☒	☐

 c)

true	false
☐	☒
- b)

☐	☒
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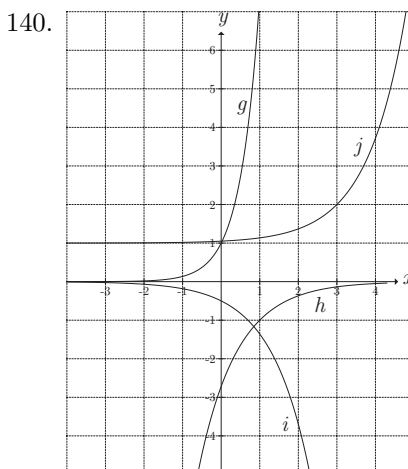
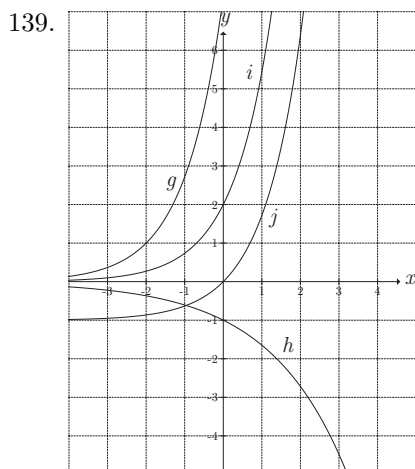
 d)

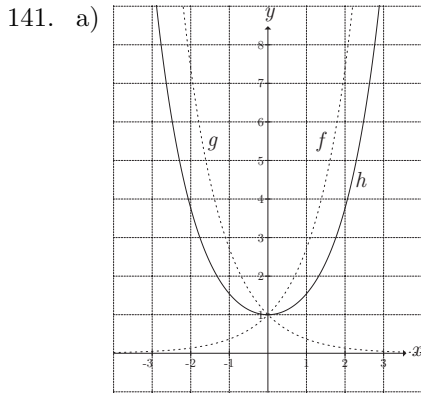
☒	☐
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106. a) $a = 2.25$
- b) $a = 0.87$
108. a) +150 %
- b) -70 %
110. a) $\frac{9}{16}$
- b) $\frac{\sqrt{3}}{2}$
- c) $\frac{3\sqrt{3}}{8}$
- d) $\frac{27}{64}$
112. a) 1
- b) $\frac{1}{2}$
114. a) $\frac{4}{5}$
- b) 48.8 %
- c) 2029
116. a) 620
- b) 41.42 %
118. a) 2679
- b) 146 600
- c) 56 %
120. a) Fr. 56 419.—
- b) Fr. 2112.—
- c) 28 %
122. a) $f(t) = 26\,350 \cdot \sqrt[30]{3}^t$
- b) 35 319
- c) 105 958
- d) 11 773
124. a) $f(t) = 500 \cdot 0.86^t$
- b) 318 mg, 174 mg
- c) 47.04 %

125. a) $f(t) = 20 \cdot 0.9^t$
 b) 13.1 Lux
 c) 1.6 Lux
 d) 34.87 %
126. a) $f(h) = 1.013 \cdot 0.88^h$
 b) 958 hPa
 c) 608 hPa
 d) 32.27 %
127. a) $f(n) = 10\,000 \cdot 1.015^n$
 b) $f(10) = 11\,605.41$,
 $f(20) = 13\,468.55$,
 $f(50) = 21\,052.42$
128. a) $f(n) = 15\,000 \cdot 1.011^n$
 b) $f(10) = 16\,734.12$,
 $f(20) = 18\,668.71$,
 $f(50) = 25\,920.84$
129. 2.5 %
130. 2.8 %

10.3 The Number e

131. a) Fr. 2.—
 b) Fr. 2.25
 c) Fr. 2.44
 d) Fr. 2.61
 e) Fr. 2.7145
 f) Fr. 2.718 12
 g) Fr. 2.718 23
 h) Fr. $1 \cdot \left(1 + \frac{1}{n}\right)^n$
132. a) Fr. 200.—
 b) Fr. 225.—
 c) Fr. 244.—
 d) Fr. 261.—
 e) Fr. 271.45
 f) Fr. 271.8 12
 g) Fr. 271.8 23
 h) Fr. $100 \cdot \left(1 + \frac{1}{n}\right)^n$
133. a) 2.25, -17.23 %
 b) 2.59, -4.58 %
 c) 2.7048, -0.495 %
 d) 2.716 92, -0.049 954 %
134. a) 2.718 145 927, -0.004 999 54 %
 b) 2.718 268 237, -0.000 499 995 %
 c) 2.718 280 469, -0.000 05 %
 d) 2.718 281 693, -0.000 005 %
135. a) $2.708\bar{3}$, -0.37 %
 b) 2.718 278 77, -0.000 11 %
136. a) $2.\bar{6}$, -1.898 82 %
 b) 2.718 281 828, ca. 0 %
137. $f: x \mapsto y_5$, $g: x \mapsto y_1$, $h: x \mapsto y_4$, $i: x \mapsto y_2$, $j: x \mapsto y_6$, $k: x \mapsto y_3$
138. $f: x \mapsto y_6$, $g: x \mapsto y_1$, $h: x \mapsto y_3$, $i: x \mapsto y_4$, $j: x \mapsto y_2$, $k: x \mapsto y_5$





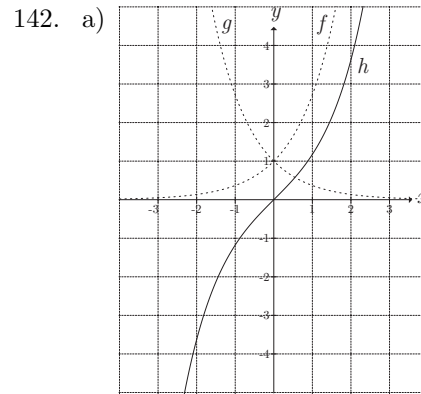
b) $\cosh(-x) = \frac{e^{-x} + e^{-(-x)}}{2} = \frac{e^{-x} + e^x}{2} = \frac{e^x + e^{-x}}{2} = \cosh(x)$

c) Monotonically decreasing on $\mathbb{R}^-$, monotonically increasing on $\mathbb{R}^+$

143. a) $\tanh(x) = \frac{\frac{e^x - e^{-x}}{2}}{\frac{e^x + e^{-x}}{2}} = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{e^{2x} - 1}{e^{2x} + 1}$

b) $x = 0$

c) odd



b) $\sinh(-x) = \frac{e^{-x} - e^{-(-x)}}{2} = \frac{e^{-x} - e^x}{2} = -\frac{e^x - e^{-x}}{2} = -\sinh(x)$

c) monotonically increasing on the entire domain $\mathbb{R}$

144. a) $\coth(x) = \frac{\frac{e^x + e^{-x}}{2}}{\frac{e^x - e^{-x}}{2}} = \frac{e^x + e^{-x}}{e^x - e^{-x}} = \frac{e^{2x} + 1}{e^{2x} - 1} = \frac{e^{2x} - 1}{e^{2x} - 1} + \frac{2}{e^{2x} - 1} = 1 + \frac{2}{e^{2x} - 1}$

b) $\coth(x) = \frac{e^x + e^{-x}}{e^x - e^{-x}}$
Both summands in the numerator are always positive

c) odd

145. a) $\cosh^2(x) - \sinh^2(x) = \left(\frac{e^x + e^{-x}}{2}\right)^2 - \left(\frac{e^x - e^{-x}}{2}\right)^2$
 $= \frac{1}{4}(e^{2x} + 2 + e^{-2x} - e^{2x} + 2 - e^{-2x})$
 $= \frac{1}{4}(2 + 2) = 1$

b) $\cosh(x)\cosh(y) + \sinh(x)\sinh(y) = \frac{e^x + e^{-x}}{2} \cdot \frac{e^y + e^{-y}}{2} + \frac{e^x - e^{-x}}{2} \cdot \frac{e^y - e^{-y}}{2}$
 $= \frac{e^{x+y} + e^{x-y} + e^{-x+y} + e^{-x-y} + e^{x-y} - e^{x+y} - e^{-x+y} + e^{-x-y}}{4}$
 $= \frac{1}{4}(2e^{x+y} + 2e^{-x-y})$
 $= \frac{1}{2}(e^{x+y} + e^{-(x+y)})$
 $= \cosh(x+y)$
 $\cosh(x)\cosh(y) - \sinh(x)\sinh(y) = \frac{e^x + e^{-x}}{2} \cdot \frac{e^y + e^{-y}}{2} - \frac{e^x - e^{-x}}{2} \cdot \frac{e^y - e^{-y}}{2}$
 $= \frac{e^{x+y} + e^{x-y} + e^{-x+y} + e^{-x-y} - e^{x-y} - e^{x+y} - e^{-x+y} + e^{-x-y}}{4}$
 $= \frac{1}{4}(2e^{x-y} + 2e^{-x+y})$
 $= \frac{1}{2}(e^{x-y} + e^{-(x-y)})$
 $= \cosh(x-y)$

$$\begin{aligned}
 146. \quad \text{a) } \cosh^2(x) + \sinh^2(x) &= \frac{1}{4} (e^{2x} + 2 + e^{-2x} + e^{2x} - 2 + e^{-2x}) \\
 &= \frac{1}{4} (2e^{2x} + 2e^{-2x}) \\
 &= \frac{1}{2} (e^{2x} + e^{-2x}) \\
 &= \cosh(2x)
 \end{aligned}$$

$$\begin{aligned}
 \text{b) } \sinh(x) \cosh(y) + \cosh(x) \sinh(y) &= \frac{e^x - e^{-x}}{2} \cdot \frac{e^y + e^{-y}}{2} + \frac{e^x + e^{-x}}{2} \cdot \frac{e^y - e^{-y}}{2} \\
 &= \frac{e^{x+y} + e^{x-y} - e^{-x+y} - e^{-x-y} + e^{x+y} - e^{x-y} + e^{-x+y} + e^{-x-y}}{4} \\
 &= \frac{1}{4} (2e^{x+y} - 2e^{-x-y}) \\
 &= \frac{1}{2} (e^{x+y} - e^{-(x+y)}) \\
 &= \sinh(x+y) \\
 \sinh(x) \cosh(y) - \cosh(x) \sinh(y) &= \frac{e^x - e^{-x}}{2} \cdot \frac{e^y + e^{-y}}{2} - \frac{e^x + e^{-x}}{2} \cdot \frac{e^y - e^{-y}}{2} \\
 &= \frac{e^{x+y} + e^{x-y} - e^{-x+y} - e^{-x-y} - e^{x+y} + e^{x-y} - e^{-x+y} + e^{-x-y}}{4} \\
 &= \frac{1}{4} (2e^{x-y} - 2e^{-x+y}) \\
 &= \frac{1}{2} (e^{x-y} - e^{-(x-y)}) \\
 &= \sinh(x-y)
 \end{aligned}$$

10.4 Logarithms and Logarithm Function

$$\begin{array}{lll}
 147. \quad \text{a) } 10^x = 100; 2 & \text{c) } 3^x = 27; 3 & 148. \quad \text{a) } 10^x = 1000; 3 \quad \text{c) } 81^x = 9; \frac{1}{2} \\
 \text{b) } 16^x = 2; \frac{1}{4} & \text{d) } 2^x = \frac{1}{8}; -3 & \text{b) } 2^x = 8; 3 \quad \text{d) } 2^x = \frac{1}{4}; -2
 \end{array}$$

$$\begin{array}{ll}
 149. \quad \text{a) } x = \log_{10} \frac{1}{10} = -1 & 150. \quad \text{a) } x = \log_{10} \frac{1}{100} = -2 \\
 \text{b) } x = \log_{\frac{1}{3}} 243 = -5 & \text{b) } x = \log_2 \frac{1}{32} = -5 \\
 \text{c) } x = \log_{64} 2 = \frac{1}{6} & \text{c) } x = \log_9 3 = \frac{1}{2} \\
 \text{d) } x = \log_4 \frac{1}{16} = -2 & \text{d) } x = \log_{\frac{1}{2}} 8 = -3
 \end{array}$$

$$\begin{array}{lll}
 151. \quad \text{a) } 2 & \text{c) } 4 & 152. \quad \text{a) } 3 \quad \text{c) } \frac{1}{4} \\
 \text{b) } \frac{1}{2} & \text{d) } 3 & \text{b) } \frac{1}{2} \quad \text{d) } 2 \\
 153. \quad \text{a) } -2 & \text{c) } -3 & 154. \quad \text{a) } -3 \quad \text{c) } -\frac{1}{5} \\
 \text{b) } -1 & \text{d) } -1 & \text{b) } -2 \quad \text{d) } -4 \\
 155. \quad \text{a) } \frac{1}{2} & \text{c) } \text{not defined} & 156. \quad \text{a) } \frac{1}{6} \quad \text{c) } \frac{1}{2} \\
 \text{b) } \text{not defined} & \text{d) } -3 & \text{b) } -\frac{1}{2} \quad \text{d) } \text{not defined} \\
 157. \quad \text{a) } 3 & \text{c) } 0 & 158. \quad \text{a) } -2 \quad \text{c) } \text{not defined} \\
 \text{b) } -\frac{1}{8} & \text{d) } -2 & \text{b) } 2 \quad \text{d) } 0
 \end{array}$$

$$\begin{array}{ll}
 159. \quad \text{vessel 1: } \mathcal{D}, \mathcal{F} & 160. \quad \text{vessel 1: } \mathcal{B}, \mathcal{G} \\
 \text{vessel 2: } \mathcal{A}, \mathcal{B} & \text{vessel 2: } \mathcal{A}, \mathcal{D}, \mathcal{E}, \mathcal{F}, \mathcal{H} \\
 \text{vessel 3: } \mathcal{C}, \mathcal{E}, \mathcal{G}, \mathcal{H} & \text{vessel 3: } \mathcal{C}
 \end{array}$$

$$\begin{array}{lll}
 161. \quad \text{a) } 3 & \text{c) } \frac{1}{2} & 162. \quad \text{a) } -\frac{1}{2} \quad \text{c) } -1 \\
 \text{b) } -2 & \text{d) } -2 & \text{b) } -2 \quad \text{d) } 3 \\
 163. \quad \text{a) } -2 & \text{c) } -1 & 164. \quad \text{a) } -\frac{1}{3} \quad \text{c) } 4 \\
 \text{b) } -4 & \text{d) } -\frac{1}{3} & \text{b) } -2 \quad \text{d) } -\frac{1}{2}
 \end{array}$$

165. a) 7 c) $\frac{1}{2}$ 166. a) 5 c) $\frac{1}{2}$
b) -4 d) $-\frac{1}{2}$ b) -3 d) $-\frac{1}{3}$
167. a) $\frac{1}{2}$ c) $\frac{2}{5}$ 168. a) $\frac{3}{2}$ c) 15
b) -4 d) 2 b) -2 d) $-\frac{4}{3}$
169. a) -108 c) $\frac{3}{8}$ 170. a) $-\frac{20}{3}$ c) $\frac{1}{2}$
b) $\frac{1}{15}$ d) -7 b) -6 d) $\frac{5}{2}$
171. a) $\frac{3}{2}$ c) $\frac{4}{3}$ 172. a) $\frac{9}{2}$ c) $\frac{3}{4}$
b) $-\frac{2}{3}$ d) $\frac{5}{2}$ b) $-\frac{3}{2}$ d) $\frac{5}{4}$
173. a) $\frac{3}{2}$ c) $-\frac{2}{3}$ 174. a) $-\frac{4}{3}$ c) $\frac{3}{2}$
b) $-\frac{4}{3}$ d) $\frac{3}{4}$ b) $\frac{3}{2}$ d) $-\frac{5}{6}$
175. a) 4 c) not defined 176. a) $\frac{3}{8}$ c) $\frac{1}{5}$
b) $\frac{18}{7}$ d) $-\frac{18}{5}$ b) 8 d) -1
177. $\mathcal{D} < \mathcal{E} < \mathcal{B} < \mathcal{C} < \mathcal{F} < \mathcal{A} < \mathcal{H} < \mathcal{G}$ 178. $\mathcal{A} < \mathcal{H} < \mathcal{D} < \mathcal{C} < \mathcal{E} < \mathcal{G} < \mathcal{F} < \mathcal{B}$
179. a) $-\frac{1}{2}$ c) $\frac{3}{2}$ 180. a) $\frac{1}{12}$ c) $-\frac{11}{16}$
b) 14 d) $\frac{4}{3}$ b) $\frac{1}{6}$ d) 10
181. a) $-\frac{5}{2}$ c) $\frac{21}{4}$ 182. a) $\frac{3}{5}$ c) $-\frac{7}{16}$
b) $\frac{4}{3}$ d) $-\frac{15}{2}$ b) $\frac{4}{3}$ d) $\frac{2}{5}$
183. a) 6 c) $\frac{5}{4}$ 184. a) 9 c) $-\frac{2}{9}$
b) -3 d) $-\frac{5}{3}$ b) $\frac{6}{7}$ d) not defined
185. a) 8 c) -12 186. a) -11 c) 22
b) $-\frac{3}{4}$ d) $\frac{5}{3}$ b) $\frac{3}{7}$ d) $\frac{5}{4}$
187. a) 3 c) 9 188. a) -4 c) 5.5
b) $-\frac{4}{5}$ d) -5 b) $\frac{9}{4}$ d) $\frac{11}{4}$
189. a) 2 c) 32 190. a) $\frac{1}{625}$ c) 625
b) 216 d) 144 b) 9 d) $\frac{1}{64}$
191. a) $\frac{1}{3}$ c) -2 192. a) $-\frac{1}{2}$ c) $-\frac{1}{4}$
b) $-\frac{1}{3}$ d) 0 b) -2 d) 1
193. a) 256 c) 529 194. a) 8 c) $\frac{1}{625}$
b) 16 d) 10 b) 0.001 d) 4.5
195. a) 1.953 45 e) 3.267 52 196. a) 2.957 05 e) 4.179 19
b) -3.095 90 f) 6.048 74 b) -2.931 16 f) -3.154 43
c) 6.643 86 g) 3.758 15 c) 8.965 78 g) -5.806 25
d) -9.965 78 h) -2.191 79 d) -7.965 78 h) 5.681 24

- | | | true | false | | true | false | | true | false | | |
|------|--|---|-------------------------------------|----|-------------------------------------|--------------------------|------|---|--|-------------------------------------|-----------------|
| 197. | a) | ☐ | ☒ | c) | ☒ | ☐ | 198. | a) | ☒ | ☐ | |
| | b) | ☐ | ☒ | d) | ☒ | ☐ | | b) | ☐ | ☒ | |
| 199. | a) | 32 | | c) | 9 | | 200. | a) | 16 | c) | 2 |
| | b) | 49 | | d) | 40 | | | b) | 125 | d) | $\frac{1}{4}$ |
| 201. | a) | $\frac{1}{5}$ | | c) | $\frac{1}{36}$ | | 202. | a) | $\frac{121}{4}$ | c) | $\frac{1}{8}$ |
| | b) | 160 | | d) | 2 | | | b) | 2 | d) | $\frac{81}{16}$ |
| 203. | a) | 0 | | c) | 27 | | 204. | a) | -1 | c) | 12 |
| | b) | 0 | | d) | 4 | | | b) | 1 | d) | 3 |
| 205. | a) | 125 | | c) | 6 | | 206. | a) | $\frac{\sqrt{3}}{3}$ | c) | $-\frac{1}{2}$ |
| | b) | $\frac{2e}{3}$ | | d) | $\frac{3}{5}$ | | | b) | 16 | d) | 4.5 |
| 207. | a) | 9 | | c) | 3 | | 208. | a) | 4 | c) | 256 |
| | b) | 17 | | d) | 15 | | | b) | 8 | d) | 49 |
| 209. | a) | $\mathbb{X} = \mathbb{R}^+ \setminus \{1\};$ | 25 | | | | 210. | a) | $\mathbb{X} = \mathbb{R}^+ \setminus \{1\};$ | 3 | |
| | b) | $\mathbb{X} = \mathbb{R}^+ \setminus \{1\};$ | $\frac{4}{x^3}$ | | | | | b) | $\mathbb{X} =] - \infty, 3[\cup] 3, 4[;$ | 4 | |
| | c) | $\mathbb{X} =] - 1, 0[\cup] 0, \infty[;$ | $\frac{3}{2}$ | | | | | c) | $\mathbb{X} = \mathbb{R}^+ \setminus \{\frac{1}{2}\};$ | $\frac{1}{2}$ | |
| | d) | $\mathbb{X} =]\frac{1}{3}, \frac{2}{3}[\cup]\frac{2}{3}, \infty[;$ | 2 | | | | | d) | $\mathbb{X} = \mathbb{R}^+ \setminus \{1\}$ | $\frac{\sqrt{x}}{6}$ | |
| 211. | a) | x | | c) | x^x | | 212. | a) | $4x$ | c) | 0 |
| | b) | 0 | | d) | 4 | | | b) | x^{x-1} | d) | 9 |
| 213. | a) | $\mathbb{X} =] - 3, \infty[;$ | -2 | | | | 214. | a) | $\mathbb{X} =] - \frac{1}{2}, \infty[;$ | 2 | |
| | b) | $\mathbb{X} =]\frac{5}{3}, \infty[;$ | 5 | | | | | b) | $\mathbb{X} = \mathbb{R}^+;$ | $5^{\frac{5}{3}}$ | |
| | c) | $\mathbb{X} = \mathbb{R} \setminus \{0\};$ | $\pm \frac{1}{3}$ | | | | | c) | $\mathbb{X} = \mathbb{R} \setminus \{-2\};$ | 2, -6 | |
| | d) | $\mathbb{X} = \mathbb{R}^+;$ | $\frac{1}{64}$ | | | | | d) | $\mathbb{X} = \mathbb{R}^+;$ | 162 | |
| 215. | wrong, $\lg 10^{1.5} = 1.5$, and $10^{1.5} \neq 45$ | | | | | | 216. | wrong, the base of every logarithm is a positive real number not equal to 1, so -3 cannot be the base of a logarithm. | | | |
| 217. | wrong, the base x must be positive. | | | | | | 218. | correct | | | |
| 219. | a) | $\log_a b + \log_a c + \log_a d$ | | | | | 220. | a) | $\log_a p + \log_a q$ | | |
| | b) | $\log_a b + \log_a c - \log_a d$ | | | | | | b) | $-\log_a p - \log_a q$ | | |
| | c) | $-\log_a b - \log_a c$ | | | | | | c) | $\log_a p + \log_a q - \log_a r - \log_a s$ | | |
| | d) | $\log_a b + \log_a c - \log_a d - \log_a f$ | | | | | | d) | $\log_a p + \log_a q + \log_a r - \log_a s$ | | |
| 221. | a) | $4 \log_a b$ | | | | | 222. | a) | $2 \log_a m$ | | |
| | b) | $-6 \log_a d$ | | | | | | b) | $-3 \log_a n$ | | |
| | c) | $\frac{1}{3} \log_a p$ | | | | | | c) | $-\frac{1}{2} \log_a r$ | | |
| | d) | $2 \log_a p + 3 \log_a q$ | | | | | | d) | $\frac{1}{2} \log_a r + 2 \log_a s$ | | |

223. a) $\log_a b - \log_a(c + d)$
 b) $\log_a b + 2\log_a(c + d)$
 c) $4\log_a b + 2\log_a c + \log_a d$
 d) $\frac{1}{2}\log_a b + \frac{1}{2}\log_a c - \frac{2}{3}\log_a d - \frac{1}{3}\log_a f$
224. a) $2\log_a(p + q) - \log_a r$
 b) $\log_a(p + q) - 3\log_a r - 2\log_a s$
 c) $\frac{1}{3}\log_a p + \frac{2}{3}\log_a q - 2\log_a r - 4\log_a s$
 d) $\frac{1}{4}\log_a p + \frac{3}{4}\log_a q - \frac{1}{2}\log_a r - \frac{1}{4}\log_a s$
225. a) $3 + \lg 4$
 b) $\lg 5 + \lg 3 + 4$
226. a) $7 + \lg 3$
 b) $\lg 4 + \lg 3 + 5$
227. a) 12
 b) 12
228. a) -13
 b) 12
229. a) $\log_a \frac{b}{c}$
 b) $\log_a bc$
230. a) $\log_a pq$
 b) $\log_a p^2$
231. a) $\log_a bc^2$
 b) $\log_a(b - c)^3$
232. a) $\log_a p^2 q^2$
 b) $\log_a \sqrt[3]{p + q}$
233. a) $\lg 300\,000$
 b) $\lg \frac{3}{50}$
234. a) $\lg 50\,000$
 b) $\lg \frac{4}{5}$
235. a)

true	false
☐	☒

 c)

true	false
☒	☐

 b)

☒	☐
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 d)

☐	☒
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236. a)

true	false
☒	☐

 c)

true	false
☐	☒

 b)

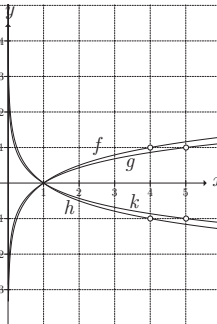
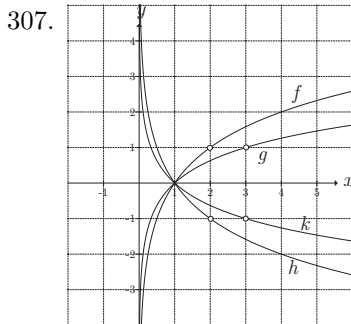
☐	☒
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 d)

☐	☒
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237. a) -1
 b) 55
238. a) 1176
 b) $-1 - \log_{17} 5$
239. 2, 3, 4, 6, 8, 9, 12, 16, 18, 24, 27, 32, 36, 48, 54
240. 2, 4, 5, 8, 10, 12, 16, 25, 32, 40
241. a) $2.782 \cdot 10^{334}$
 b) $1.911 \cdot 10^{2184}$
 c) $2.661 \cdot 10^{-458}$
242. a) $1.207 \cdot 10^{530}$
 b) $1.341 \cdot 10^{154}$
 c) $2.238 \cdot 10^{-1454}$
243. $\mathcal{D} < \mathcal{A} < \mathcal{F} < \mathcal{E} < \mathcal{B} < \mathcal{C}$
244. $\mathcal{D} < \mathcal{B} < \mathcal{C} < \mathcal{A} < \mathcal{F} < \mathcal{E}$
245. a) 22 338 618 digits
 b) 23 249 425 digits
246. a) 24 862 048 digits
 b) 41 024 320 digits
247. a) $\Pi(5) = 3$,
 $f(5) \approx 3.106\,67$,
 $\Delta = 3.6\%$
 b) $\Pi(10) = 4$,
 $f(10) \approx 4.342\,945$,
 $\Delta = 8.6\%$
 c) $\Pi(20) = 8$,
 $f(20) \approx 6.676\,164$,
 $\Delta = 16.5\%$
 d) $\Pi(100) = 25$,
 $f(100) \approx 21.714\,72$,
 $\Delta = 13.1\%$
248. a) $\Pi(1000) = 168$,
 $f(1000) \approx 144.764\,83$,
 $\Delta = 13.8\%$
 b) $\Pi(10\,000) = 1229$,
 $f(10\,000) \approx 1085.736\,21$,
 $\Delta = 11.7\%$
 c) $\Pi(100\,000) = 9592$,
 $f(100\,000) \approx 8685.889\,64$,
 $\Delta = 9.4\%$
 d) $\Pi(1\,000\,000) = 78\,498$,
 $f(1\,000\,000) \approx 72\,382.413\,65$,
 $\Delta = 7.8\%$

249. a) $\frac{\lg 12}{\lg 4}$ c) $\frac{\log_6 5}{\log_6 13}$ 250. a) $\frac{\ln 16}{\ln 21}$ c) $\frac{2}{1+\log_4 3}$
 b) $\frac{1}{\log_{18} 7}$ d) $\frac{\lg 19}{3}$ b) $\frac{\lg 11}{\lg 6}$ d) $\frac{\log_{17} 21}{\log_{17} 19}$
251. a) $\log_9 4$ c) $\log_3 12$ 252. a) $\log_{\sqrt{3}} 17$ c) $\lg 2$
 b) $\log_{14} 18$ d) $\log_9 20$ b) $\log_7 11$ d) $\log_9 24$
253. a) $\frac{1}{2}$ c) $\frac{3}{4}$ 254. a) 4 c) 2
 b) $\frac{4}{3}$ d) $\frac{5}{4}$ b) $\frac{1}{4}$ d) $\frac{8}{3}$
255. a) $\log_a b \cdot \log_b a = \frac{\lg b}{\lg a} \cdot \frac{\lg a}{\lg b} = 1$ $\square$ 256. a) $\log_{a^m} b : \log_a b = \frac{\lg b}{m \lg a} \cdot \frac{\lg a}{\lg b} = \frac{1}{m}$ $\square$
 b) $\log_a b^m : \log_a b = \frac{m \lg b}{\lg a} \cdot \frac{\lg a}{\lg b} = m$ $\square$ b) $\log_{a^m} b^m = \frac{m \lg b}{m \lg a} = \frac{\lg b}{\lg a} = \log_a b$ $\square$
257. ①-A, ②-C, ③-E, ④-D, ⑤-F, ⑥-B 258. ①-C, ②-A, ③-D, ④-F, ⑤-E, ⑥-B
259. a) 2 c) 9 260. a) 9 c) 8
 b) $\{\}$ d) 44 b) 2 d) 16
261. a) $\frac{1}{100}$ c) 3 262. a) $\frac{1}{8}$ c) 25
 b) $\sqrt{3}$ d) ± 121 b) 200 d) $2\sqrt{3}$
263. a) $\frac{1}{3}$ c) $\frac{1}{3}, \frac{8}{3}$ 264. a) $\frac{1}{2}$ c) -1
 b) $\frac{11}{7}$ d) 12 b) $\{\}$ d) 97
265. a) 4 c) $10^{2.6}$ 266. a) 2 c) $10^{1.2}$
 b) $\frac{2}{3}$ d) $\frac{1}{4}$ b) $-\frac{1}{3}$ d) 9
267. a) $\sqrt[3]{100}$ c) 8 268. a) $\sqrt{5}$ c) 9
 b) 729 d) 32 b) 49 d) 729
269. a) 1, 100 c) $\frac{1}{32}, 16$ 270. a) 1, 10 000 c) $\frac{1}{4}, 1024$
 b) 10 d) $\frac{1}{1000}, 1\,000\,000$ b) 9 d) $\frac{1}{512}, 2$
271. a) $\log_4 28$ c) $\log_{11} 33$ 272. a) $\log_6 108$ c) $\log_{15} \frac{72}{3}$
 b) $\log_3 \frac{9}{5}$ d) $\log_{\frac{1}{5}} \frac{11}{125}$ b) $\log_9 \frac{9}{7}$ d) $\log_{\frac{2}{3}} \frac{1}{9}$
273. a) $\log_7 \frac{27}{2}$ c) $\log_{15} \frac{4\sqrt{3}}{3}$ 274. a) $\log_5 \frac{12}{5}$ c) $\log_{\sqrt{6}} (5\sqrt{2})$
 b) $\log_{\frac{12}{5}} \frac{4}{25}$ d) $\log_{\frac{1}{12}} \frac{7}{9}$ b) $\log_{\frac{9}{4}} \frac{81}{2}$ d) $\log_{\frac{4}{35}} \frac{49}{2}$
275. a) $\log_3 10 \approx 2.096$ 276. a) $\log_6 4 \approx 0.7737$
 b) $\log_4 \frac{7}{4} \approx 0.4037$ b) $\log_5 \sqrt{15} \approx 0.8413$
 c) $\log_{0.6} \frac{3}{20} \approx 3.714$ c) $-\log_{0.8} 3 \approx 4.923$
 d) $\ln 5 \approx 1.609$ d) $\ln \frac{9}{e^3} \approx -0.8028$
277. a) $\frac{1}{\log_5 6} = \log_6 5 \approx 0.8982$ 278. a) $\frac{2}{\log_6 5} = \log_5 36 \approx 2.227$
 b) $\ln \sqrt[3]{\frac{7}{e^4}} \approx -0.6847$ b) $\ln \left(\frac{e^2 \sqrt{3}}{3} \right) = 2 - \frac{1}{2} \ln 3 \approx 1.451$
 c) $\log_6 2 \approx 0.3869$ c) $\log_3 4 \approx 1.262$
 d) $\log_3 2.4 \approx 0.7969$ d) $\lg \frac{16}{3} \approx 2.415$

279. a) $-4 + \lg 9 \approx -0.8301$
 b) $\frac{\lg 5}{5} = \lg \sqrt[5]{5} \approx 0.4644$
 c) $(\log_{11} 81)^2 \approx 3.359$
 d) $\log_3 \frac{1}{27 \cdot \sqrt[3]{2}} = -3 - \frac{1}{4} \log_3 2 \approx -3.158$
280. a) $-2 + \log_5 2 \approx -1.569$
 b) $\frac{\log_6 12}{6} = \log_6 \sqrt[6]{12} \approx 0.2311$
 c) $(\lg 12)^2 \approx 1.165$
 d) $\frac{2}{3} - \frac{2}{3} \log_5 3 = \log_5 \sqrt[3]{\frac{25}{9}} \approx 0.2116$
281. a) $\log_{\frac{3}{4}} 3 \approx -3.819$
 b) 1
 c) $\log_{\frac{3}{16}} 2 \approx -0.4141$
 d) $-\frac{1}{2 + \lg 3} \approx -0.2789$
282. a) $\log_9 6 \approx 0.8155$
 b) $\log_{\frac{3}{10}} \frac{80}{3} \approx -2.727$
 c) $\log_{\frac{4}{5}} \frac{16}{5} \approx -5.213$
 d) $\frac{5}{3}$
283. a) $\frac{1}{125}, 125$
 b) $\frac{1}{10\,000}, 10\,000$
 c) $\frac{1}{32}, 32$
 d) $\frac{1}{8}, 8$
284. a) $\frac{1}{36}, 36$
 b) $\frac{1}{10^{10}}, 10^{10}$
 c) $3^{\frac{3}{2}}, 3^{-\frac{3}{2}}$
 d) $\frac{1}{225}, 225$
285. a) 2, $\lg 9$
 b) $\frac{3}{2}$
 c) $\{\}$
 d) $\log_9 2, \log_9 6$
286. a) 4
 b) $\log_3 5, \log_3 8$
 c) 0
 d) 1, $\log_4 5$
287. a) $\square : \frac{1}{4}$
 solution: $\log_a \frac{1}{2}$
 b) $\square : \text{any number } y > 1.5$
 solution: $\log_a (4y - 6) - 3$
288. a) $\square : 1$
 solution: 0
 b) $\square : \text{any positive number } y$
 solution: $\frac{1}{2} (\log_a (5y + 1) + 1)$
289. a) 1, 4 b) 1, $3\sqrt{3}$
290. a) 1, 2 b) 1, $\sqrt{3}$
291. 10.5 years
292. 10.3 years
293. at the very end of 2042
294. 2040 (from mid-February)
295. 7.2 years
296. towards the end of 2040
297. 39.6 years
298. 43.9 years
299. 2039
300. 2048
301. 2.8 h
302. 0.93 h
303. 3.4 h
304. 1.35 h
305. 6.94 years
306. 9 years



309. $0 < a < \frac{1}{3} \vee a > 3$

310. $\frac{1}{4} < a < 1 \vee 1 < a < 2$

311. a) ② c) ③

312. a) ③ c) ①

b) ③ d) ③

b) ④ d) ③

313. a) $\frac{1}{2}$ b) $\sqrt{6}$

314. a) 3 b) 25

315. a) $\mathbb{X} = \mathbb{R}^+$
 $\mathbb{Y} = \mathbb{R}_0^+$ b) $\mathbb{X} = \mathbb{R} \setminus \{0\}$
 $\mathbb{Y} = \mathbb{R}$

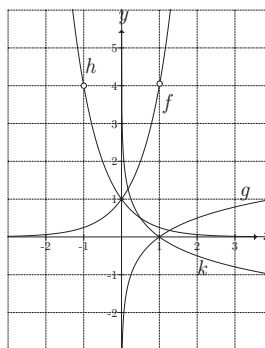
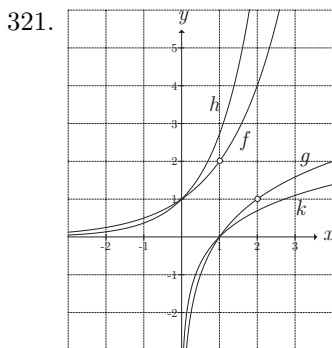
316. a) $\mathbb{X} = \mathbb{R}^-$ b) $\mathbb{X} = \mathbb{R} \setminus \{0\}$
 $\mathbb{Y} = \mathbb{R}$

317. a) $] -2, \infty[$ c) $] -\infty, 2[$
b) $] -\frac{1}{4}, \infty[$ d) $\mathbb{R} \setminus [-1, 1]$

318. a) $] -\infty, 5[$ c) $] -2, 2[$
b) $] -\frac{3}{2}, \infty[$ d) $] -\infty, 4[$

319. a) zero: $\frac{1}{2}$
 y -intercept: not defined
b) zero: -2
 y -intercept: $\log_4(5)$

320. a) zero: -1
 y -intercept: $-3 \lg(2)$
b) zero: $\frac{3}{4}$
 y -intercept: not defined



323. $a(x) = 2^x$, $b(x) = -2^x$,
 $c(x) = \left(\frac{1}{2}\right)^x$, $d(x) = -\left(\frac{1}{2}\right)^x$
 $f(x) = \lg(x)$, $g(x) = \lg(-x)$,
 $h(x) = \log_{\frac{1}{2}}(x)$, $i(x) = \log_{\frac{1}{2}}(-x)$

324. $a(x) = 3^x$, $b(x) = \log_3(x)$,
 $c(x) = \left(\frac{1}{3}\right)^x$, $d(x) = \log_3(-x)$
 $f(x) = -3^x$, $g(x) = -\left(\frac{1}{3}\right)^x$,
 $h(x) = \log_{\frac{1}{3}}(-x)$, $i(x) = \log_{\frac{1}{3}}(x)$

325. a) $f^{-1}(x) = \log_{\frac{2}{3}} x$
b) $f^{-1}(x) = 4^x$
c) $f^{-1}(x) = -\frac{1}{3} + \frac{1}{3} \log_8 x$
d) $f^{-1}(x) = \frac{1}{2} \cdot 5^x - \frac{7}{2}$

326. a) $f^{-1}(x) = \left(\frac{1}{2}\right)^x$
b) $f^{-1}(x) = \log_8 x$
c) $f^{-1}(x) = 2 - \lg x$
d) $f^{-1}(x) = -\frac{1}{5} + \frac{1}{5} \cdot 4^x$

327. ①- $\mathcal{D}$, ②- $\mathcal{A}$, ③- $\mathcal{E}$, ④- $\mathcal{F}$, ⑤- $\mathcal{B}$, ⑥- $\mathcal{C}$
328. ①- $\mathcal{E}$, ②- $\mathcal{B}$, ③- $\mathcal{E}$, ④- $\mathcal{A}$, ⑤- $\mathcal{D}$, ⑥- $\mathcal{C}$
329. g : shift by 2 in x -direction; $y = \lg(x - 2)$
 h : shift by 3 in y -direction; $y = \lg(x) + 3$
 i : reflection on the straight line $y = x$;
 $y = 2^x$
 j : mirroring on the x -axis and stretching on the x -axis with factor $\frac{1}{2}$;
 $y = \log_{\frac{1}{4}} x = -\frac{1}{2} \lg x$
330. g : shift by -1 in the x -direction;
 $y = \log_{\frac{1}{3}}(x + 1)$
 h : reflection on the y -axis; $y = \log_{\frac{1}{3}}(-x)$
 i : reflection on the straight line $y = x$;
 $y = \left(\frac{1}{3}\right)^x$
 j : mirroring on the x -axis and stretching on the x -axis with a factor of 2;
 $y = 2 \log_3 x = -2 \log_{\frac{1}{3}}(x)$
331. a) $\frac{1}{2}$
 b) $-\frac{3}{2}$
332. a) $\frac{2}{3}$
 b) $-\frac{1}{2}$
333. Compression in x -direction with factor $\frac{1}{2}$
 Shift by $\frac{1}{2}$ in the y -direction
334. Stretching in x -direction with factor 9
 Shift by -2 in the y -direction
335. $f(x) = \log_a x = \frac{\lg x}{\lg a} = \frac{1}{\lg a} \cdot \lg x$
 stretch factor: $k = \frac{1}{\lg a}$
- 336. $\log_a x = 0 \Rightarrow x = 1$ for every base a . Based on the previous task, each graph is a stretch with its own stretch factor on the x -axis of the function $y = \lg x$, so the graphs can only intersect on the x -axis.
337. a) $f(x) = 16 \cdot \ln(x) + 31$
 b) 53.2 human years
338. a) $f(x) = e^{\frac{x-31}{16}}$
 b) 1.28 dog years

10.5 Applications

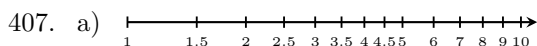
339. a) $f(t) = 6 \cdot 1.021^t$
 b) 10.09 million people
 c) 33.35 years
 d) 23.10 %
 e) 3.18 million
 f) 3.92 million
340. a) $f(t) = 3 \cdot 1.018^t$
 b) 6.70 million people
 c) 38.85 years
 d) 19.53 %
 e) 1.71 million
 f) 2.04 million
341. a) 0.17 %
 b) $f(t) = 81.8 \cdot \left(\frac{832}{818}\right)^{0.1t}$
 c) 86.07 million
 d) 85.30 to 86.85 million
342. a) 0.83 %
 b) $f(t) = 195.7 \cdot \left(\frac{2126}{1957}\right)^{0.1t}$
 c) 86.07 million
 d) 248.67 to 253.15 million
343. a) 1.38 %
 b) $f(t) = 1.05 \cdot e^{0.01366t}$
 c) 1.58 billion
 d) 2051
344. a) 0.89 %
 b) $f(t) = 7.2 \cdot e^{0.00888t}$
 c) 9.40 million
 d) 2079

345. a) 1.38 %
b) $f(t) = 3.7 \cdot e^{-0.0139 t}$
c) 2.12 million
d) 49.74 years
347. a) 1960-1970: 0.69 billion
1970-1980: 0.73 billion
1980-1990: 0.88 billion
1990-2000: 0.81 billion
2000-2010: 0.80 billion
2010-2020: 0.86 billion
b) 1960-1970: 2.079 %
1970-1980: 1.812 %
1980-1990: 1.825 %
1990-2000: 1.427 %
2000-2010: 1.234 %
2010-2020: 1.177 %
c) 11.06 billion
349. a) 304 007, 8541, 73
b) 6.07 h
c) 16.22 h
d) 3.49 h
351. a) $f(t) = 100 \text{ g} \cdot 0.97729^t$
b) 2.27 %
c) 100.2 years
d) 19.1 g
353. a) 38 213 years
b) 2.4 g
c) 0.045 g
355. a) 11.84 mg
b) 10.58 years
357. a) 2.27 %
b) 40.82 %
c) 130.39 years
359. a) 39.12 %
b) 103.25 years
361. a) 161.37 units
b) 12.62 days
363. 41.03 years
346. a) 3.45 %
b) $f(t) = 6000 \cdot e^{-0.035 t}$
c) 2972
d) 19.74 years
348. a) the largest: 1980-1990
the smallest: 1960-1970
b) the largest: 1960-1970
the smallest: 2010-2020
c) The forecast probably overestimates actual growth, as the table shows that the percentage increase has decreased over the decades. The world population does not grow exponentially indefinitely, e.g. there are not unlimited resources available.
350. a) $2.616 \cdot 10^9$, $2.56 \cdot 10^8$, 8
b) 1.34 h
c) 15.36 h
d) 50.66 min
352. a) $f(t) = 50 \text{ mg} \cdot 0.9172^t$
b) 8.28 %
c) 32.55 days
d) 17.72 mg
354. a) 8.936 billion years
b) 75 %
c) 10.37 billion years
356. a) 4.19 mg
b) 26.47 days
358. a) 49.649 996 kg
b) in 292.1 million years
c) 1627.04 million years
360. a) 34.79 %
b) mid-2079
362. a) 86.05 %
b) 321.35 days
364. 3.08 %

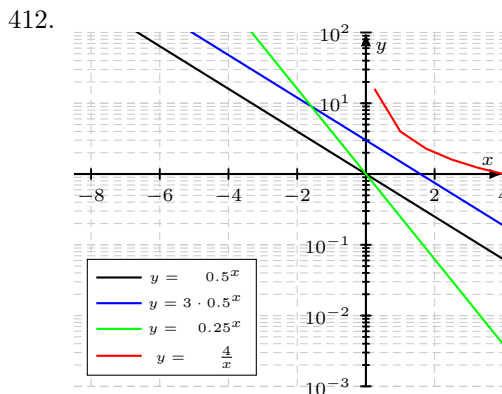
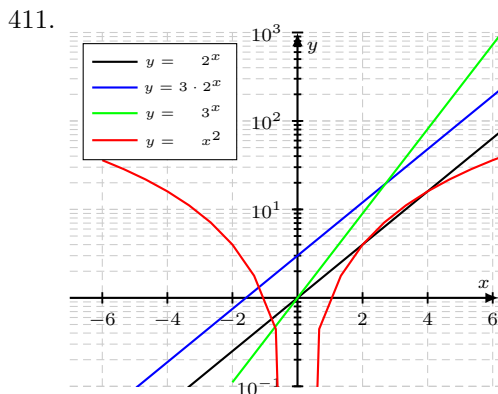
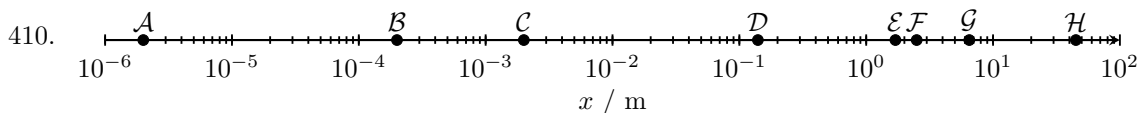
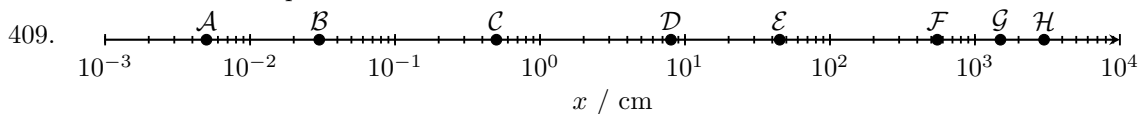
365. a) $1.79 \cdot 10^{10}$
b) 6600 years
c) 6000 years
d) The time of the dinosaurs is too long ago, the bones no longer contain a sufficient amount of ^{14}C . On the other hand, it is unclear how many ^{14}C atoms were contained in one gram of carbon at the time of the dinosaurs, and without such a starting value it is impossible to calculate.
367. around 3300 B.C.
369. a) 2.4 million years
b) 99.8917 %
371. a) 0.631 ppm
b) 74.92 %
c) 2.105 ppm, 2.445 ppm
d) 138.98 years
373. a) 1.82°C , 2.73°C
b) 23.13 years
375. a) 1600, 25 600
b) 23.25 days
377. a) $f(t) = 50 \cdot 2^{0.27844 t}$
b) 3.59 days
379. a) 2400
b) 6.84 days
c) 8 times
d) $f(t) = 150 \cdot 2^{0.4t}$
381. a) $f(t) = 80 \cdot 2^{\frac{t}{13.3}}$
b) 1824
c) 35.16 days
d) 227
383. a) $f(t) = 1500 \cdot 1.6^{\frac{t}{5}}$
b) 3840
c) 20.18 days
d) 156 %
e) 24.50 days
366. a) $5.3 \cdot 10^9$
b) 3560 years
c) $3.27 \cdot 10^{10}$
d) Stones contain at most tiny traces of ^{14}C .
368. with uncertainty ± 30 years: died between 3337 and 3282 B.C.
with Libby's figures: died between 3187 and 3132 B.C.
370. a) 2.9 million years
b) 0.009 75 %
372. a) 0.011 91 ppm
b) 80.52 %
c) 0.0169, 0.0910
d) 20.61 years
374. a) 4.51 years
b) 15.51 years
376. a) 400, 1600
b) 46.51 days
378. a) $f(t) = 50 \cdot 1.2129^t$
b) 8.34 days
380. a) 1920
b) 7.65 days
c) 6 times
d) $f(t) = 120 \cdot e^{0.277 t}$
382. a) $f(t) = 60 \cdot 2^{\frac{t}{19.6}}$
b) 501
c) 79.55 days
d) 122
384. a) $f(t) = 10 \cdot 1.35^{\frac{t}{3}}$
b) 74
c) 29.95 days
d) 171.92 %
e) 23.02 days

385. a) 51.2 %, 26.21 %
b) 3.11 years
387. a) $1.344 \cdot 10^9$, 9.15 years after 2000
b) 1500 %
c) 26.58 years
389. a) $f(t) = 10\,000 \cdot 1.0125^t$
b) Fr. 12 048.29
c) after 8 years
391. Fr. 2724.37
393. Fr. 222 741.13
395. Fr. 4414.71
397. Fr. 1 340 095.64
399. Fr. 3 747 610.28
401. Fr. 719 956 085.90
386. a) 78.37 %
b) 22.76 days
388. a) $f(t) = 8.5 \cdot 10^9 \cdot 2^{\frac{t}{2}}$
b) 2026 or 2027
c) 300 %
390. a) $f(t) = 20\,000 \cdot 1.0175^t$
b) Fr. 25 944.56
c) after 6 years
392. Fr. 5780.39
394. Fr. 186 166.17
396. Fr. 45 017.37
398. Fr. 1 851 453.47
400. Fr. 1 827 604.35
402. Fr. 171 227 409.70

403. a) linear: $m = 0.5, q = 5$,
 $f(t) = 0.5t + 5$
 exponential: $a = \left(\frac{3}{2}\right)^{0.1}, b = \frac{20}{3}$,
 $g(t) = \frac{20}{3} \cdot \left(\frac{3}{2}\right)^{\frac{t}{10}}$
 quadratic: $a = -\frac{1}{40}, b = \frac{5}{4}$,
 $h(t) = -\frac{1}{40}t^2 + \frac{5}{4}t$
 bounded: $a = \left(\frac{2}{3}\right)^{0.1}, b = \frac{45}{2}$,
 $i(t) = 25 - \frac{45}{2} \cdot \left(\frac{2}{3}\right)^{\frac{t}{10}}$
- b) $g(15) \approx 12.247, f(15) = 12.5$,
 $i(15) \approx 12.753, h(15) = 13.125$
- c) the sharpest slowdown in the quadratic model; growth also slowed slightly in the limited growth scenario
- d) Exponential model with $6.\bar{6}$
- e) $f(30) = 20, g(30) = 22.5$,
 $h(30) = 15, i(30) = 18.\bar{3}$
- f) approaches the growth bound S asymptotically
- g) Peak after 25 days, symmetrical progression, i.e. the culture would decline to 0 by day 50.
- h) Exponential model
- i) L.: 0.5 cm^2 , E.: 17.1 days, B.: 54.2 days
- j) The growth rate is initially proportional to the existing amount of algae, as there are sufficient nutrients and space for growth; therefore, the exponential model is the best fit.
 The limited model initially shows a rapid increase, similar to exponential growth. When space and nutrients become scarce, growth slows down and approaches the growth limit.
 Due to environmental conditions, the culture does not grow uniformly as assumed in the linear model. The culture's growth does not become negative, as in the quadratic model.
404. a) linear: $m = 4.8, q = -16$
 $f(t) = 4.8t - 16$
 exponential: $a = 2^{0.4}, b = 2$,
 $g(t) = 2 \cdot 2^{0.4t}$
 quadratic: $a = \frac{8}{25}, b = 0$
 $h(t) = \frac{8}{25}t^2$
 bounded: $a = 2^{-0.4}, b = 128$,
 $i(t) = 40 - 128 \cdot 2^{-0.4t}$
- b) $g(8) = 18.38$,
 $h(8) = 20.48$,
 $f(8) = 22.4$,
 $i(8) = 26.07$
- c) linear: 4.8 cm^2 per day, quadratic: 6.72 cm^2 per day.
 In the linear model, the growth rate is constant; in the quadratic model, growth accelerates.
- d) only in the restricted model
- e) The exponential model shows the fastest growth; by day 12, the calculated area is already greater than the area of the Petri dish.
- f) The negative initial values in the linear model ($f(0) = -16$) and in the bounded model ($i(0) = -88$) are unrealistic. The initial value of 0 in the quadratic model is also unrealistic.
- g) after 17.5 days
- h) The growth rate is initially proportional to the existing amount of algae, as there are sufficient nutrients and space for growth.
 At the very beginning, the limited model is unsuitable due to the negative starting value; thereafter, there is rapid growth, similar to exponential growth. When space and nutrients become scarce, growth slows down and approaches the growth limit.
405. a) same size; $\lg 2 - \lg 1 = \lg 2 - 0 = \lg 2$ and $\lg 4 - \lg 2 = \lg 2^2 - \lg 2 = 2 \lg 2 - \lg 2 = \lg 2$
 b) 4 times
 c) The logarithm of 0 is not defined.
 d) 10
 e) Add distance of 1 to 2 to position 3
406. a) Same size; $\lg 20 - \lg 10 = \lg\left(\frac{20}{10}\right) = \lg\left(\frac{2}{1}\right) = \lg 2 - \lg 1 = \lg 2$
 b) 5 times
 c) The logarithm of negative values is not defined.
 d) 1000
 e) Mirror the number 1 on the number 3



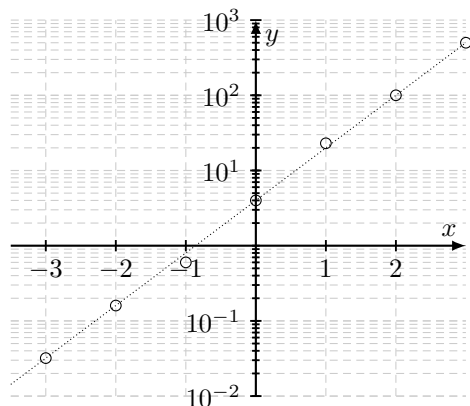
- b) the same division. For example, because $\lg 15 = \lg(10 \cdot 1.5) = \lg 10 + \lg 1.5 = 1 + \lg 1.5$ and $\lg 25 = \lg 10 + \lg 2.5 = 1 + \lg 2.5$, 15 and 25 have a distance of $\lg 2.5 - \lg 1.5$, just like the numbers 1.5 and 2.5 in the previous subtask.



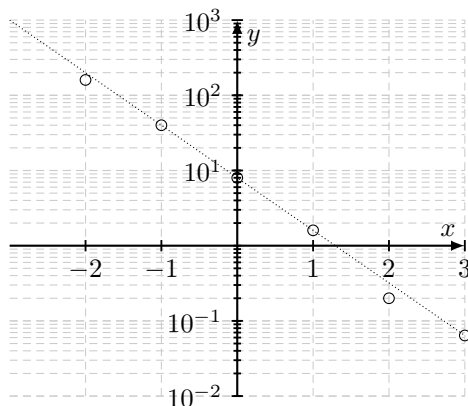
413. a) $\lg y = \lg(2^x) = x \cdot \lg 2$ and $\lg 2 > 0$
 b) $\lg y = \lg 3 + \lg(2^x) = x \cdot \lg 2 + \lg 3$
 c) $\lg y = \lg(x^2) = 2 \lg |x|$

414. a) $\lg y = \lg(0.5^x) = x \cdot \lg 0.5 = x \lg\left(\frac{1}{2}\right) = x(\lg 1 - \lg 2) = -x \cdot \lg 2$
 b) $\lg y = \lg(0.25^x) = \lg(0.5^{2x}) = 2 \lg(0.5^x)$
 c) $\lg y = \lg\left(\frac{4}{x}\right)$; Furthermore, the numerator of each logarithm must be positive.

415. No, they are not on a straight line.



416. No, they are not on a straight line.



417. a) a - h , b - g , c - f
 b) Exponential function; in x -direction the values $\lg x$ are mapped, so write as a function of $\lg x$, then get
 $h : y = x = 10^{\lg x}$
 c) b represents $g : y = \lg(x)$, conversion to logarithm base 10: $g : y = \frac{\lg x}{\lg 2}$
 c represents $f : y = \log_4(x)$, conversion to logarithm base 10:
 $g : y = \frac{\lg x}{\lg 4} = \frac{\lg x}{2 \lg 2} = \frac{1}{2} \lg(x)$
 the gradient of the straight line c is therefore half that of the straight line b.
418. a) a - g , b - h , c - f
 b) Descending straight line through the origin symmetrical to the graph of $x \mapsto \log_2 x$, because
 $f : x \mapsto \log_{\frac{1}{2}} x = \frac{\lg x}{\lg 2} = -\lg x$
 c) a represents
 $g : x \mapsto \sqrt{x} = (10^{\lg x})^{\frac{1}{2}} = 10^{\frac{1}{2} \lg x}$, i. e. a function of $x \mapsto x$ stretched in the x -direction by a factor of 2, b represents
 $h : x \mapsto x^2 = (10^{\lg x})^2 = 10^{2 \lg x}$, i. e. a function of $x \mapsto x$ stretched in the x -direction by a factor of $\frac{1}{2}$.
419. a) 4
 b) $39\,811\,\mu\text{m}$
 c) 1995 times
421. a) 7.33
 b) 7.08
 c) 1000 times
423. a) $7.94 \cdot 10^{15}$
 b) 1000 times
 c) 6.51
425. a) $2.51 \cdot 10^{-3} \text{ mol}/\ell$
 b) 6.60
 c) 100 000 times
 d) 1.49
427. a) $10^{-5} \text{ mol}/\ell$
 b) $10^{-6} \text{ mol}/\ell$
 c) +1
429. a) 1
 b) 100 times
 c) increases by 1
431. a) $1.05 \cdot 10^{-7} \text{ W m}^{-2}$
 b) $3.945 : 1$
 c) $39.81 : 1$
 d) $m = -2.75 \text{ mag}$
433. 0.017 mag
435. $3.31 : 1$
437. $\Delta m = -1.75$
420. a) 4.70
 b) $1\,584\,893.19\,\mu\text{m}$
 c) 100 times
422. a) 6.0
 b) 5.67
 c) 63.10 times
424. a) $1.78 \cdot 10^8$
 b) 5.62 times
 c) 6.13
426. a) $5.01 \cdot 10^{-5} \text{ mol}/\ell$
 b) $3.98 \cdot 10^{-8} \text{ mol}/\ell$
 c) 10 times
 d) $1.59 \cdot 10^{-6} \text{ mol}/\ell$
428. a) $5.01 \cdot 10^{-5} \text{ mol}/\ell$
 b) $1.00 \cdot 10^{-5} \text{ mol}/\ell$
 c) +0.70
430. a) 2
 b) 1000 times
 c) increases by 0.8
432. a) $2.74 \cdot 10^{-14} \text{ W m}^{-2}$
 b) $398\,107.17 : 1$
 c) 560-fold
 d) $1.75 : 100\,000$
434. 1.91 mag
436. $2.51 : 1$
438. factor 100

439. a) 3
b) from 3 to 3.30, from 3 to 2
c) from 10^3 to 10^6
441. a) 200 times
b) $5 : 2.70$
443. a) Moonlight -0.60 , candlelight 1
b) 1.60
c) 40 times
445. a) 0 dB
b) 20 dB
c) 30 dB
d) 90 dB
447. a) $10^{-11} \text{ W m}^{-2}$
b) 10^{-8} W m^{-2}
449. a) 10 000 times
b) 40 dB
451. a) 16 times
b) 2 times
453. 6.99 dB
440. a) 2.90
b) from 2.90 to 3.20, from 2.90 to 1.90
c) from 800 to 640 000
442. a) 16 times
b) $2.90 : 1.70$
444. a) Neon sign 2.18, computer screen 1.30
b) 1.68 times
c) 7.5 times
446. a) 60 dB
b) 80 dB
c) 100 dB
d) 130 dB
448. a) 10^{-5} W m^{-2}
b) 1 W m^{-2}
450. a) $6 : 5$
b) $10 : 1$
452. a) 32 times
b) 4 times
454. 1.46 dB